Institute of Actuaries of India

ACET December 2021

Mathematics

1. The domain of the function $\sin^{-1} x^2$ is

   A. $(0,1)$.

   B. $[0,1]$.

   C. $(-1,1)$.

   D. $[-1,1]$.

1 mark

2. The value of $3^{1/2} \times 9^{1/4} \times 27^{1/8} \times \cdots$ is

   A. 3.

   B. 9.

   C. 27.

   D. 81.

2 marks

3. A is a square matrix of order 4, then the adjoint of $3A$, i.e. $\text{adj}(3A)$, is equal to

   A. $27\text{adj}(A)$.

   B. $81\text{adj}(A)$.

   C. $3\text{adj}(A)$.

   D. $9\text{adj}(A)$.

1 mark

4. The mapping $h: N \to N$ is defined by $h(x) = x - (-1)^x, x \in N$, the set of natural numbers. It is

   A. not a 1-1 mapping.

   B. not an onto mapping.

   C. both 1-1 and onto mapping.

   D. none of these.

1 mark

5. If $f(x) = \left(1 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$, then at $x = 0$

   A. $f'(x)$ does not exist but $f(x)$ is maximum.

   B. $f'(x)$ does not exist but $f(x)$ is minimum.

   C. $f'(x)$ exists.

   D. $f'(x)$ does not exist and $f(x)$ is neither maximum nor minimum.

2 marks

6. The equation $3\sin\theta + 4\cos\theta = c$ is solvable for

   A. $c = -5$ and $c = 5$ only.

   B. $-5 < c < 5$ only.

   C. $c \leq -5$ and $c \geq 5$ only.

   D. $-5 \leq c \leq 5$ only.

1 mark

7. The remainder when 7^{103} is divided by 25 is

   A. 3.

   B. 18.

   C. 24.

   D. 1.

2 marks

8. The system of equations $2x - y = 5,\ x - 2y = 4,\ 2x + y = 3$ has

   A. a unique solution.

   B. exactly two solutions.

   C. no solution.

   D. infinitely many solutions.

1 mark

9. If $\mu = \int_3^5 \frac{\log x^2}{\log x^2 + \log(64 - 16x + x^2)}\,dx$, the value of μ is

   A. 2.

   B. 3.

   C. 8.

   D. 1.

2 marks

10. The value of a, for which the system of equations

$$\begin{aligned} 7x + 2y - z &= 0 \\ 2x + 3y - 5z &= 0 \\ 5x - y + az &= 0 \end{aligned}$$

    have more than one solution, is

    A. -4.

    B. -6.

    C. 6.

    D. 4.

1 mark

11. A circle whose centre is at $(-\frac{7}{2}, 0)$ and radius is $\frac{7}{2}$ can be represented by

    A. $r = 7\cos\theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

B. $r = -7\cos\theta$ for $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$.

C. $r = 7\cos\theta$ for $0 \leq \theta < 2\pi$.

D. $r = -7\cos\theta$ for $0 \leq \theta < 2\pi$.

1 mark

12. $x = \sqrt{t-3}, y = \sqrt{4-t}\,, t \in [3,4]$ represent an equation of

A. a parabola.

B. an ellipse.

C. a hyperbola.

D. none of these.

1 mark

13. The limit $\lim\limits_{x\to 0} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$

A. is 1.

B. is 0.

C. approaches ∞.

D. does not exist.

2 marks

14. The number of points at which the function $f(x) = (\log|x|)^{-1}$ is not continuous is

A. 3.

B. 1.

C. 2.

D. ∞.

1 mark

15. The identity $\text{cosec}^{-1}(\text{cosec}\, x) = x$ holds for values of x belonging to

A. the set of real numbers.

B. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

D. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

1 mark

16. The number of real roots of the equation $e^{\sin x} - e^{-\sin x} = 2.5$ is

A. 2.

B. 1.

C. 0.

D. infinite.

3 marks

17. The solutions of the equation $\sin x = -1$ are given by

A. $x = 2n\pi$, where n is any integer.

B. $x = 2n\pi - \frac{\pi}{2}$, where n is any integer.

C. $x = 2n\pi + \frac{\pi}{2}$, where n is any integer.

D. $x = n\pi$, where n is any integer.

1 mark

18. The value of $\int_{-3}^{3} |1 - x^2| \, dx$ is

A. $\frac{44}{3}$.

B. 12.

C. $\frac{22}{3}$.

D. 0.

2 marks

19. Based on approximation by polynomial of degree three, the missing term in the following data

x	0	1	2	3	4
$y = f(x)$	1	3	9	-	81

is

A. 27.

B. 31.

C. 29.

D. 16.

3 marks

20. The cosine of the angle between $\vec{\imath} + \vec{\jmath} + \vec{k}$ and its projection on the plane containing the vectors $\vec{\imath}$ and $\vec{\jmath}$ is

A. $\sqrt{\frac{1}{3}}$.

B. $\sqrt{\frac{1}{6}}$.

C. $\sqrt{\frac{2}{3}}$.

D. $\sqrt{\frac{1}{2}}$.

1 mark

21. The arithmetic mean of the 4 consecutive integers starting with x is y. What is the arithmetic mean of 8 consecutive integers that start with $x + 2$?

A. $y + 1$.

B. $y + 2$.

C. $y + 3$.

D. $y + 4$.

1 mark

22. Positive integers from 1 to 15, inclusive are placed in 3 groups of 5 each. What is the highest possible average of the medians of these 3 groups?

A. 8.

B. 9.

C. 10.

D. 11.

3 marks

23. An analysis of the annual revenues by 5 companies in a year are done. The mean and median of the revenues is Rs. 600 Crores. The only mode among the observations is Rs 1000 crores. All revenues are in hundreds of crores only. What is the difference between the highest and the lowest revenue earned by the 5 companies in the year?

A. 800.

B. 900.

C. 1000.

D. 1100.

2 marks

24. Given the sum of four numbers is 28 and the sum of their squares is 232, what is the standard deviation of the four numbers?

A. 3.

B. 4.

C. 5.

D. 6.

1 mark

25. The mean time required to complete the ACET examination is 60 minutes and completion times follow an exponential distribution. If the duration of the examination is 90 minutes, what is the probability that a student will not be able to complete the examination?

A. $e^{-3/2}$.

B. $e^{-2/3}$.

C. $1 - e^{-3/2}$.

D. $1 - e^{-2/3}$.

1 mark

26. A local drugstore owner knows that, on average, people enter his store at the rate of five per hour. Assuming that number of people entering in a given 3-minute period follows a Poisson distribution, find the probability that nobody enters the store during this period.

A. $e^{-1/12}$.

B. $e^{-3/5}$.

C. $e^{-1/20}$.

D. $e^{-1/4}$.

1 mark

27. The number of errors made per page by a copy typist has a Poisson distribution with a mean of 0.2. Errors occur in each page independently of the other pages. Find the coefficient of variation of the number of errors in an assignment of 8 pages.

A. 1.6.

B. $\sqrt{1.6}$.

C. $1/\sqrt{1.6}$.

D. 1.

1 mark

28. An answering machine can record a message perfectly with probability 0.8, satisfactorily with probability 0.15 and poorly with probability 0.05. Out of 10 such recordings, what is the probability that at least one is perfect?

A. 0.8^{10}.

B. $1 - 0.2^{10}$.

C. $1 - 0.05^{10}$.

D. $1 - 0.15^{10}$.

1 mark

29. It is found that the distribution of marks scored by each candidate on a particular subject follows the normal distribution with mean 65 and standard deviation 15. What is the score of the 87^{th} percentile approximately? [Given $\Phi(-1.13) = 0.13$.]

A. 48.

B. 87.

C. 82.

D. 67.

1 mark

30. A machine fills 100 grams (according to the label) boxes with some cereal. The amount poured into the box is normally distributed with a standard deviation of 1 gram. What does the *population mean* have to be in order for 99.5% of the boxes to contain 100 grams or more of cereal? [Given $\Phi(2.58) = 0.995$.]

A. 100.

B. 97.42.

C. 99.5.

D. 102.58.

2 marks

31. Given $Var(X) = 1$, $Var(Y) = 9$ and $Corr\,(X, Y) = 1/4$, what is the value of $Cov(Y, X + Y)$?

A. $37/4$.

B. $39/4$.

C. $40/4$.

D. $41/4$.

1 mark

32. Suppose X has the uniform distribution over $[1,3]$ and that, for given $X = x$, Y has the exponential distribution with rate parameter x. Then $Cov(X, Y)$ equals

A. $1 - 4\log 3$.

B. $1 - 3\log 3$.

C. $1 - 2\log 3$.

D. $1 - \log 3$.

3 marks

33. A portfolio has two stocks. The return of one stock has standard deviation of 5, and that of the other has standard deviation of 3. The correlation coefficient between returns of the two stocks is 0.5. The standard deviation of the total return from the portfolio is

A. $\sqrt{34}$.

B. $\sqrt{41.5}$.

C. 7.

D. 15.5%.

1 mark

34. The correlation coefficient between X and Y as obtained from the two regression lines

$$4X - 5Y + 33 = 0$$
$$20X - 9Y - 107 = 0$$

is

A. $1/9$.

B. $1/5$.

C. $5/9$.

D. $3/5$.

2 marks

35. The means of the two random variables X and Y as obtained from the two regression lines

$$X - 3Y - 8 = 0$$
$$Y - 5X + 7 = 0$$

are

A. $(-13/14, 33/14)$.

B. $(-33/14, 13/14)$.

C. $(13/14, -33/14)$.

D. $(33/14, -13/14)$.

2 marks

36. There are two series of index numbers P for price index and S for stock of the community. The mean and standard deviation of P are 50 and 4 and those of S are 60 and 2, respectively. The correlation coefficient of the two series is 0.5. The regression line of P on S will be

    A. $P = S - 10$.

    B. $P = S + 10$.

    C. $P = 35 - S/4$.

    D. $P = 35 + S/4$.

2 marks

37. If a polygon has 170 diagonals, then the number of its sides is

    A. 34.

    B. 17.

    C. 20.

    D. 18.

1 mark

38. The number of permutations of 10 distinct objects taken 5 at a time in which 3 particular objects occur together is

    A. 756.

    B. 378.

    C. 126.

    D. 2016.

2 marks

39. For two sets X and Y, $(Y - X \cap Y) \cap (X \cap Y)$ is equivalent to

    A. Y.

    B. $X - Y$.

    C. $\{0\}$.

    D. none of these.

1 mark

40. If X and Y are independent events and $P(Z) = 0$, then X, Y, Z are

    A. independent.

    B. pairwise independent but not mutually independent.

    C. not pairwise independent.

    D. mutually exclusive.

1 mark

Data Interpretation

In a factory, there are four working units, named X, Y, Z and K. A year-wise production level for each of these four units has been listed in the following table. Answer the questions 41 and 42 based on your interpretation of the table.

Year	X	Y	Z	K
2016	150	188	173	139
2017	241	138	207	309
2018	205	290	296	236
2019	275	130	365	127
2020	140	99	232	158

41. The year in which the X unit had a total contribution of approximately 20% was

    A. 2016.

    B. 2017.

    C. 2018.

    D. 2019.

2 marks

42. The unit which never had the maximum production level (among all the units) in any year and the unit which never had the minimum production level in any year, respectively, were

    A. X, K.

    B. Y, Z.

    C. Z, X.

    D. X, Z.

1 mark

The following table gives number of mobile phones (in thousand units) in Ahmedabad and Jaipur. The mobile phones are sold in different brands (B1, B2, B3 and B4) with different model variants as shown in the table. Seven models of each brand are matched by price range and functionality. Answer questions 43-44 based on this table.

	Ahmedabad				Jaipur			
	B1	B2	B3	B4	B1	B2	B3	B4
Model 1	160	159	168	154	164	194	165	191
Model 2	175	169	169	194	175	188	164	180
Model 3	170	188	191	164	160	169	159	171
Model 4	149	162	175	193	168	159	168	165
Model 5	197	198	153	181	193	163	174	173
Model 6	160	173	174	153	191	188	184	171
Model 7	189	150	175	150	185	192	193	196

43. The model variant, which has the same average sales (taking together all brands) in both the cities, is

    A. Model 1.

B. Model 2.

C. Model 3.

D. Model 4.

2 marks

44. The model variant, which has the largest excess in number of units sold (taking together all brands) in Jaipur over that in Ahmedabad, is

A. Model 7.

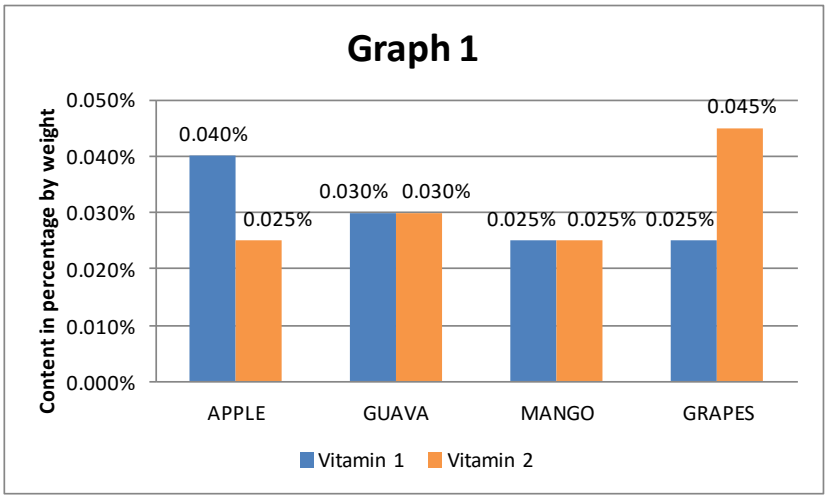
B. Model 6.

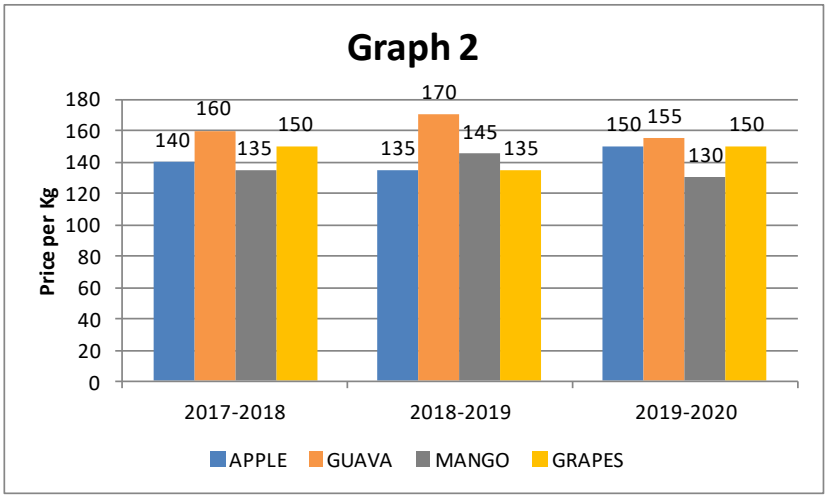
C. Model 5.

D. Model 4.

1 mark

Refer to the graphs below and answer to the questions 45-48.





The first graph shows the Vitamin 1 and Vitamin 2 content in each fruit whereas the second graph shows the price per kg. (in Rupees) of Apple, Guava, Mango and Grapes for 2017-2018, 2018-2019 and 2019-2020, respectively.

45. In the year 2017-18 the fruit, where 1 gram of vitamin 1 was the costliest, was

A. Apple.

B. Guava.

C. Mango.

D. Grapes.

1 mark

46. The ratio of the cost of 1 gram of Vitamin 2 from the costliest source to that from the cheapest source in the year 2017-18 was

A. 1.68.

B. 2.0.

C. 2.2.

D. 2.4.

1 mark

47. If 0.25 grams of Vitamin 1 from each fruit is required, then its approximate cost in rupees in the year 2019-20 was

A. 501.

B. 502.

C. 503.

D. 504.

2 marks

48. If Apple, Guava, Mango and Grapes are mixed in the ratio 1: 2: 3: 4 in terms of weight, then the total percentage of Vitamin (1 and 2) in this mixture is

A. 0.0615%.

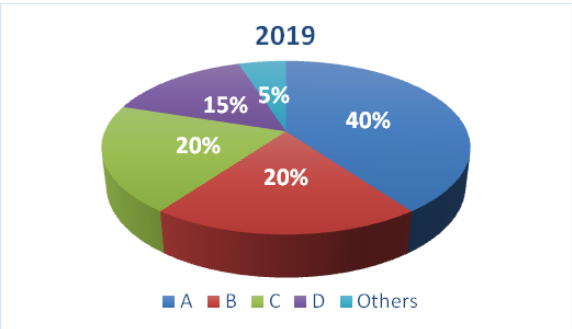
B. 0.615%.

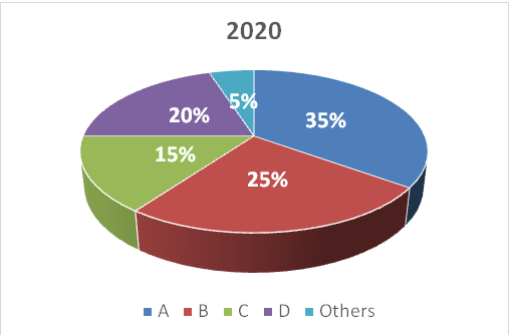
C. 6.15%.

D. 61.5%.

2 marks

The two pie charts below show the percentage market share on value basis of the companies A to D and others for 2019 and 2020. The market size is 100 crores in 2019 and 200 Crores in 2020. Answer questions 49-51 based on this information.





49. Identify the company which had the minimum rate of sales growth from 2019 to 2020.

A. A.

B. B.

C. C.

D. D.

1 mark

50. If companies in the Others category had doubled their sales growth rate from the current rate, then the market size in 2020 would have been

A. 202.

B. 203.

C. 204.

D. 205.

1 mark

51. If the market size becomes 400 Crores in 2020 instead of 200 Crores with all the proportion of business remaining the same. What would be ratio of sales of other companies in 2020 to that in 2019.

A. 4.

B. 3.

C. 2.

D. 1.

1 mark

52. Amateur is a person who
    A. does a thing for pleasure and not as a profession.
    B. is not matured in doing anything.
    C. is not an expert in whatever he does.
    D. is the jack of all trades.

1 mark

53. Egotist is a person who
    A. always talks with pride.
    B. belittles achievements of others.
    C. often talks of his achievements.
    D. does not have any ego.

1 mark

54. Choose the word closest in meaning to the word 'Humdrum'.
    A. Abnormal.
    B. Uninteresting.
    C. Exciting.
    D. Impossible.

1 mark

55. Select the word that is closest in meaning to the word ‘Progressive’.
    A. Gradual.
    B. Promotional.
    C. Supportive.
    D. Tolerant.

1 mark

56. Select the correct sentence.
    A. He made into an office the living room.
    B. He made the living room into an office.
    C. He made into the living room to an office.
    D. He made into an office of the living room.

1 mark

57. Meaning of the phrase “to let up” is
    A. to become less intense.
    B. to allow someone to go.
    C. to allow someone in.
    D. to ask for the permission to go.

1 mark

58. Meaning of the phrase “to put up with” is

A. to close the matter.

B. to resolve a conflict.

C. to tolerate.

D. to end a relation.

1 mark

59. Meaning of the phrase “to call it a day” is

A. to compromise.

B. to finish the day’s work.

C. to mark a day as special.

D. to stop what you are doing.

1 mark

Questions 60 to 61: The phrases are jumbled up. Choose the correct sequence from the options to complete the given sentence.

60. One cannot recall any movement _____.

i. which has gripped the imagination of the entire human race
ii. in world history
iii. which started nearly twenty-five years ago
iv. so completely and so rapidly as the Green Movement

A. ii, i, iv, iii.

B. i, ii, iii, iv.

C. ii, iii, iv, i.

D. iv, iii, i, ii.

2 marks

61. While the government offered _____.

i. the farmers stuck to their demand
ii. to send a proposal regarding the amendments
iii. that it was willing to make in the new farm laws
iv. for repeal of the laws

A. i, ii, iii, iv.

B. ii, iii, i, iv.

C. i, iii, iv, ii.

D. ii, iv, i, iii.

2 marks

Read the passage below and answer Question No. 62.

The origins of tea as a beverage can be traced back more than 5,000 years. Chinese mythology first addresses the drink in 3,000BC, when the emperor Nin Song was said to have discovered it. Nin Song was something of a visionary. Among other innovations, he believed that water should

be boiled before drinking as a health precaution. As the story goes, he was travelling with some members of the court when they stopped to rest. Some leaves from a bush fell into the water being boiled for the weary travellers, and thus was tea born.

In 800AD, a man named Lu Yu wrote the first known book on tea cultivation and preparation. The work, called the Ch'a Ching, melded Zen Buddhist teachings with the art and craft of tea, forever linking the drink to spirituality. In 1191AD, the cultivation and brewing of the leaves spread to Japan when a monk named Yeisei returned from pilgrimage, bringing seeds back with him. Yeisei had observed tea being used in and enhancing meditation and spiritual awareness in China. He shared this discovery with his peers and the tradition quickly caught on—all the way to the highest levels of society, including the imperial court.

Tea was so well-received in Japan that it was elevated to an art form, culminating in the creation of the well-known Japanese Tea Ceremony. The ceremony evolved and grew both more intricate and more exclusive, with students of the art receiving years of practice and training before they were allowed to perform it. The once-lowly leaf had been raised to the pinnacle of spiritual and social grace. In the words of Lafcadio Hearn, an historian and writer of Irish origin who emigrated to Japan in the late 19th Century, "The Tea ceremony requires years of training and practice to graduate in art . . . yet the whole of this art, as to its detail, signifies no more than the making and serving of a cup of tea. The supremely important matter is that the act be performed in the most perfect, most polite, most graceful, most charming manner possible."

I. The main purpose of this passage is to
   i. Give brief highlights from the history of the cultivation of tea.
   ii. Provide an anecdotal account how tea became a drink.
   iii. Highlight some important elements of the history of preparing and drinking tea.

II. The last sentence of the first paragraph serves to illustrate which of the following about tea?
   i. The mistake that led to tea drinking's ultimate elevation as a social grace.
   ii. The accidental and fortunate nature of how tea was discovered.
   iii. The link between tea and Zen Buddhist practice of pilgrimage.

III. Which of the following inferences may be drawn from the discussion of Lu Yu's work?
   i. Before 800AD, it was largely unknown how to cultivate tea.
   ii. Some people even today drink tea for reasons other than its physical benefits.
   iii. Drinking tea was primarily a Zen Buddhist practice until the late 700s.

62. The correct answers to I, II and III are
   A. i, ii, iii, respectively.
   B. iii, ii, ii, respectively.
   C. ii, i, iii, respectively.
   D. iii, ii, i, respectively.

3 marks

63. A goes to the market and meets a man. The man is the husband of the sister of his mother. How is the man related to A?

A. Brother.

B. Nephew.

C. Uncle.

D. Father.

1 mark

64. The second day of a month is Sunday. What will be the last day of the next month which has 31 days?

A. Friday.

B. Saturday.

C. Monday.

D. Information is inadequate.

1 mark

65. There are five buildings on the same side of a straight stretch of road, P,Q,R,S,T. Building P is to the right of Q. Building T is to the left of R and to the right of P. Building Q is to the right of S. Which building is exactly in the middle?

A. T.

B. P.

C. S.

D. Q.

1 mark

66. If CHAMPION is coded as HCMAIPNO, how can NEGATIVE be coded in that code?

A. ENAGITEV.

B. NEAGVEIT.

C. MGAETVIE.

D. EGAITEVN.

1 mark

67. On the basis of the given statement and few conclusions, choose the right statement which gives the correct conclusions.

Statements:

Some cabbages are tomatoes.

All tomatoes are guavas.

Conclusions:

I. All guavas are cabbages.

II. Some guavas are cabbages

A. Only I follows.

B. Only II follows.

C. Neither of I and II follows.

D. Both I and II follow.

1 mark

68. An accurate clock shows 7 a.m. Through how many degrees will the hour hand rotate when the clock shows 1 p.m.?

A. 154°.

B. 180°.

C. 170°.

D. 160°.

1 mark

69. A cubical cardboard box has edges of 1 meter. An ant wants to crawl from one corner on the floor of the box to the opposite corner on the roof. The shortest path for the ant is

A. $\sqrt{2}$ meters.

B. $\sqrt{3}$ meters.

C. $\sqrt{5}$ meters.

D. $\left(\sqrt{2}+1\right)$ meters.

2 marks

70. In a class of 100 students, the numbers of students participating in art, cricket, trekking and combinations of these are reported below.

Art: 40

Cricket: 28

Trekking: 48

Art and Trekking: 16

Trekking and Cricket: 14

Cricket and Art: 9

What is the maximum number of students taking part in all the three activities?

A. 23.

B. 16.

C. 9.

D. 14.

2 marks

****