Institute of Actuaries of India

ACET November 2022 Solutions

Mathematics

1. B. It is known that $-1 \le \cos x \le 1$. this implies that $-1 \le 1 - 2\cos x \le 3$ and $\frac{1}{1-2\cos x}$ must be less than or equal to -1 or greater than or equal to $\frac{1}{3}$.

   Hence the range of $f(x) = \frac{1}{1-2\cos x}$ $x \in R$ is $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$.

2. C. Given that $f(x) = (4 - x^4)^{\frac{1}{4}}$. Then

$$f \circ f(x) = f\left((4-x^4)^{\frac{1}{4}}\right) = \left[4 - \left\{(4-x^4)^{\frac{1}{4}}\right\}^4\right]^{\frac{1}{4}} = [4 - 4 + x^4]^{\frac{1}{4}} = x.$$

3. D. Let $\cos^{-1}\frac{\sqrt{3}}{2} = \theta \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6} \epsilon [0, \pi]$.

   Hence the principal value of $\cos^{-1}\frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$.

4. B. $x = +\sqrt{x+6}$ is defined if $x + 6 > 0$, so $x > 0$. Now, $x^2 = x + 6$ or $x^2 - x - 6 = 0$, giving $(x-3)(x+2) = 0$.

   Since $x > 0$, the solution of the equation is 3.

5. C. One can write $\frac{8}{n^2-4n+3} = \frac{8}{(n-3)(n-1)} = \frac{A}{n-3} + \frac{B}{n-1}$.

   This implies $8 = A(n-1) + B(n-3)$ giving $A = 4, B = -4$. Hence,

$$\sum_{n=4}^{\infty} \frac{8}{n^2 - 4n + 3} = \sum_{n=4}^{\infty} 4\left[\frac{1}{n-3} - \frac{1}{n-1}\right] = 4\left\{\left[1 - \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{1}{3} - \frac{1}{5}\right] + \cdots\right\}$$
$$= 4\left(1 + \frac{1}{2}\right) = 6.$$

6. A. $S_n = \frac{1+2+\cdots+n}{3n} = \frac{n(n+1)}{6n} = \frac{(n+1)}{6}$; $S_n^2 = \frac{(n+1)^2}{36}$. Hence,

$$\sum_{n=1}^{20} S_n^2 = \sum_{n=1}^{20} \frac{(n+1)^2}{36} = \frac{2^2 + 3^2 + \cdots + 21^2}{36} = \frac{(1^2 + 2^2 + 3^2 + \cdots + 21^2) - 1}{36}$$
$$= \frac{1}{36}\left(\frac{21 \times 22 \times 43}{6} - 1\right) = \frac{1}{36} \times 3310 = \frac{1}{36} \times S.$$

7. D. Given that $\sin x = \frac{4}{5}$ (in I quadrant) and $\cos y = -\frac{12}{13}$ (in II quadrant).

   $\cos x = \pm\sqrt{1 - \sin^2 x} = \pm\sqrt{1 - \frac{16}{25}} = \pm\frac{3}{5} = \frac{3}{5}$ (cos x is positive in I quadrant).

$\sin y = \pm\sqrt{1-\cos^2 y} = \pm\sqrt{1-\frac{144}{169}} = \pm\frac{5}{13} = \frac{5}{13}$ (sin$y$ is positive in II quadrant).

Hence, $\sin(x+y) = \sin x\cos y + \cos x \sin y = \frac{4}{5}.\frac{-12}{13} + \frac{3}{5}.\frac{5}{13} = -\frac{33}{65}$.

8. B. Given that $x^2 - 2x + c = 0$ has roots α and β. Then $\alpha + \beta = 2$ and $\alpha\beta = c$.

Similarly, $x^2 - 3x + c = 0$ has roots $\frac{\alpha}{2}$ and 2β, which implies $\frac{\alpha}{2} + 2\beta = 3$ and $\alpha\beta = c$.

Solving $\alpha + \beta = 2$ and $\frac{\alpha}{2} + 2\beta = 3$, one has $\alpha = \frac{2}{3}$ and $\beta = \frac{4}{3}$ giving $c = \alpha\beta = \frac{8}{9}$.

9. A. The direction ratios of the vector $\vec{a} = 3\vec{\imath} - 4\vec{\jmath} + \vec{k}$ are $(3, -4, 1)$.

$$|\vec{a}| = \sqrt{3^2 + (-4)^2 + 1^2} = \sqrt{26}$$

Hence, the direction cosines of the vector $\vec{a}$ are $\left(\frac{3}{\sqrt{26}}, \frac{-4}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right)$.

10. C. The two adjacent sides of a parallelogram are

$\vec{a} = 3\vec{\imath} - 5\vec{\jmath} + 6\vec{k}$ and $\vec{b} = 2\vec{\imath} - 3\vec{\jmath} - 4\vec{k}$.

Hence, the diagonal of the parallelogram

$\vec{c} = \vec{a} + \vec{b} = (3\vec{\imath} - 5\vec{\jmath} + 6\vec{k}) + (2\vec{\imath} - 3\vec{\jmath} - 4\vec{k}) = 5\vec{\imath} - 8\vec{\jmath} + 2\vec{k}$.

$|\vec{c}| = \sqrt{5^2 + (-8)^2 + 2^2} = \sqrt{93}$. Hence a unit vector parallel to the diagonal is $\frac{\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{93}}\left(5\vec{\imath} - 8\vec{\jmath} + 2\vec{k}\right)$.

11. D.

$$\lim_{x\to 0}\frac{(1-\cos 2x)(2+\cos x)}{x\tan 3x} = \lim_{x\to 0}\frac{(2\ \sin^2 x)(2+\cos x)}{x\frac{\tan 3x}{3x}3x}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2} \times \lim_{x\to 0}\frac{2+\cos x}{3} \times \lim_{x\to 0}\frac{1}{\frac{\tan 3x}{3x}} = 2\times 1\times 1 = 2.$$

12. B. Given that $\log(x+y) = 3xy$. Differentiating with respect to x, we have $\frac{1}{(x+y)}\left(1 + \frac{dy}{dx}\right) = 3x\frac{dy}{dx} + 3y$. This gives $\frac{1}{(x+y)} + \frac{dy}{dx}\left(\frac{1}{x+y} - 3x\right) = 3y$.

Hence, $\frac{dy}{dx} = \frac{3y - \frac{1}{x+y}}{\frac{1}{x+y} - 3x} = \frac{3y(x+y)-1}{1-3x(x+y)}$.

Now, when $x = 0$, $\log(x+y) = 3xy$ implies $\log y = 0$ and $y = 1$.

Hence, $\frac{dy}{dx} = 2$.

13. A. If $f(x) = xe^{x(1-x)}$ then

$$\begin{aligned} f'(x) &= e^{x(1-x)} + xe^{x(1-x)}(1-2x) = e^{x(1-x)}[1 + x(1-2x)] \\ &= e^{x(1-x)}[1 + x - 2x^2] = -e^{x(1-x)}[2x^2 - x - 1] \\ &= -e^{x(1-x)}(x-1)(2x+1). \end{aligned}$$

The right-hand side will be positive only if $(x-1)(2x+1) < 0$ and it is possible only if one of them is positive and the other is negative.

Both $(x-1)>0$ and $(2x+1)<0$ are not possible. The set of x for which $(x-1)<0$ and $(2x+1)>0$ is $\left(-\frac{1}{2},1\right)$.

14. B.

$$\int(\tan x+\cot x)^2dx=\int(\tan^2 x+\cot^2 x+2\tan x\cot x)dx$$
$$=\int\{(\sec^2 x-1)+(\operatorname{cosec}^2 x-1)+2\}dx$$
$$=\int(\sec^2 x+\operatorname{cosec}^2 x)dx=\tan x-\cot x+c.$$

15. D.

$$\int_{-1}^{3}|x^3-x|\,dx=\int_{-1}^{0}|x^3-x|\,dx+\int_{0}^{1}|x^3-x|\,dx+\int_{1}^{3}|x^3-x|\,dx$$

Now, $x^3-x\begin{cases}\geq 0 \text{ on } [-1,0)\\ \leq 0 \text{ on } [0,1)\\ \geq 0 \text{ on } [1,3].\end{cases}$

Hence,

$$\int_{-1}^{3}|x^3-x|\,dx=\int_{-1}^{0}(x^3-x)\,dx+\int_{0}^{1}-(x^3-x)dx+\int_{1}^{3}(x^3-x)\,dx$$
$$=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]\Bigg|_{-1}^{0}+\left[\frac{x^2}{2}-\frac{x^4}{4}\right]\Bigg|_{0}^{1}+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]\Bigg|_{1}^{3}$$
$$=-\left[\frac{1}{4}-\frac{1}{2}\right]+\left[\frac{1}{2}-\frac{1}{4}\right]+\left[\frac{81}{4}-\frac{9}{2}\right]-\left[\frac{1}{4}-\frac{1}{2}\right]=\frac{33}{2}.$$

16. A. The given relation

$$\begin{vmatrix}3i & -4i & 1\\ 2 & 4i & -1\\ 10 & 4 & i\end{vmatrix}=x+iy$$

implies

$$-4i\begin{vmatrix}3i & 1 & 1\\ 2 & -1 & -1\\ 10 & i & i\end{vmatrix}=x+iy\Rightarrow x+iy=0\implies x=0 \text{ and } y=0..$$

17. C. Given that

$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix},$$

then

$$A^2=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & -\sin\theta\cos\theta-\cos\theta\sin\theta\\ \sin\theta\cos\theta+\cos\theta\sin\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}=\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}$$
$$=\begin{bmatrix}\cos\pi & -\sin\pi\\ \sin\pi & \cos\pi\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}.$$

18. D. Given that

$$A = \begin{bmatrix} 0 & 3q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}.$$

Hence

$$A^T = \begin{bmatrix} 0 & p & p \\ 3q & q & -q \\ r & -r & r \end{bmatrix}.$$

Now, $AA^T = I$ implies

$$\begin{bmatrix} 9q^2 + r^2 & 3q^2 - r^2 & -3q^2 + r^2 \\ 3q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -3q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This gives $9q^2 + r^2 = 1;\ 3q^2 = r^2;\ p^2 - q^2 - r^2 = 0.$

These give $p^2 = 4q^2$ and $9q^2 + 3q^2 = 12\,q^2 = 1.$

Hence $p^2 = \frac{1}{3}$ giving $|p| = \frac{1}{\sqrt{3}}$.

19. A. Given that $f(x) = e^x - 2x - 1$. Hence $f'(x) = e^x - 2$.

With the given initial value $x_0 = 1$,

$$f(1) = e^1 - 2 \times 1 - 1 = e - 3 = 2.7183 - 3 = -0.2817.$$

$$f'(1) = e^1 - 2 = 2.7183 - 2 = 0.7183.$$

Hence, the first iteration, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-0.2817}{0.7183} = 1 + 0.3922 = 1.3922.$

$$\begin{aligned} \text{Now, } f(1.3922) &= e^{1.3922} - 2 \times 1.3922 - 1 \\ &= e^1 \times e^{0.3922} - 2 \times 1.3922 - 1 \\ &= 2.7183 \times 1.4802 - 2 \times 1.3922 - 1 \\ &= 4.0236 - 2.7844 - 1 = 0.2392. \end{aligned}$$

Similarly, $f'(1.3922) = e^{1.3922} - 2 = 4.0237 - 2 = 2.0236.$

Hence, the second iteration,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.3922 - \frac{0.2393}{2.0236} = 1.3922 - 0.1182 = 1.2740.$$

20. A. $y = \frac{1}{1+x}$ with $h = \frac{1}{2}$ we have the following table.

x	0	$\frac{1}{2}$	1
y	1	$\frac{2}{3}$	$\frac{1}{2}$

$$\int_0^1 \frac{1}{1+x}\,dx = \frac{h}{2}[(y_0 + y_2) + 2y_1] = \frac{1}{4}\left[\left(1 + \frac{1}{2}\right) + 2 \times \frac{2}{3}\right] = \frac{17}{24}.$$

Statistics

21. D. Total number of arrangements of the letters of the word "ATTITUDE" is $\frac{8!}{3!}$.
Other than three T's, there are five letters which can be arranged in 5! ways. In each arrangement of these 5 letters there are 6 places, 4 between the 5 letters and one on extreme left and the other on extreme right. To separate 3 T's, arrange them in 6 places. This can be done in $\binom{6}{3}$ ways.
So the number of ways in which no two T's are together is $5! \times \binom{6}{3}$.

Required probability is

$$\frac{5! \times \binom{6}{3}}{\frac{8!}{3!}} = \frac{5! \times 6! \times 3!}{3! \times 3! \times 8!} = \frac{5}{14}.$$

22. B. $P(F) = \frac{3}{5}$. $P(E|F) = \frac{1}{2} \Rightarrow P(E \cap F) = \frac{1}{2} \times P(F) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$.

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

$\Rightarrow \frac{4}{5} = P(E) + \frac{3}{5} - \frac{3}{10} \Rightarrow P(E) = \frac{1}{2}$.

23. C. $\bar{x}_{n+1} = \frac{\text{sum of } n+1 \text{ observations}}{n+1} = \frac{\text{sum of } n \text{ observations} + x}{n+1} = \frac{n\bar{x}_n + x}{n+1}$.

24. A. Suppose the ordered observations are $a \leq b \leq c \leq d$. Median = $\frac{b+c}{2}$. We have

Mean $= \frac{a+b+c+d}{4} = 12$, Range $= d - a = 20$ and smallest observation $= a = 5$.

So $d = 20 + a = 20 + 5 = 25$.

$\frac{a+b+c+d}{4} = 12 \Rightarrow b + c = 48 - a - d = 48 - 5 - 25 = 18$.

Median $= \frac{b+c}{2} = \frac{18}{2} = 9$.

25. B. Probability distribution of X.

x	23	24	25	26	27
$P(X = x)$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{1}{12}$

Probability that the age is more than 25 is

$P(X > 25) = P(X = 26) + P(X = 27) = \frac{3}{12} + \frac{1}{12} = \frac{1}{3}$.

26. D. Let A_i be the event that order i is not shipped on time.

$P(A_i) = 0.10, i = 1, 2, 3$

Probability that exactly one order is not shipped on time, computed from the binomial distribution with parameters 3 and 0.10, is

$\binom{3}{1} \times 0.9 \times 0.9 \times 0.1 = 3 \times 0.081 = 0.243.$

27. D. Let the observations be $x_1, x_2, \dots, x_{20}$.

$\sum_{i=1}^{20}(x_i - 10) = 50$. This implies $\sum_{i=1}^{20} x_i - 20 \times 10 = 50$. So $\sum_{i=1}^{20} x_i = 250$.

Mean of the observations is $\frac{\sum_{i=1}^{20} x_i}{20} = \frac{250}{20} = 12.5$.

28. A. The smallest observation is 1 and largest observation is 37. Range $= 37 - 1 = 36$.

Observations are ordered. Median = (25th obs. + 26th obs.) $/2 = (4+5)/2 = 4.5$.

The distribution is positively skewed.

The mean should be larger than median for a positively skewed distribution.

29. B. C = event that a claim is filed

A_1 = event that the client is high-risk

A_2 = event that the client is medium-risk

A_3 = event that the client is low-risk

$$P(C|A_1) = 0.02, P(C|A_2) = 0.01, P(C|A_3) = 0.0025.$$

$$P(A_1) = 0.10, P(A_2) = 0.30, P(A_3) = 0.60.$$

$$\begin{aligned} P(C) &= P(C|A_1)P(A_1) + P(C|A_2)P(A_2) + P(C|A_3)P(A_3) \\ &= 0.02 \times 0.10 + 0.01 \times 0.3 + 0.0025 \times 0.60 \\ &= 0.002 + 0.003 + 0.0015 = 0.0065. \end{aligned}$$

$$P(A_1|C) = \frac{0.02 \times 0.10}{0.0065} = \frac{4}{13}.$$

30. A. The expected number of red lights she hit each day is

$$E(X) = \sum_{x=0}^{5} xP(X = x)$$

$= 0 \times 0.05 + 1 \times 0.25 + 2 \times 0.30 + 3 \times 0.20 + 4 \times 0.15 + 5 \times 0.05 = 2.3.$

31. C. $P(-a < X < a) = \int_{-a}^{a} \frac{1}{4} dx = \frac{a}{2} = 0.95$. So $a = 1.90$.

32. B. $P(X = 3) = P(X \le 3) - P(X \le 2) = F(3) - F(2) = 0.6 - 0.3 = 0.3.$

$P(X \ge 3) = 1 - P(X < 3) = 1 - P(X \le 2) = 1 - F(2) = 1 - 0.3 = 0.7.$

$$P(X = 3|X \ge 3) = \frac{P(X = 3, X \ge 3)}{P(X \ge 3)} = \frac{P(X = 3)}{P(X \ge 3)} = \frac{3}{7}.$$

33. A. $Y = 5 + 0.5X$. $E(Y) = 5 + 0.5E(X)$.

$E(X) = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 0$. So $E(Y) = 5 + 0.5 \times 0 = 5$.

$$E\big(Y-E(Y)\big)^{25} = E(5+0.5X-5)^{25} = E(0.5X)^{25} = (0.5)^{25}E(X^{25}).$$

$$E(X^{25}) = \int_{-\infty}^{\infty} x^{25}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx = 0.$$

So $E(Y-5)^{25} = 0$.

34. B. Mean of the distribution is = $\int_a^b \frac{1}{b-a}dx = \frac{a+b}{2}$.

$x_{0.25}$: 25th percentile of the distribution.

$P(X \le x_{0.25}) = 0.25 \Rightarrow \int_a^{x_{0.25}} \frac{1}{b-a}dx = 0.25 \Rightarrow \frac{x_{0.25}-a}{b-a} = 0.25$

$\Rightarrow x_{0.25} = a + 0.25(b-a) = 0.75a + 0.25b$

So $\frac{a+b}{2} = 8$ and $0.75a + 0.25b = 6$. This implies $a = 4, b = 12$.

90th percentile of the distribution = $a + 0.90(b-a) = 4 + 0.90 \times 8 = 11.2$.

35. C. $PX = 2, Y \ge 1 = P(X=2)P(Y \ge 1)$ (since X and Y are independent)

$$= \frac{1}{3}\big(P(Y=1)+P(Y=2)\big) = \frac{1}{3}\left(\frac{1}{3}+\frac{1}{3}\right) = \frac{2}{9}.$$

36. A. Let X be the number of cracks in 6 kilometres of highway.

X ~Poisson(18).

Required probability = $P(X=0) = e^{-18}$.

37. D. Let X be the random variable denoting the time to failure of the component.

Required probability = $P(X > 200) = \int_{200}^{\infty} \frac{1}{100}e^{-\frac{x}{100}}dx = e^{-\frac{200}{100}} = e^{-2}$.

38. B. Assume that Regression of x on y is $x + 4y + 3 = 0$. This implies $x = -4y - 3$

Regression of y on x is $4x + 9y + 1 = 0$. This implies $y = -\frac{4}{9}x - \frac{1}{9}$.

Then r^2 = the product of regression coefficients is $-4 \times \left(-\frac{4}{9}\right) = \frac{16}{9}$ >1

This is not possible. So the regression of x on y is $4x + 9y + 1 = 0$ and the regression of y on x is $x + 4y + 3 = 0$.

Regression coefficient of x on y is $-\frac{9}{4}$.

Regression coefficient of y on x is $-\frac{1}{4}$.

$r^2 = \left(-\frac{9}{4}\right) \times \left(-\frac{1}{4}\right) = \frac{9}{36}$.

$r = -\frac{3}{4} = -0.75$. (Sign is negatives as sign of regression coefficient is negative.)

Note that $\bar{x}$ and $\bar{y}$ satisfy both the regression equations. Therefore, by solving $x + 4y + 3 = 0$ and $4x + 9y + 1 = 0$, we get $\bar{x} = \frac{23}{7}$ and $\bar{y} = -\frac{11}{7}$.

39. B. $P(Y=3|X=1)=\frac{P(X=1,Y=3)}{P(X=1)}$.

$P(X=1)$

$=P(X=1,Y=0)+P(X=1,Y=1)+P(X=1,Y=2)+P(X=1,Y=3)$

$=0+1/8+2/8+1/8=1/2.$

$P(Y=3|X=1)=\frac{1/8}{1/2}=\frac{1}{4}$.

40. C. $\text{corr}(Y_1,Y_2)=\frac{\text{cov}(Y_1,Y_2)}{\sqrt{\text{var}(Y_1)\text{var}(Y_2)}}$.

X_1 and X_2 are independent, so $\text{cov}(X_1,X_2)=0$.

$\text{cov}(Y_1,Y_2)=\text{cov}(a_{11}X_1+a_{12}X_2,a_{21}X_1+a_{22}X_2)$

$=a_{11}a_{21}.\text{var}(X_1)+a_{11}a_{22}\text{cov}(X_1,X_2)+a_{12}a_{21}\text{cov}(X_2,X_1)+a_{12}a_{22}\text{var}(X_2)$

$=a_{11}a_{21}\sigma_1^2+a_{12}a_{22}\sigma_2^2$.

$\text{var}(Y_1)=a_{11}^2\text{var}(X_1)+2a_{11}a_{12}\text{cov}(X_1,X_2)+a_{12}^2\text{var}(X_2)=a_{11}^2\sigma_1^2+a_{12}^2\sigma_2^2$.

$\text{var}(Y_2)=a_{21}^2\text{var}(X_1)+2a_{21}a_{22}\text{cov}(X_1,X_2)+a_{22}^2\text{var}(X_2)=a_{21}^2\sigma_1^2+a_{22}^2\sigma_2^2$.

41. B. Number of total drivers under 20 = $51 + 9 = 60$.

Total number of drivers = $968 + 432 = 1400$.

The percentage of total drivers under 20 = $(60/1400) \times 100 = 4.3$ (approx.).

42. B. The number of male drivers in the age group 25-44 = $92 + 95 + 97 + 99 = 383$. Similarly, the number of female drivers in the age group 25-44 = $43 + 45 + 63 + 61 = 212$.

43. B. Number of male drivers in the age group 30-59 $= 95 + 97 + 99 + 106 + 111 + 87 = 595$.

Number of female drivers in the age group 30-59 $= 45 + 63 + 61 + 54 + 49 + 33 = 305$.

Total number of drivers in the age group 30-59 $= 595 + 305 = 900$.

Percentage of drivers in the age group 30-59 = $\frac{900\times100}{1400} = 64.3$ (approximately).

44. D. Total number of gold medals = 280.

Percentage of gold medals won by Australia and England = $(67 + 57) \times 100/280$

$= 124 \times 100/280 = 44.3$ (approx.).

Number of countries which won at least 6 gold medals is 13.

Number of countries which won less than 4 gold medals is 12.

Percentage of gold medals won by Australia, England, Canada and India

$= (67 + 57 + 26 + 22) \times 100/280 = \left(\frac{172}{280}\right) \times 100 = 61.4$ (approx.).

45. A. Chart shows that the maximum number of fatalities occur in 1995 and the minimum number of fatalities occur in 2010.

46. D. There is negative correlation in (b).

No linear relationship in (d). So 0.022 must be related to (d).

Positive correlation is related to (e) and (f). There is strong positive correlation in (e). So 0.974 is related to (e) and 0.333 must be related to (f).

47. B. Negative correlation is related to (a), (b) and (c).

There is strong negative correlation in (a). -0.987 must be related to (a).

Correlation in (c) is stronger than (b) but weaker than (a).

Therefore, -0.889 must be related to (c) and -0.522 must be related to (b).

There is a weak positive correlation in (f).

48. C. The chart shows that C3, C5, C10 and C12 have no industrial portion.

49. A. The chart shows that C2, C9, C15 and C16 have no vacant sites portion.

50. D. From the chart, it is seen that C3 is having highest percentage of residential property and C17 is having second highest.

51. D. The chart shows that C13 is having highest percentage of vacant sites – about 55%.

52. A.
53. A.
54. D.
55. C.
56. C.
57. A.
58. D.
59. C.
60. C.
61. C.
62. A.

Logical reasoning

63. D. The only sister of the brother of the woman will be the woman herself and she is the mother of that man. Thus, the woman is the daughter of the maternal grandmother of that man.

64. B. 1896 was a leap year. The next leap year came in 1904 as 1900 was not a leap year.

65. C. In 12 hours, the hand turns 360 degrees. In 7 hours it will be $(360/12) \times 7 = 210.$ degrees.

66. B. A $5 \times 5 \times 5$ cube has 5 cubes of dimension $1 \times 1 \times 1$ along its length, breadth and width. If we add a layer of $1 \times 1 \times 1$ cubes (the smaller cubes) then the new cube will have following dimensions:
Length $= 5 + 1 + 1 = 7$; breadth$= 5 + 1 + 1 = 7$ and width$= 5 + 1 + 1 = 7.$
So the number of $1 \times 1 \times 1$ cubes in it $= 7 \times 7 \times 7 = 343.$

67. A. Only one message is true and the other two are false. If the second box has the gift, then there will be two true messages, which would be on the first and third boxes. If the third box has the gift, then there will be two true messages, which would be on the first and second boxes. If the first box has the gift, then there will be one true message, which would be on the second box. Hence the gift is in the first box.

68. D. The ocean is lake and no sea is a ocean. So no lake can be sea which is Conclusion II. Some rivers are sea means some rivers are not seas and can be ocean and some ocean is water therefore water being river can be a possibility which is Conclusion III.

69. C. The number listening to music only is $113 - 43 - 29 - 18 = 23.$

70. B. The movement towards West in the second leg of the journey is cancelled by an equal movement towards East in the fourth leg. The final position is 17 meters to the South of the initial position.

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