IIT-JEE

ANSWERS & HINTS
for
WBJEE - 2011

MULTIPLE CHOICE QUESTIONS
SUB : MATHEMATICS

1. The eccentricity of the hyperbola $4x^2 - 9y^2 = 36$ is

(A) $\frac{\sqrt{11}}{3}$ (B) $\frac{\sqrt{15}}{3}$ (C) $\frac{\sqrt{13}}{3}$ (D) $\frac{\sqrt{14}}{3}$

Ans : (C)

Hints : $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$a = 3,\ b = 2$

$\therefore e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$

2. The length of the latus rectum of the ellipse $16x^2 + 25y^2 = 400$ is

(A) 5/16 unit (B) 32/5 unit (C) 16/5 unit (D) 5/32 unit

Ans : (B)

Hints : Length of latus rectum = $2\frac{b^2}{a} = \frac{2 \times 16}{5} = \frac{32}{5}$

$16x^2 + 25y^2 = 400$

$\frac{x^2}{25} + \frac{y^2}{16} = 1;$

$a^2 = 25;\ b^2 = 16$

3. The vertex of the parabola $y^2 + 6x - 2y + 13 = 0$ is

$(y-1)^2 = -6x - 12$

$(y-1)^2 = -6(x+2) = 4\left(\frac{-6}{4}\right)(x+2)$

Vertex $\rightarrow (-2, 1)$

(A) (1, –1) (B) (–2, 1) (C) $\left(\frac{3}{2}, 1\right)$ (D) $\left(-\frac{7}{2}, 1\right)$

Ans : (B)

Hints :

4. The coordinates of a moving point p are $(2t^2 + 4, 4t + 6)$. Then its locus will be a

(A) circle (B) straight line (C) parabola (D) ellipse

Ans : (C)

Hints : $x = 2t^2 + 4,\ y = 4t + 6$, $y = 4t + 6 \rightarrow t = \left(\frac{y-6}{4}\right)$

$$x = 2\left(\frac{y-6}{4}\right)^2 + 4 \Rightarrow \frac{(y-6)^2}{8} = x - 4$$

$$(y-6)^2 = 4(2)(x-4)$$

5. The equation $8x^2 + 12y^2 - 4x + 4y - 1 = 0$ represents

(A) an ellipse (B) a hyperbola (C) a parabola (D) a circle

Ans : (A)

Hints : $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

represents ellipse if $h^2 - ab < 0$

$$3x^2 + 12y^2 - 4x + 4y - 1 = 0$$

$$h = 0, a = 3, b = 12$$

$$h^2 - ab < 0$$

6. If the straight line $y = mx$ lies outside of the circle $x^2 + y^2 - 20y + 90 = 0$, then the value of m will satisfy

(A) $m<3$ (B) $|m|<3$ (C) $m>3$ (D) $|m|>3$

Ans : (B)

Hints : $x^2 + m^2x^2 - 20mx + 90$

$$x^2\left(1+m^2\right) - 20mx + 90 = 0$$

$$D < 0$$

$$400m^2 - 4\times 90\left(1+m^2\right) < 0$$

$$40m^2 < 360$$

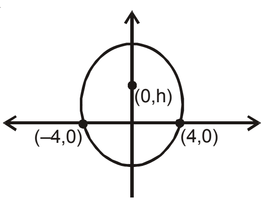
$$m^2 < 9\ ;\ |m| < 3$$

7. The locus of the centre of a circle which passes through two variable points (a, 0), (–a, 0) is

(A) $x=1$ (B) $x+y=a$ (C) $x+y=2a$ (D) $x=0$

Ans : (D)

Hints :



Centre lies on y-axis locus $x = 0$

8. The coordinates of the two points lying on $x + y = 4$ and at a unit distance from the straight line $4x + 3y = 10$ are

(A) $(-3, 1), (7, 11)$ (B) $(3, 1), (-7, 11)$ (C) $(3, 1), (7, 11)$ (D) $(5, 3), (-1, 2)$

Ans : (B)

Hints : Let $p(h, 4-h)$

$$\left|\frac{4h+3(4-h)-10}{5}\right| = 1$$

$$|h+2| = 5$$

$$h = 3, -7; \ p = 1, 1$$

$$(3,1), (-7,11)$$

9. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as diameter is

(A) $x^2 + y^2 = 1$ (B) $x(x-1) + y(y-1) = 0$

(C) $x^2 + y^2 = 2$ (D) $(x-1)(x-2) + (y-1) + (y-2) = 0$

Ans : (B)

Hints : $2x^2 - 2x = 0 \quad x(x+1) = 0 \quad x = 0,1; \ y = 0,1$

$(0,0), (1,1)$ as diametric ends

$$(x-0)(x-1) + (y+0)(y-1) = 0$$

$$x^2 + y^2 - x - y = 0$$

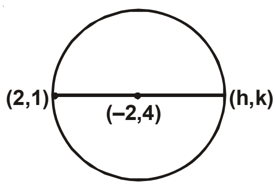
10. If the coordinates of one end of a diameter of the circle $x^2+y^2+4x-8y+5=0$, is $(2,1)$, the coordinates of the other end is

(A) $(-6, -7)$ (B) $(6, 7)$ (C) $(-6, 7)$ (D) $(7, -6)$

Ans : (C)

Hints : $x^2 + y^2 + 9x - 8y + 5 = 0$

Centre circle $(-2, 4)$



$$\frac{h+2}{2} = -2$$

$$h = -4 - 2 = -6$$

$$\frac{k+1}{2} = 4 \Rightarrow k = 7$$

$$(h,k) \rightarrow (-6, 7)$$

11. If the three points $A(1,6)$, $B(3, -4)$ and $C(x, y)$ are collinear then the equation satisfying by x and y is

(A) $5x + y - 11 = 0$ (B) $5x + 13y + 5 = 0$ (C) $5x - 13y + 5 = 0$ (D) $13x - 5y + 5 = 0$

Ans : (A)

Hints : $\begin{vmatrix} 1 & 1 & 6 \\ 1 & 3 & -4 \\ 1 & x & y \end{vmatrix} = 0$

$\Rightarrow 1(3y+4x)-(y-6x)+1(-4-18)=0 \quad \Rightarrow 3y+4x-y+6x-12=0$

$\Rightarrow 2y+10x-22=0$

$y+5x=11$

12. If $\sin\theta = \dfrac{2t}{1+t^2}$ and θ lies in the second quadrant, then cosθ is equal to

(A) $\dfrac{1-t^2}{1+t^2}$ (B) $\dfrac{t^2-1}{1+t^2}$ (C) $\dfrac{-|1-t^2|}{1+t^2}$ (D) $\dfrac{1+t^2}{|1-t^2|}$

Ans : (C)

Hints : θ in 2nd quad Cosθ < 0

$|\cos\theta| = \left|\dfrac{1-t^2}{1+t^2}\right| = \dfrac{|1-t^2|}{1+t^2}$

$\cos\theta = -\dfrac{|1-t^2|}{1+t^2}$

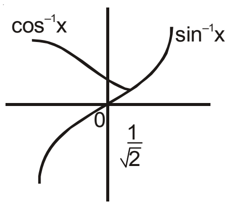
13. The solutions set of inequation $\cos^{-1}x < \sin^{-1}x$ is

(A) [–1, 1] (B) $\left[\dfrac{1}{\sqrt{2}}, 1\right]$ (C) [0, 1] (D) $\left(\dfrac{1}{\sqrt{2}}, 1\right]$

Ans : (D)

Hints : $\cos^{-1}x < \sin^{-1}x$

$x \in \left(\dfrac{1}{\sqrt{2}}, 1\right]$, $\cos^{-1}x < \sin^{-1}x$



14. The number of solutions of 2sinx + cos x = 3 is

(A) 1 (B) 2 (C) infinite (D) No solution

Ans : (D)

Hints : $\sqrt{5} < 3$ No solution

15. Let $\tan\alpha = \dfrac{a}{a+1}$ and $\tan\beta = \dfrac{1}{2a+1}$ then $\alpha+\beta$ is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{3}$ (C) $\dfrac{\pi}{2}$ (D) π

Ans : (A)

Hints : $\tan\alpha = \frac{a}{a+1}$, $\tan\beta = \frac{1}{2a+1}$

$$\tan(\alpha+\beta) = \frac{\frac{a}{a+1}+\frac{1}{2a+1}}{1-\frac{a}{(a+1)(2a+1)}} = \frac{\frac{a(2a+1)+a+1}{(a+1)(2a+1)}}{\frac{(a+1)(2a+1)-a}{(a+1)(2a+1)}} = \frac{2a^2+2a+1}{2a^2+2a+1} = 1$$

$$\alpha+\beta = \frac{\pi}{4}$$

16. If $\theta+\phi = \frac{\pi}{4}$, then $(1+\tan\theta)(1+\tan\phi)$ is equal to

(A) 1 (B) 2 (C) 5/2 (D) 1/3

Ans : (B)

Hints : $(1+\tan\theta)\left(1+\frac{(1-\tan\theta)}{1+\tan\theta}\right)$

$$= (1+\tan\theta)\frac{2}{1+\tan\theta} = 2$$

17. If sinθ and cosθ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation

(A) $a^2 + b^2 + 2ac = 0$ (B) $a^2 - b^2 + 2ac = 0$

(C) $a^2 + c^2 + 2ab = 0$ (D) $a^2 - b^2 - 2ac = 0$

Ans : (B)

Hints : $\sin\theta + \cos\theta = \frac{b}{a}$

$$\sin\theta.\cos\theta = \frac{c}{a}$$

$$\left(\frac{b}{a}\right)^2 = 1 + \frac{2c}{a}$$

$$b^2 = a^2 + 2ac$$

$$a^2 - b^2 + 2ac = 0$$

18. If A and B are two matrices such that A+B and AB are both defined, then

(A) A and B can be any matrices (B) A, B are square matrices not necessarily of the same order

(C) A, B are square matrices of the same order (D) Number of columns of A = number of rows of B

Ans : (C)

Hints : Addition is defined if order of A is equal to order of B

$\underset{n\times m}{A}\ \underset{n\times m}{B}$ is defined if m = n

$\Rightarrow$ A, B are square matrices of same order

19. If $A = \begin{pmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{pmatrix}$ is a symmetric matrix, then the value of x is

(A) 4 (B) 3 (C) –4 (D) –3

Ans : (C)

Hints : $A = A^T$

$$\begin{pmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{pmatrix} = \begin{pmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{pmatrix}$$

$\Rightarrow x-1 = 2x+3$ or $x = -4$

20. If $z = \begin{vmatrix} 1 & 1+2i & -5i \\ 1-2i & -3 & 5+3i \\ 5i & 5-3i & 7 \end{vmatrix}$ then $\left(i = \sqrt{-1}\right)$

(A) z is purely real
(B) z is purely imaginary
(C) $z + \bar{z} = 0$
(D) $(z - \bar{z})\, i$ is purely imaginary

Ans : (A)

Hints : $z = \begin{vmatrix} 1 & 1+2i & -5i \\ 1-2i & -3 & 5+3i \\ 5i & 5-3i & 7 \end{vmatrix} = 1(-21-64) - \left((1-2i)\left(7(1+2i) + 5i(5-3i)\right)\right) + 5i\left((1+2i)(5+3i) - 15i\right)$

= Real

21. The equation of the locus of the point of intersection of the straight lines x sin θ + (1 – cos θ) y = a sin θ and x sin θ – (1 + cos θ) y + a sin θ = 0 is

(A) $y \pm ax$
(B) $x = \pm ay$
(C) $y^2 = 4x$
(D) $x^2 + y^2 = a^2$

Ans : (D)

Hints : $y = a \sin\theta$

$x = a\cos\theta$.

$\boxed{x^2 + y^2 = a^2}$

22. If $\sin\theta + \cos\theta = 0$ and $0 < \theta < \pi$, then θ

(A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) $\frac{3\pi}{4}$

Ans : (D)

Hints : $\sin\theta + \cos\theta = 0$

$\Rightarrow \tan\theta = -1$ $\qquad \theta = \frac{3\pi}{4}$

23. The value of cos 15° – sin 15° is

(A) 0
(B) $\frac{1}{\sqrt{2}}$
(C) $-\frac{1}{\sqrt{2}}$
(D) $\frac{1}{2\sqrt{2}}$

Ans : (B)

Hints : $\cos 15^\circ - \sin 15^\circ = \sqrt{2}\cos 60^\circ = \frac{1}{\sqrt{2}}$

24. The period of the function f(x) = cos 4x + tan 3x is

(A) π
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{4}$

Ans : (A)

Hints : $\text{LCM}\left(\frac{2\pi}{4}, \pi/3\right) = \pi$

25. If $y = 2x^3 - 2x^2 + 3x - 5$, then for $x = 2$ and $\Delta x = 0.1$ value of Δy is

(A) 2.002 (B) 1.9 (C) 0 (D) 0.9

Ans : (B)

Hints : $\frac{dy}{dx} = 6x^2 - 4x + 3$ $\quad \Delta y = \left(\frac{dy}{dx}\right)_{x=2} \Delta x = 1.9$

26. The approximate value of $\sqrt[5]{33}$ correct to 4 decimal places is

(A) 2.0000 (B) 2.1001 (C) 2.0125 (D) 2.0500

Ans : (C)

Hints : $y = x^{1/5}$ $\quad \Delta y = \left(\frac{dy}{dx}\right)\Delta x = 1/80 \times 1$

$y = 2 + 1/80$

27. The value of $\int_{-2}^{2}(x\cos x + \sin x + 1)\,dx$ is

(A) 2 (B) 0 (C) -2 (D) 4

Ans : (D)

Hints : $\int_{-2}^{2}(x\cos x + \sin x + 1)\,dx \quad = \int_{-2}^{2} dx = 4$

28. For the function $f(x) = e^{\cos x}$, Rolle's theorem is

(A) applicable when $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

(B) applicable when $0 \le x \le \frac{\pi}{2}$

(C) applicable when $0 \le x \le \pi$

(D) applicable when $\frac{\pi}{4} \le x \le \frac{\pi}{2}$

Ans : (A)

Hints : $f\left(\pi/2\right) = f\left(3\pi/2\right)$

29. The general solution of the differential equation $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$ is

(A) $(A + Bx)e^{5x}$ (B) $(A + Bx)e^{-4x}$ (C) $(A + Bx^2)e^{4x}$ (D) $(A + Bx^4)e^{4x}$

Ans : (B)

Hints : $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$

auxilary equation $m^2 + 8m + 16 = 0 \Rightarrow m = -4$

Solution $\boxed{y = (ax + b)e^{-4x}}$

30. If $x^2 + y^2 = 4$, then $y\frac{dy}{dx} + x =$

(A) 4 (B) 0 (C) 1 (D) -1

Ans : (B)

Hints : $x + y\frac{dy}{dx} = 0$

31. $\int \frac{x^3 dx}{1+x^8} =$

(A) $4\tan^{-1}x^3 + c$ (B) $\frac{1}{4}\tan^{-1}x^4 + c$ (C) $x + 4\tan^{-1}x^4 + c$ (D) $x^2 + \frac{1}{4}\tan^{-1}x^4 + c$

Ans : (B)

Hints : $\int \frac{x^3 dy}{1+\left(x^4\right)^2} = \frac{1}{4}\tan^{-1}\left(x^4\right)$

32. $\int_{\pi}^{16\pi} |\sin x|\, dx =$

(A) 0 (B) 32 (C) 30 (D) 28

Ans : (C)

Hints : $15\int_0^{\pi} \sin x\, dx = 15\left(-\cos x\right)_0^{\pi} = 30$

33. The degree and order of the differential equation $y = x\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}$ are respectively

(A) 1,1 (B) 2, 1 (C) 4, 1 (D) 1, 4

Ans : (C)

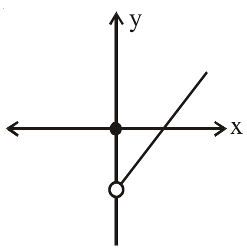
Hints : $y\left(\frac{dy}{dx}\right)^2 = x\left(\frac{dy}{dx}\right)^4 + 1$

34. $f(x) = \begin{cases} 0 & , \quad x = 0 \\ x-3 & , \quad x > 0 \end{cases}$ The function $f(x)$ is

(A) increasing when $x \ge 0$
(B) strictly increasing when $x > 0$
(C) Strictly increasing at $x = 0$
(D) not continuous at $x = 0$ and so it is not increasing when $x > 0$

Ans : (B)

Hints :



35. The function $f(x) = ax + b$ is strictly increasing for all real x if

(A) $a > 0$ (B) $a < 0$ (C) $a = 0$ (D) $a \le 0$

Ans : (A)

Hints : $f'(x) = a$

$f'(x) > 0 \Rightarrow a > 0$

36. $\int \frac{\cos 2x}{\cos x} dx =$

(A) $2\sin x + \log|\sec x + \tan x| + C$ (B) $2\sin x - \log|\sec x - \tan x| + c$

(C) $2\sin x - \log|\sec x + \tan x| + C$ (D) $2\sin x + \log|\sec x - \tan x| + C$

Ans : (C)

Hints : $\int \frac{2\cos^2 x - 1}{\cos x} dx \;\; = 2\sin x - \log|\sec x + \tan|$

37. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$

(A) $-\frac{1}{2}\sin 2x + C$ (B) $\frac{1}{2}\sin 2x + C$ (C) $\frac{1}{2}\sin x + C$ (D) $-\frac{1}{2}\sin x + C$

Ans : (A)

Hints : $\int\left(\sin^2 x - \cos^2 x\right)dx = -\int \cos^{2x} dx = -\frac{1}{2}\sin 2x + C$

38. The general solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = x + y$ is

(A) $e^x + e^{-y} = C$ (B) $e^x + e^y = C$ (C) $e^y + e^{-x} = C$ (D) $e^{-x} + e^{-y} = C$

Ans : (A)

Hints : $\frac{dy}{dx} = e^x . e^y \Rightarrow \int e^{-y} dy. = \int e^x dx \Rightarrow \boxed{e^x + e^{-y} = c}$

39. If $y = \frac{A}{x} + Bx^2$, then $x^2 \frac{d^2y}{dx^2} =$

(A) $2y$ (B) y^2 (C) y^3 (D) y^4

Ans : (A)

Hints : $\frac{x^2 d^2 y}{dx^2} = 2\left(A/x + Bx^2\right) = 2y$

40. If one of the cube roots of 1 be ω, then

$$\begin{vmatrix} 1 & 1+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2 - 1 \\ -i & -1+\omega & -1 \end{vmatrix} =$$

(A) ω (B) i (C) 1 (D) 0

Ans : (D)

Hints : $C_2 \to C_2 - C_3$

$C_3 \to C_3 + C_2$

$C_3 \to C_3 + \omega C_1$

$C_2 \to C_2 - C_1$

41. 4 boys and 2 girls occupy seats in a row at random. Then the probability that the two girls occupy seats side by side is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$

Ans : (C)

Hints : $n(e) = \lfloor 5 . \lfloor 2$

$n(s) = \lfloor 6$

$p = \frac{\lfloor 5 . \lfloor 2}{\lfloor 6} = \frac{2}{6} = \frac{1}{3}$

42. A coin is tossed again and again. If tail appears on first three tosses, then the chance that head appears on fourth toss is

(A) $\frac{1}{16}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

Ans : (B)

Hints : $p = 1.1.1.\frac{1}{2} = \frac{1}{2}$

43. The coefficient of x^n in the expansion of $\frac{e^{7x}+e^x}{e^{3x}}$ is

(A) $\frac{4^{n-1}-(-2)^{n-1}}{\lfloor n}$ (B) $\frac{4^{n-1}-2^{n-1}}{\lfloor n}$ (C) $\frac{4^n-2^n}{\lfloor n}$ (D) $\frac{4^n+(-2)^n}{\lfloor n}$

Ans : (D)

Hints : $\frac{e^{7x}+e^x}{e^{3x}} = e^{4x}+e^{-2x}$

Co-efficient of x^n

$$\frac{(4)^n}{n!}+\frac{(2)^n}{n!}(-1)^n = \frac{4^n+(-2)^n}{n!}$$

44. The sum of the series $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-.....\infty$ is

(A) $2\log_e 2+1$ (B) $2\log_e 2$ (C) $2\log_e 2-1$ (D) $\log_e 2-1$

Ans : (C)

Hints : $s = \frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}......\infty$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right).....$$

$$=\frac{1}{1}-\frac{1}{2}-\frac{1}{2}+\frac{1}{3}+\frac{1}{3}-\frac{1}{4}-\frac{1}{4}+\frac{1}{5}....$$

$$=\frac{1}{1}-\frac{2}{2}+\frac{2}{3}-\frac{2}{4}+\frac{2}{5}-...$$

$$=2\left[\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}....\right]-1 = 2\log 2-1$$

45. The number $(101)^{100}-1$ is divisible by

(A) 10^4 (B) 10^6 (C) 10^8 (D) 10^{12}

Ans : (A)

Hints : $(101)^{100}-1 = {}^{100}C_1 100+{}^{100}C_2 100^2+{}^{100}C_3 100^3+..................+{}^{100}C_{100}100^{100}$

$= 100^2\,[\,1+{}^{100}C_2+{}^{100}C_3 100+.......\,]$

$=(10^4)$

46. If A and B are coefficients of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then A/B is equal to

(A) 4 (B) 2 (C) 9 (D) 6

Ans : (B)

Hints : $A = {}^{2n}C_n$.

$B = {}^{2n-1}C_n$

$$\frac{A}{B} = \frac{^{2n}C_n}{^{2n-1}C_n} = \frac{2n}{n} = 2$$

47. If n > 1 is an integer and x ≠0, then $(1 + x)^n - nx - 1$is divisible by

(A) nx^3 (B) n^3x (C) x (D) nx

Ans : (C)

Hints : $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x3 + ...$

$= 1 + nx + x^2 ({}^nC_2 + {}^nC_3 x + ...)$

$(1 + x)^n - nx - 1 = x^2 ({}^nC_2 + {}^nC_3x + ...)$

48. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n is

(A) 7 or 14 (B) 7 (C) 14 (D) 14 or 21

Ans : (A)

Hints : ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in AP

$2.\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$$\frac{2}{5(n-5)} = \frac{1}{(n-4)} + \frac{1}{30}$$

by solving n = 14 or 7

49. The number of diagonals in a polygon is 20. The number of sides of the polygon is

(A) 5 (B) 6 (C) 8 (D) 10

Ans : (C)

Hints : ${}^nC_2 - n = 20$

n = 8

50. ${}^{15}C_3 + {}^{15}C_5 + ... + {}^{15}C_{15} =$

(A) 2^{14} (B) $2^{14} - 15$ (C) $2^{14} + 15$ (D) $2^{14} - 1$

Ans : (B)

Hints : ${}^{15}C_3 + {}^{15}C_5 + ... + {}^{15}C_{15} = 2^{14} - {}^{15}C_1 = 2^{14} - 15$

51. Let a , b, c be three real numbers such that a + 2b + 4c = 0. Then the equation $ax^2 + bx + c = 0$

(A) has both the roots complex

(B) hat its roots lying within $-1 < x < 0$

(C) has one of roots equal to $\frac{1}{2}$

(D) has its roots lying within $2 < x < 6$

Ans : (C)

Hints : $\frac{1}{4}a + \frac{1}{2}b + c = 0$

$$\left(\frac{1}{2}\right)^2 a + \left(\frac{1}{2}\right)b + c = 0$$

$$\therefore\ x = \frac{1}{2}$$

52. If the ratio of the roots of the equation $px^2 + qx + r = 0$ is a : b, then $\frac{ab}{(a+b)^2} =$

(A) $\frac{p^2}{qr}$ (B) $\frac{pr}{q^2}$ (C) $\frac{q^2}{pr}$ (D) $\frac{pq}{r^2}$

Ans : (B)

Hints : Let roots are aα and bα

$$\Rightarrow (a+b)\alpha = \frac{-q}{p}$$

$$ab\alpha^2 = \frac{r}{p}$$

$$\frac{ab\alpha^2}{(a+b)^2\alpha^2} = \frac{r}{p}.\frac{p^2}{q^2}$$

$$\frac{ab}{(a+b)^2} = \frac{rp}{q^2}$$

53. If α and β are the roots of the equation $x^2 + x + 1 = 0$, then the equation whose roots are α^{19} and β^7 is

(A) $x^2 - x - 1 = 0$ (B) $x^2 - x + 1 = 0$ (C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

Ans : (D)

Hints : α and β are the roots of $x^2 + x + 1 = 0$

$\alpha = \omega$ $\beta = \omega^2$

$\alpha^{19} = \omega$ $\beta^7 = \omega^2$

$x^2 - (\alpha^{19} + \beta^7)x + \alpha^{19}\beta^7 = 0$

Thou,

$x^2 - (\omega + \omega^2)x + \omega . \omega^2 = 0$

$x^2 + x + 1 = 0$

54. For the real parameter t, the locus of the complex number $z = (1 - t^2) + i\sqrt{1 + t^2}$ in the complex plane is

(A) an ellipse (B) a parabola (C) a circle (D) a hyperbola

Ans : (B)

Hints : Given $z = (1 - t^2) + i\sqrt{1 + t^2}$

Let $z = x + iy$

$x = 1 - t^2$

$y^2 = 1 + t^2$

Thus, $x + y^2 = 2$

$y^2 = 2 - x$

$y^2 = -(x - 2)$

Thus parabola

55. If $x + \frac{1}{x} = 2\cos\theta$, then for any integer n, $x^n + \frac{1}{x^n} =$

(A) $2 \cos n\theta$ (B) $2 \sin n\theta$ (C) $2i \cos n\theta$ (D) $2i \sin n\theta$

Ans : (A)

Hints : $x + \frac{1}{x} = 2\cos\theta$

Let $x = \cos\theta + 1 \sin\theta$

$\frac{1}{x} = \cos\theta - 1\sin\theta$

Thus $x^n + \frac{1}{x^n} = 2\cos n\theta$

56. If $\omega \neq 1$ is a cube root of unity, then the sum of the series $S = 1 + 2\omega + 3\omega^2 + ... + 3n\omega^{3n-1}$ is

(A) $\frac{3n}{\omega - 1}$ (B) $3n(\omega - 1)$ (C) $\frac{\omega - 1}{3n}$ (D) 0

Ans : (A)

Hints : $s = 1 + 2\omega + 3\omega^2 + ... + 3n\,\omega^{3n-1}$

$s\omega = \omega + 2\omega^2 + ... + (3n-1)\omega^{3n} + 3n\omega^{3n}$

$s(1-\omega) = 1 + \omega + \omega^2 + ...+ \omega^{3n-1} - 3n\omega^{3h}$

$= 0 - 3n$

$s = \frac{-3n}{1-\omega} = \frac{3n}{\omega - 1}$

57. If $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_3(x+y) = 2$, then

(A) $x = 1, y = 8$ (B) $x = 8, y = 1$ (C) $x = 3, y = 6$ (D) $x = 9, y = 3$

Ans : (C)

Hints : $\log_3 x + \log_3 y = 2 + \log_3 2$

$\Rightarrow x.y = 18$

$\log(x+y) = 2 \Rightarrow x + y = 9$

we will get $x = 3$ and $y = 6$

58. If $\log_7 2 = \lambda$, then the value of $\log_{49}(28)$ is

(A) $(2\lambda + 1)$ (B) $(2\lambda + 3)$ (C) $\frac{1}{2}(2\lambda + 1)$ (D) $2(2\lambda + 1)$

Ans : (C)

Hints : $\log_{49} 28 = \log_{7^2} 4 \times 7$

$= \frac{1}{2}\left[2\log_7 2 + \log_7 7\right] = \frac{1}{2}\left[2\lambda + 1\right]$

59. The sequence log a, $\log\frac{a^2}{b}, \log\frac{a^3}{b^2}$, ... is

(A) a G.P. (B) an A.P. (C) a H.P. (D) both a G.P. and a H.P

Ans : (B)

Hints : log a . (2log a – log b)(3log a – 2 log b)

$= T_2 - T_1 = \log a - \log b$

$= T_3 - T_2 = \log a - \log b$

60. If in a triangle ABC, sin A, sin B, sin C are in A.P., then

(A) the altitudes are in A.P. (B) the altitudes are in H.P.

(C) the angles are in A.P. (D) the angles are in H.P.

Ans : (B)

Hints : $\frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3 = \Delta$

$a = \frac{2\Delta}{p_1} \mid b = \frac{2\Delta}{p_2} \mid c = \frac{2\Delta}{p3}$

H.P.

61. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} =$

(A) 0 (B) – 1 (C) 1 (D) 2

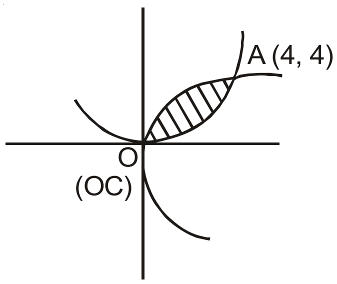
Ans : (A)

Hints : $c_1 \to c_1 + c_2 + c_3$

62. The area enclosed between $y^2 = x$ and $y = x$ is

(A) $\frac{2}{3}$ sq. units (B) $\frac{1}{2}$ units (C) $\frac{1}{3}$ units (D) $\frac{1}{6}$ units

Ans : (D)



Hints : $A = \int_0^1 \left(\sqrt{x} - x\right)dx$

$= \frac{2}{3}\left(x^{3/2}\right)_0^1 - \frac{1}{2}(x^2)_o^1 = \frac{2}{3}[1-0] - \frac{1}{2}[1-0]$

$= \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$

63. Let $f(x) = x^3e^{-3x}$, $x > 0$. Then the maximum value of f(x) is

(A) e^{-3} (B) $3e^{-3}$ (C) $27e^{-9}$ (D) ∞

Ans : (A)

Hints : $f(x) = x^3.e^{-3x}$

$= f'(x) = 3x^2e^{-3x} + x^3\, e^{-3x}\,(-3)$

$= x^2 3e^{-3x}[1-x] = 0, x = 1$

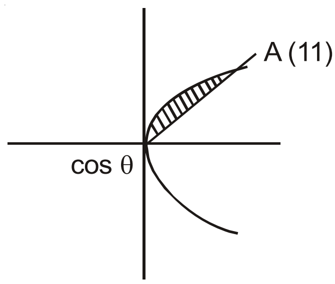
Maximum at $x = 1$

$f(1) = e^{-3}$

64. The area bounded by $y^2 = 4x$ and $x^2 = 4y$ is

(A) $\frac{20}{3}$ sq. unit (B) $\frac{16}{3}$ sq. unit (C) $\frac{14}{3}$ sq. unit (D) $\frac{10}{3}$ sq. unit

Ans : (B)



Hints : $A = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right)dx$

$= 2.\frac{2}{3}\left(x^{3/2}\right)_0^4 - \frac{1}{4.3})(x^3)_0^4$

$= \frac{4}{3}\left[4^{3/2} - 0\right] - \frac{1}{4.3}[4^3 - 0]$

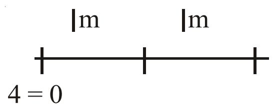
$= \frac{4^{5/2}}{3} - \frac{16}{3} = \frac{32-16}{3} = \frac{16}{3}$

65. The acceleration of a particle starting from rest moving in a straight line with uniform acceleration is $8m/sec^2$. The time taken by the particle to move the second metre is

(A) $\frac{\sqrt{2}-1}{2}$ sec (B) $\frac{\sqrt{2}+1}{2}$ sec (C) $(1+\sqrt{2})$ sec (D) $\left(\sqrt{2}-1\right)$ sec

Ans : (A)

Hints :



$S = ut + \frac{1}{2}at^2$	$S = uT + \frac{1}{2}aT^2$
$1 = \frac{1}{2}.8.t^2$	$2 = \frac{1}{2}8.T^2$
$\frac{1}{4} = t^2$	$T^2 = \frac{1}{2}$
$t = \frac{1}{2}$	$T = \frac{1}{\sqrt{2}}$

$$\text{Time} = \frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{2}{2} = \frac{\sqrt{2}-1}{2}$$

66. The solution of

$$\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x} \text{ is}$$

(A) $x = c\sin(y/x)$ (B) $x = c\sin(xy)$ (C) $y = c\sin(y/x)$ (D) $xy = c\sin(x/y)$

Ans : (A)

Hints : $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

Put $\frac{y}{x} = \theta$, $y = \theta x$

$$\frac{dy}{dx} = \theta + \frac{xd\theta}{dx}$$

$$\theta + x.\frac{d\theta}{dx} = \theta + \tan\theta, \; \frac{d\theta}{\tan\theta} = \frac{dy}{x}$$

$$\int \cot\theta \, d\theta = \int \frac{dx}{x}$$

$\log\sin\theta = \log x + \log c$

$\sin\theta = x.c.$, $\sin\frac{y}{x} = x.c$

$x = c.\sin\frac{y}{x}$

67. Integrating Factor (I.F.) of the defferential equation

$$\frac{dy}{dx} - \frac{3x^2y}{1+x^3} = \frac{\sin^2(x)}{1+x} \text{ is}$$

(A) e^{1+x^3} (B) $\log(1+x^3)$ (C) $1+x^3$ (D) $\frac{1}{1+x^3}$

Ans : (D)

Hints : If $e^{\int pdx} = e^{-\int\frac{3x^2dx}{1+x^3}} = e^{-\log(1+x^3)} = e^{\log(1+x^3)^{-1}}$

$= (1+x^3)^{-1} = \frac{1}{1+x^3}$

68. The differential equation of $y = ae^{bx}$ (a & b are parameters) is

(A) $yy_1 = y_2^2$ (B) $yy_2 = y_1^2$ (C) $yy_1^2 = y_2$ (D) $yy_2^2 = y_1$

Ans : (B)

Hints : $y = a.e^{bx}$ .... (i)

$y_1 = abe^{bx}$

$y_1 = by$ .... (ii)

$y_2 = by_1$ .... (iii)

Dividing (ii) & (iii) $\frac{y_1}{y_2} = \frac{y}{y_1} \Rightarrow y_1^2 = yy_2$

69. The value of

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^3}{r^4 + n^4} \text{ is}$$

(A) $\frac{1}{2}\log_e(1/2)$ (B) $\frac{1}{4}\log_e(1/2)$ (C) $\frac{1}{4}\log_e 2$ (D) $\frac{1}{2}\log_e 2$

Ans : (C)

Hints : $\underset{n\to\infty}{\text{Lt}}.\Sigma \dfrac{n^3\left(\frac{r}{n}\right)^3}{n^4\left[\left(\frac{r}{n}\right)^4 + 1\right]}$

$= \frac{1}{4}.4\int_0^1 \frac{x^3}{1+x^4}\,dx = \frac{1}{4}\left[\log(1+x^4)\right]_0^1$

$= \frac{1}{4}(\log 2 - \log 1) = \frac{1}{4}\log 2$

70. The value of $\int_0^{\pi} \sin^{50} x \cos^{49} x\ dx$ is

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) 1

Ans : (A)

Hints : $I = \int_0^{\pi} \sin^{50} x.\cos^{49} x\ dx \quad \int_0^4 f(x) = \int_0^4 f(a-x)$

$I = \int_0^{\pi} \sin^{50} x(-\cos^{49}(x)) = -\int_0^{\pi} \sin^{50} x.\cos^{49} x$

$= I = -I$

$I = 0$

71. $\int 2^x (f'(x) + f(x)\log 2)dx$ is

(A) $2^x f'(x) + C$ (B) $2^x f(x) + C$ (C) $2^x(\log 2)f(x) + C$ (D) $(\log 2)\, f(x) + C$

Ans : (B)

Hints : $I = \int 2^x\, f'(x)dx + \int 2^x f(x)\log 2dx$

$= 2^x f(x)$

72. Let $f(x) = \tan^{-1}x$. Then $f'(x) + f''(x)$ is = 0, when x is equal to

(A) 0 (B) + 1 (C) i (D) – i

Ans : (B)

Hints : $f(x) = \tan^{-1}x$

$$f'(x) = \frac{1}{1+x^2}$$

$$f''(x) = \frac{-1}{(1+x^2)}.2x \ , \ \frac{1}{1+x^2} = \frac{2x}{(1+x^2)^2}$$

$1 + x^2 = 2x, \ (x-1)^2 = 0$

$x = 1$

73. If $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, then $y'(1) =$

(A) 1/4 (B) 1/2 (C) – 1/4 (D) – 1/2

Ans : (A)

Hints : $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ Put $x = \tan\theta$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \tan^{-1}\tan\frac{\theta}{2}$$

$$= \frac{\theta}{2} = \frac{1}{2}.\tan^{-1}x, \ y' = \frac{1}{2(1+x^2)}$$

$$y'(1) = \frac{1}{2.2} = \frac{1}{4}$$

74. The value of $\lim_{x\to 1}\frac{x + x^2 + ... + x^n - n}{x-1}$ is

(A) n (B) $\frac{n+1}{2}$ (C) $\frac{n(n+1)}{2}$ (D) $\frac{n(n-1)}{2}$

Ans : (C)

Hints : $\operatorname{Lt}_{x\to 1}\frac{(x-1)+(x^2-1)+(x^3-1).............(x^4-1)}{x-1}$

$$= 1 + 2 + 3 .... + n = \frac{n(n+1)}{2}$$

75. $\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{x^2} =$

(A) π^2 (B) 3π (C) 2π (D) π

Ans : (D)

Hints : $= \underset{x\to 0}{\text{Lt}} \frac{\sin\left(\pi \sin^2 x\right)}{x^2} = \underset{x\to 0}{\pi} \frac{\sin^2 x}{x^2} = \pi$

$= \pi$

76. If the function

$$f(x) = \begin{cases} \frac{x^2 - (A+2)x + A}{x-2} & \text{for } x \neq 2 \\ 2 & \text{for } x = 2 \end{cases}$$

is continuous at x = 2, then

(A) $A = 0$ (B) $A = 1$ (C) $A = -1$ (D) $A = 2$

Ans : (A)

Hints : $\frac{4-(A+2)2+A}{0} = \frac{-A}{0}$ Put A = 0.

77. $f(x) = \begin{cases} [x]+[-x], & \text{when } x \neq 2 \\ \lambda & \text{when } x = 2 \end{cases}$

If f(x) is continuous at x = 2, the value of λ will be

(A) – 1 (B) 1 (C) 0 (D) 2

Ans : (A)

Hints : LHL = $\underset{h\to 0}{\text{Lt}} [2-h]+[-(2-h)]$

$= \underset{h\to 0}{\text{Lt}}\ 1+(-2+h) = 1-2 = -1$

RML = $\underset{h\to 0}{\text{Lt}} [2+h]+(-(2+h))$

$= 2 + (-2-h) = 2-3 = -1$

$\lambda = -1$

78. The even function of the following is

(A) $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$ (B) $f(x) = \frac{a^x + 1}{a^x - 1}$

(C) $f(x) = x.\frac{a^x - 1}{a^x + 1}$ (D) $f(x) = \log_2\left(x + \sqrt{x^2+1}\right)$

Ans : (C)

Hints : $f(-x) = (-x)\frac{a^{-x} - 1}{a^{-x} + 1}$

$= (-n)\ \frac{1-a^x}{1+a^x}$

$= n\frac{(a^x - 1)}{(a^x + 1)} = f(x)$

79. If f(x + 2y, x – 2y) = xy, then f(x, y) is equal to

(A) $\frac{1}{4}xy$ (B) $\frac{1}{4}(x^2 - y^2)$ (C) $\frac{1}{8}(x^2 - y^2)$ (D) $\frac{1}{2}(x^2 + y^2)$

Ans : (C)

Hints :

$$\begin{aligned} x + 2y &= a \\ x - 2y &= b \\ \hline 2x &= a + b \\ 4y &= a - b \end{aligned}$$

$$f(a,b) = \left(\frac{a+b}{2}\right)\left(\frac{a-b}{4}\right) = \frac{a^2 - b^2}{8}$$

80. The locus of the middle points of all chords of the parabola $y^2 = 4ax$ passing through the vertex is

(A) a straight line (B) an ellipse (C) a parabola (D) a circle

Ans : (C)

Hints : $2h = x,\ 2k = y$

$y^2 = 4ax$

$k^2 = 2ah$

$y^2 = 2ax$

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DESCRIPTIVE TYPE QUESTIONS

SUB : MATHEMATICS

1. The harmonic mean of two numbers is 4. Their arithemetic mean A and the geometric mean G satisfy the relation $2A + G^2 = 27$. find the numbers.

Ans. $(3, 6)$

Sol : Let the number be a, b

$\boxed{A.H = G^2}$ $\boxed{H = 4}$

$\Rightarrow G^2 = 4A$

$2A + G^2 = 27$ $\Rightarrow 2A + 4A = 27$

$\Rightarrow A = \frac{27}{6} = \frac{9}{2}$

$\Rightarrow G^2 = 18 \Rightarrow G = \sqrt{18}$

$\Rightarrow$ $a.b = 18$

$a + b = 9$ $\Rightarrow \boxed{\begin{matrix} a=6 & \text{or} & a=3 \\ b=3 & & b=6 \end{matrix}}$

2. If the area of a rectangle is 64 sq. unit, find the minimum value possible for its perimeter.

Ans. 32

Sol. Let the dimesions be a, b

Area = ab

Paimeter $= 2(a + b)$

We have $ab = 64 \Rightarrow b = \frac{64}{a}$

Perimeter as function of a

$P(a) = 2\left(a + \frac{64}{a}\right)$

for maxima or minimum

$P'(a) = 2\left(1 - \frac{64}{a^2}\right) = 0 \Rightarrow a = \pm 8 = 8$

$P''(a) = 2 \times \frac{64}{a^3} = \frac{2 \times 64}{8^3} > 0$

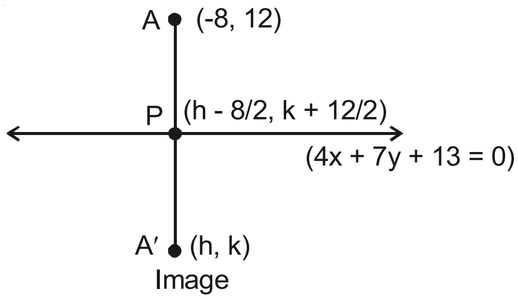
P (8) is minimum

Minimum $P(8) = 2(8 + 8) = 32$

3. Find the image of the point (–8, 12) with respect to the line 4x + 7y + 13 = 0

Ans. (–16, –2)

Sol.



$$4\left(\frac{h-8}{2}\right)+7\left(\frac{k+12}{2}\right)+13=0$$

$$2h-16+\frac{7k}{2}+42+13=0$$

$$4h + 7k + 78 = 0$$

$$\boxed{4h+7k=-78}\ ....(i)$$

2 nd equation, we can get

Slope of $AA' = \frac{7}{4}$

$$\frac{k-12}{n+18}=\frac{7}{4}$$

$\Rightarrow 4k-48=75+56$

$4k-7h=104$ ....(ii)

Solving (i) & (ii)

Equation (i) × 7 + Equation (ii) × 4

$$\Rightarrow \begin{array}{r} 28h+49k=-546 \\ -28h+16k=416 \\ \hline 65k=-130 \end{array}$$

$$\boxed{k=-2}$$

$$\boxed{h=-16}$$

A′ (– 16, –2) is the image of (–8, 12)

4. How many triangles can be formed by joining 6 points lying on a circle ?

Ans. 20

Sol. Number of triangle

$$^6C_3=\frac{\lfloor 6}{\lfloor 3\lfloor 3}=20$$

5. If $r^2 = x^2 + y^2 + z^2$, then prove that

$$\tan^{-1}\left(\frac{yz}{rx}\right)+\tan^{-1}\left(\frac{zx}{ry}\right)+\tan^{-1}\left(\frac{xy}{rz}\right)=\frac{\pi}{2}$$

A. $\theta=\tan^{-1}\left(\frac{S_1-S_3}{1-S_2}\right)$

$$1 - S_2 = 1 - \left(\frac{z^2}{r^2} + \frac{x^2}{r^2} + \frac{y^2}{r^2}\right) = 0 \qquad \Rightarrow \theta = \pi/2$$

6. Determine the sum of imaginary roots of the equation

$$(2x^2 + x - 1)(4x^2 + 2x - 3) = 6$$

Ans. $-\frac{1}{2}$

Sol. Put $2x^2 + x = y$

$\Rightarrow (4 - 1)\ (24 - 3) = 6$, on solving

$\Rightarrow 2x^2 + x + 1/2 = 0$

$\boxed{\alpha + \beta = -1/2}$

7. If cos A + cos B + cos C = 0, prove that

$$\cos 3A + \cos 3B + \cos 3C = 12 \cos A \cos B \cos C$$

A. $\text{L.H.S} = \sum 4\cos^3 A - 3\cos A$

$= 4\sum \cos^3 A - 3\sum \cos A$

$= 12 \cos A . \cos B. \cos C$

8. Let IR be the set of real numbers and $f : IR \to IR$ be such that for all $x, y \in IR$, $|f(x) - f(y)| \le |x - y|^3$. Prove that f is a constant function.

A. $\left|\frac{f(x) - f(y)}{x - y}\right|_{x \to y} \le |x - y|^3$

$= |f'(x)| \le |0| \quad \Rightarrow |f'(x)| = 0$

$\Rightarrow f(x) = \text{constant}$

9. Find the general solution of

$$(x + \log y)\, dy + y\, dx = 0$$

Ans. $xy + y \ln y - y = 0$

Sol. $x\, dy + y\, dx + \log y\, dy = 0$

$\int d(xy) + \int \log y\, dy = 0$

$xy + y \ln y - y = 0$

10. Prove that $I = \int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\operatorname{cosec} x} + \sqrt{\sec x}} dx = \frac{\pi}{4}$

A. $I = \int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\operatorname{cosec} x} + \sqrt{\sec x}} dx = \int_0^{\pi/2} \frac{\sqrt{\operatorname{cosec} x}}{\sqrt{\operatorname{cosec} x} + \sqrt{\sec x}}$

$2I = \int_0^{\pi/2} dx \quad \Rightarrow \boxed{I = \pi/4}$

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