Institute of Actuaries of India

ACET January 2021

Mathematics

1. The value of $\sum ij$ where the summation is over all i, j such that $0 \leq i, j \leq 10$ is

   A. 3025.

   B. 55.

   C. 110.

   D. 6050.

   1 mark

2. Let R be the relation defined on the set $A = \{1,2,3,4,5,6\}$ as $R = \{(x, y): y = x + 1\}$ then, R is

   A. Reflexive.

   B. Symmetric.

   C. Transitive.

   D. neither reflexive nor symmetric nor transitive.

   1 mark

3. If $f(x) = [x]$, the greatest integer less than or equal to x and $g(x) = |x - 1|$, then $f°g(-\frac{7}{2})$ is

   A. 2.

   B. 3.

   C. 4.

   D. 5.

   1 mark

4. Let $f: R \to R$ defined as

$$f(x) = \begin{cases} -1 \ if \ x < 0, \\ 0 \ if \ x = 0, \\ +1 \ if \ x < 0. \end{cases}$$

   Then $f(x)$ is

   A. one to one.

   B. onto.

C. one to one and onto.

D. neither one to one nor onto.

1 mark

5. A root of $x^3 - 2x + 5 = 0$ lies in the interval

A. $(-1,0)$.

B. $(-2,-1)$.

C. $(-3,-2)$.

D. $(-4,-3)$.

1 mark

6. The set S of real numbers such that $|1 - 2x| - 2x \geq 0$ is

A. $S = \left(-\infty, \frac{1}{4}\right)$.

B. $S = \left(-\frac{1}{4}, \frac{1}{4}\right)$.

C. $S = (-\infty, 0)$.

D. $S = \left(-\infty, \frac{1}{4}\right]$.

2 marks

7. Suppose $f(x)$ is a function having values as given in the following table.

x	1	2	3	4	5
$f(x)$	2	5	7	-	32

Based on the above information the value of $f(x)$ at $x = 4$, estimated from a cubic polynomial approximation, is

A. 10.

B. 14.

C. 16.

D. 20.

2 marks

8. The sum of squares of the zeroes of the polynomial expression $p(x) = x^2 + bx - 2$ is 5. Then, the possible values of b are

A. ± 1.

B. ± 2.

C. ± 3.

D. ± 4.

1 mark

9. The constant term in the expansion of $\left(\frac{4x^2}{3} - \frac{3}{2x}\right)^9$ is

A. $\binom{9}{6}\left(\frac{4}{3}\right)^6\left(-\frac{3}{2}\right)^3$.

B. $\binom{9}{3}\left(\frac{4}{3}\right)^3\left(-\frac{3}{2}\right)^6$.

C. $\binom{9}{3}\left(-\frac{4}{3}\right)^3\left(\frac{3}{2}\right)^6$.

D. $\binom{9}{6}\left(-\frac{4}{3}\right)^6\left(\frac{3}{2}\right)^3$.

2 marks

10. If $0 < x < \pi$ and $16^{\sin^2 x} + 16^{\cos^2 x} = 10$, then $\tan x$ is equal to

A. 0 or ∞.

B. 1.

C. $\frac{\pi}{6}$ or $\frac{\pi}{3}$.

D. $\frac{1}{\sqrt{3}}$ or $\sqrt{3}$.

3 marks

11. $\lim_{x\to\infty} \frac{\frac{1}{x^2} - 2\tan^{-1}\frac{1}{x}}{\frac{1}{x}}$ is

A. 2.

B. -2.

C. 0.

D. $\frac{1}{2}$.

1 mark

12. If $y = (\log_e x)^{\cos x}$, then $\frac{dy}{dx}$ is equal to

A. $(\log_e x)^{\cos x}\left[\frac{\cos x}{x\log_e x} - \sin x \log_e(\log_e x)\right]$.

B. $(\log_e x)^{\cos x}\left[\frac{\cos x}{x\log_e x} + \sin x \log_e(\log_e x)\right]$.

C. $(\log_e x)^{\cos x}\left[\frac{\cos x}{x\log_e x} - \cos x \log_e(\log_e x)\right]$.

D. $(\log_e x)^{\cos x}\left[\frac{\cos x}{x\log_e x} + \cos x \log_e(\log_e x)\right]$.

2 marks

13. Let $f(x) = x^2 - x + 1$. Then, $f(x)$ is

A. strictly increasing in $(-1,1)$.

B. strictly decreasing in $(-1,1)$.

C. neither strictly increasing nor strictly decreasing in $(-1,1)$.

D. a constant function in $(-1,1)$.

1 mark

14. The value of $\int \frac{x}{(x+1)(x+2)}\,dx$ is

A. $\log\frac{x^2}{|x+1|} + C$.

B. $\log\frac{(x+2)^2}{|x+1|} + C$.

C. $\log\frac{(x+2)^2}{(x+1)^2} + C$.

D. $\log\frac{x+2}{x+1} + C$.

2 marks

15. The value of $\int_{e^{-1}}^{e^2}\left|\frac{\log x}{x}\right|dx$ is

A. $\frac{5}{2}$.

B. e.

C. $\frac{2}{5}$.

D. $\frac{3}{2}$.

3 marks

16. If $\left|\vec{a}+\vec{b}\right| = \sqrt{29}$ and $\vec{a}\cdot\vec{b} = 5$, then $\left|\vec{a}-\vec{b}\right|$ is

A. -3.

B. 4.

C. $\sqrt{3}$.

D. 3.

1 mark

17. The area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \vec{\imath} - \vec{\jmath} + 3\vec{k}$ and $\vec{b} = 2\vec{\imath} - 7\vec{\jmath} + \vec{k}$ is

A. $15\sqrt{2}$ sq. units.

B. 450 sq. units.

C. $\sqrt{20}$ sq. units.

D. 65 sq. units.

1 mark

18. If P is a non-singular matrix of order n, then $det(kP)$ for $k \neq n$ is

A. $k\,\det(P)$.

B. $n^k\,\det(P)$.

C. $k^n\,\det(P)$.

D. $\det(P)$.

1 mark

19. If M is the matrix such that $|M| \neq 0$ and $M^2 - M + I = 0$, then M^{-1} is

A. $I + M$.

B. $I - M$.

C. $M^2 - I$.

D. $M^2 + I$.

1 mark

20. If

$$P(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

then $P(x)P(y)$ is equal to

A. $P(xy)$.

B. $P(x - y)$.

C. $P(x + y)$.

D. the identity matrix.

2 marks

Statistics

21. In a class of 56 students, 28 opted for Mathematics, 30 opted for Biology and 22 opted for both Mathematics and Biology. If one of these students is selected at random, then probability that the student has opted neither Mathematics nor Biology is

    A. $\frac{9}{14}$.

    B. $\frac{5}{14}$.

    C. $\frac{3}{28}$.

    D. $\frac{1}{7}$.

1 mark

22. A man decided to visit four cities C_1, C_2, C_3 and C_4 in a random order. The probability that he visits C_1 before C_2 and C_2 before C_3 is

    A. $\mathbf{\frac{1}{6}}$.

    B. $\mathbf{\frac{1}{8}}$.

    C. $\mathbf{\frac{1}{3}}$.

    D. $\mathbf{\frac{1}{12}}$.

1 mark

23. If A and B are two events such that $(A \cap B) = \frac{1}{4}$, $P(B) = \frac{1}{3}$. Then $P(A \cup B^c)$

    A. is $\frac{5}{12}$.

    B. is $\frac{1}{12}$.

    C. is $\frac{11}{12}$.

    D. cannot be computed from the given information.

1 mark

24. For two events E and F, $P(E|F) > P(E)$, then which of the following is correct?

A. $P(F|E) < P(F)$.

B. $P(F|E) = P(F)$.

C. $P(E|F) + P(F|E) < P(E) + P(F)$.

D. $P(F|E) > P(F)$.

1 mark

25. Let A_1, A_2, A_3 be independent events with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$, respectively. Then $P(A_1 \cup A_2 \cup A_3)$ equals

A. $\frac{9}{32}$.

B. $\frac{23}{32}$.

C. $\frac{31}{32}$.

D. 1.

1 mark

26. A man is known to speak truth 80% of the times. He throws a die and reports that the number appeared is greater than 4. The probability that it is actually a number greater than 4 is

A. $\frac{4}{5}$.

B. $\frac{2}{5}$.

C. $\frac{2}{3}$.

D. $\frac{4}{7}$.

3 marks

27. The stem and leaf plot of lifetimes in days of a set of 40 light bulbs is shown below.

Stem	Leaf
2	1 3
3	0 2 3 5 7 8
4	0 1 2 3 4 5 5 6 7 8 9
5	0 3 4 6 7 8 9
6	2 3 4 7 8
7	0 4 7 9
8	1 4 6
9	3 2

The median lifetime of light bulbs is

A. 49 days.

B. 49.5 days.

C. 50 days.

D. 51.5 days.

1 mark

28. Suppose there are n observations on the variable x, out of which n_1 observations are one and remaining are zero. Let $y = x^2$, then the variance of the values of y is

A. n_1/n^2.

B. n_1^2/n^2.

C. $n_1(n - n_1)/n^2$.

D. $n_1^2(n - n_1)^2/n^2$.

2 marks

29. Suppose there are three departments D_1, D_2 and D_3 in an organization. The numbers of workers employed in these departments are 50, 50 and 100, respectively. The mean earnings in a certain month per worker in the three departments are Rs. 15000, Rs. 13000 and Rs. 18500, respectively. The mean earning per worker of the whole organization is

A. Rs. 15500.

B. Rs. 16250.

C. Rs. 16500.

D. Rs. 17000.

1 mark

30. Consider the frequency distribution of weights (in kg) of 250 persons.

Weights in Kg	Number of persons
35.5 – 40.5	8
40.5 – 45.5	12
45.5 – 50.5	25
50.5 – 55.5	40
55.5 – 60.5	45
60.5 – 65.5	60
65.5 – 70.5	46
70.5 – 75.5	12
75.5 – 80.5	2

Which of the following statement is true for the distribution of weight?

A. Mode of the distribution is 60.5.

B. The distribution is positively skewed.

C. The median value lies in the class interval 60.5 – 65.5.

D. The coefficient of skewness is negative.

2 marks

31. If the range of a set of observations is 20, which of the following statements is true?

A. If each observation is multiplied by -3, then the range of the new observations becomes 60.

B. If each observation is increased by 5, then the range of the new observations becomes 25.

C. If each observation is multiplied by 3 and then 5 added to it, then the range of the new observations becomes 65.

D. If each observation is multiplied by 5 and then 3 subtracted from it, the range of the new observations will be 97.

1 mark

32. A discrete random variable X follows uniform distribution over the values 7, 12, 15, 20, 23, 25, 30. Then $P(X > 18|X < 26)$ equals

A. $\frac{1}{2}$.

B. $\frac{4}{7}$.

C. $\frac{2}{3}$.

D. $\frac{3}{4}$.

1 mark

33. A vendor supplies an electronic item in lots of 25. A buyer, before taking a lot, tests a random sample of 4 items and if all are good, he accepts the lot. The probability that he accepts a lot containing 5 defective items is

A. 0.2.

B. 0.8.

C. 0.0016.

D. 0.4096.

1 mark

34. The number of misprints per page of a book (X) follows the Poisson distribution such that $P(X = 1) = P(X = 2)$. If the book contains 500 pages, the expected number of pages containing at most one misprint is

A. $500e^{-2}$.

B. $1000e^{-2}$.

C. $1500e^{-2}$.

D. $500(1-3e^{-2})$.

2 marks

35. Let X be a continuous random variable with probability density function

$$f(x)=\begin{cases}\lambda e^{-\lambda(x-\alpha)}, & x\geq\alpha\\ 0, & x<\alpha\end{cases}$$

The mean of the distribution is

A. $\lambda+\alpha$.

B. $\frac{\alpha\lambda+1}{\lambda}$.

C. $\frac{\alpha\lambda+1}{\alpha}$.

D. $\frac{\alpha+\lambda}{\alpha\lambda}$.

2 marks

36. If the 90th percentile of the standard normal distribution is 1.65. Then the 90th percentile of the normal distribution with mean 20 and variance 4 is

A. 21.65.

B. 23.30.

C. 26.60.

D. 36.30.

2 marks

37. Let X be a continuous random variable with pdf $f(x)=0.5,\ -1\leq x\leq 1$. The skewness of the distribution is

A. 1.

B. 0.5.

C. -1.

D. 0.

1 mark

38. The marginal distributions of X and Y are given in the following table.

y \ x	1	2	Total
0			0.7
1		p	0.3
Total	0.7	0.3	1

The value of p for which X and Y have correlation 0.1 is

A. 0.111.

B. 0.09.

C. 0.15.

D. 0.22.

3 marks

39. Let ρ_{XY} be the correlation coefficient between X and Y. If $U = aX + b$ and $V = cY + d$, where a, b, c and d are constants. Let ρ_{XV} be the correlation coefficient between X and V. Then

A. $\rho_{XV} = \rho_{XY}$ if $c > 0$.

B. $\rho_{XV} = \rho_{XY}$ if $c < 0$.

C. $\rho_{XV} = c\rho_{XY}$.

D. $c\rho_{XV} = \rho_{XY}$.

1 mark

40. For the two variables x and y the equations of two regression lines are $4x - 5y + c = 0$ and $20x - 9y - d = 0$, where c and d are some constants. The ratio of variances of y and x is

A. $4c/d$.

B. d/c.

C. $16/9$.

D. $25/16$.

2 marks

Data Interpretation

Answer Questions 41 and 42 based on the data given in the following table. The table shows the numbers (f) of candidates obtaining marks x or higher in a certain examination, where the total number of candidates is 250.

x	25	30	40	50	60	70	80	90	100
f	250	240	200	150	120	80	20	10	0

41. The number of candidates who have scored less than 60 is

    A. 100.

    B. 120.

    C. 130.

    D. 170.

1 mark

42. Suppose the pass mark is 40. Then the percentage of candidates who have passed the examination is

    A. 40.

    B. 65.

    C. 70.

    D. 80.

1 mark

Answer Questions 43-45 based on the data given the following table. The table shows the number of students appeared in class X board examination in a state for the years from 2011 to 2019.

Year	Male (in lakhs)	Female (in lakhs)	Total (in lakhs)
2011	3.40	3.20	6.60
2012	3.38	3.32	6.70
2013	3.45	3.40	6.85
2014	3.53	3.52	7.05
2015	3.55	3.65	7.20
2016	3.60	3.85	7.45
2017	3.70	4.00	7.70
2018	3.85	4.20	8.05
2019	3.95	4.25	8.20

43. The annual percentage increase of candidates was the largest from

    A. 2013 to 2014.

B. 2015 to 2016.

C. 2016 to 2017.

D. 2017 to 2018.

2 marks

44. More than 52% of the students appeared for the exam were female in the year

A. 2016.

B. 2017.

C. 2018.

D. 2019.

2 marks

45. The number of female candidates surpassed the number of male candidates by 20000 or more in the years

A. 2016, 2017, 2018, 2019.

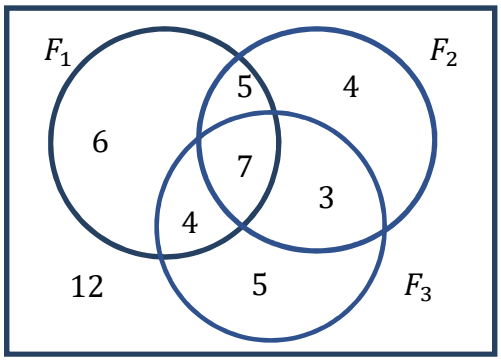
B. 2017, 2018, 2019.

C. 2016, 2017, 2018.

D. 2018, 2019.

1 mark

Answer Questions 46-48 based on the data in the Venn diagram given below. A group of students were surveyed to find out which of the three types of food F_1, F_2 and F_3, they liked.



46. The number of students who liked exactly one type of food is

A. 34.

B. 22.

C. 7.

D. 15.

1 mark

47. The number of students who liked F_1 and F_2 but not F_3 is

A. 5.

B. 7.

C. 9.

D. 12.

1 mark

48. The number of students who liked F_1 or F_3 is

A. 41.

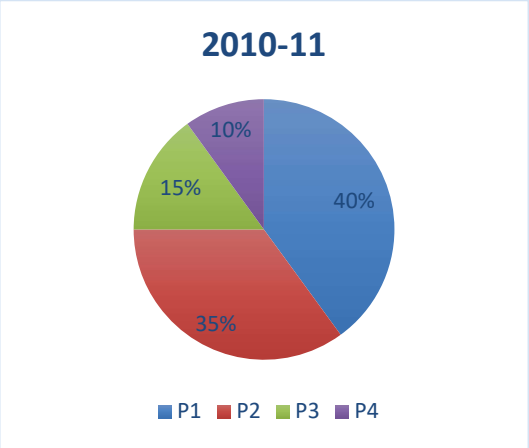
B. 30.

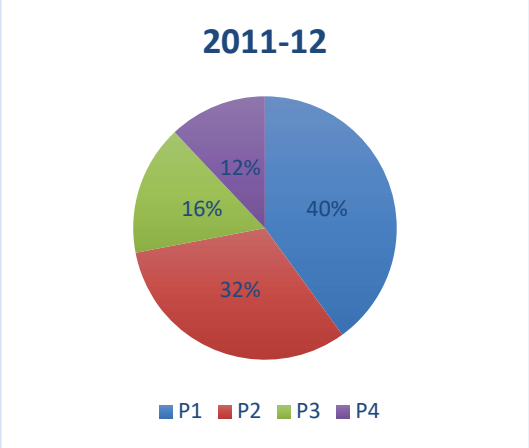
C. 19.

D. 11.

1 mark

Answer Questions 49 and 50 based on the pie charts given below. The pie charts show export of four products for the years 2010-11 and 2011-12. The total export in 2010-11 was Rs. 5000 crore and increased by 7% in 2011-12.





49. Total export of the products P3 and P4 in two years is

A. Rs. 2846 crores.

B. Rs. 2748 crores.

C. Rs. 2737 crores.

D. Rs. 2825 crores.

2 marks

50. From 2010-11 to 2011-12, total value of export of the products P1 and P2

A. increased by 2.72%.

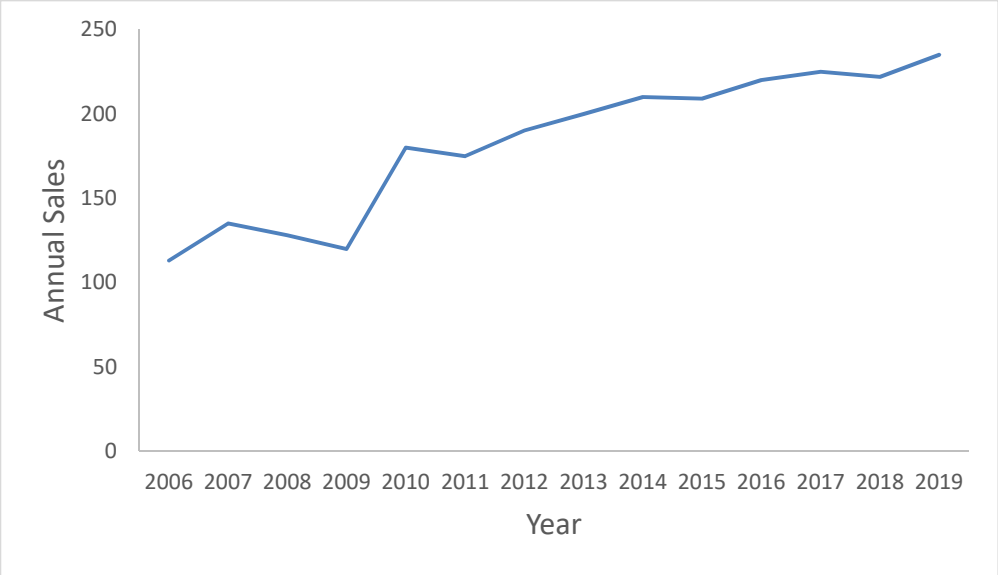
B. increased by 1.25%.

C. decreased by 3.00%.

D. decreased by 2.50%.

2 marks

51. The line diagram shows annual sales (in crores of rupees) of a company from 2006 to 2019.



The number of years in which annual sales decreased over those of the corresponding previous years is

A. 2.

B. 3.

C. 4.

D. 5.

1 mark

English

52. Agnostic is a person who
    A. is a non-believer in the existence of God.
    B. is not sure about the existence of God.
    C. is a firm believer in the existence of God.
    D. leads a life full of agony.

1 mark

53. Fanatic is a person who
    A. has excessive zeal for a cause.
    B. acts against religion.
    C. cannot express himself freely.
    D. is mentally ill.

1 mark

54. A person who studies the elections and trends in voting is called
    A. an electologist.
    B. a psephologist.
    C. an electorate.
    D. a statistician.

1 mark

55. A hidden storage of items of the same type is called a
    A. locker.
    B. vault.
    C. burrow.
    D. cache.

1 mark

56. A synonym of “Acumen” is
    A. Preference.
    B. Perfection.
    C. Accuracy.
    D. Sharpness.

1 mark

57. A synonym of “Endeavour” is

A. Effort.

B. Ending.

C. Endurance.

D. Cease.

1 mark

58. An antonym of “Precarious” is

A. Post facto.

B. Confused.

C. Stable.

D. Unsure.

1 mark

59. An antonym of “Quarantine” is

A. Proliferate.

B. Celebrate.

C. Desegregate.

D. Entertain.

1 mark

60. Select the incorrect sentence.

A. I have seen him yesterday.

B. I saw him yesterday.

C. I have seen him.

D. He was seen by me yesterday.

2 marks

61. Select the incorrect sentence.

A. Let us not pluck the flowers.

B. Let the flowers not be plucked.

C. Let the flowers not been plucked.

D. Let the flowers be plucked.

2 marks

Read the passage below and answer Question No. 11.

Coronavirus is a group of Ribonucleic Acid (RNA) viruses that cause diseases in mammals and birds. These viruses cause respiratory tract infections. Mild illnesses include some cases of the common cold, while more lethal varieties can cause SARS, MERS and COVID-19. Symptoms in different species vary: in chickens, they cause an upper respiratory tract disease, while in cows and pigs they cause diarrhea. The name "coronavirus" is derived from Latin corona, meaning "crown" or "wreath". Coronaviruses were first discovered in the 1930s when an acute respiratory infection of domesticated chickens was shown to be caused by the infectious bronchitis virus. The infection of new-born chicks was characterized by gasping and listlessness. Human coronaviruses were discovered in the 1960s. Infected carriers are able to shed viruses into the environment. The interaction of the coronavirus spike protein with its complementary cell receptor is central in determining the tissue tropism, infectivity, and species range of the released virus. They are transmitted from one host to another host, depending on the coronavirus species, by either an aerosol, fomite, or fecal-oral route. Human coronaviruses infect the epithelial cells of the respiratory tract, while animal coronaviruses generally infect the epithelial cells of the digestive tract. For example, SARS coronavirus infects, via an aerosol route, the human epithelial cells of the lungs. Transmissible gastroenteritis coronavirus (TGEV) infects, via a fecal-oral route, the pig epithelial cells of the digestive tract by binding to the alanine aminopeptidase receptor.

I. COVID-19 and SARS are caused by
   i. The same Coronavirus
   ii. RNA viruses
   iii. Acute Respiratory Infection

II. SARS infects
   i. human epithelial cells
   ii. pig epithelial cells
   iii. both

III. Coronavirus may cause
   i. diarrhea in pigs
   ii. respiratory tract disease in cows
   iii. both

62. The correct answers to I, II and III are
   A. i, ii, iii, respectively.
   B. ii, i, i, respectively.
   C. ii, i, iii, respectively.
   D. i, iii, iii, respectively.

3 marks

Logical Reasoning

63. A man says to a woman, "You are the only child of the father- in- law of my father's only child." How is the woman related to the man?

    A. Sister in law.

    B. Granddaughter.

    C. Wife.

    D. Daughter.

1 mark

64. In a certain substitution code language, "NUMBER" is written as "156897" and "BARREN" is written as "847791". How is "RUBBER" written in that

    A. 756697.

    B. 758897.

    C. 795957.

    D. 795579.

1 mark

65. Statements:

    All Forests are Cars.

    All Lights are Cars.

    All Vegetables are Lights.

    Conclusions:

    I. Some Vegetables are Forests
    II. Some Cars are Lights
    III. Some Cars are Forests
    IV. All Vegetables are Cars

    A. All conclusions follow.

    B. Only II, III and IV follow.

    C. Only I, II and III follow.

    D. Only I, II and IV follow.

1 mark

66. Identify a meaningful order of the following.

1. Index
2. Bibliography
3. Chapters
4. Preface

A. 1, 3, 2, 4.

B. 3, 1, 4, 2.

C. 4, 3, 2, 1.

D. 4, 1, 3, 2.

1 mark

67. A cube of side n is painted brown on all its sides. It is then cut into n^3 identical cubes. How many of the smaller cubes will have three of its sides painted?

A. 4.

B. $4n$.

C. 8.

D. $8n$.

1 mark

68. Year 1895 is having a same calendar as that of the year X. Which of the following is a possible value of X?

A. 1900.

B. 1901.

C. 1902.

D. 1903.

1 mark

69. Six persons are sitting around a circular table. S is facing R who is to the left of A and to the right of P. A is to the left of D. Y is to the left of P. If D exchanges his seat with Y and P exchanges with R, who will be sitting to the left of D?

A. Y.

B. R.

C. A.

D. S.

2 marks

70. Wasim, Aaqib and Imran belong to three different tribes of people. The three tribes are *Sachhas* (those who always speak the truth), *Jhootas* (who always lie) and *Badlas* (those who alternatively speak the truth and lie).

Wasim said that Aaqib is not a *Badla*.

Imran said that Wasim is a *Badla*.

Who among the three is a *Jhoota*?

A. Aaqib.

B. Wasim.

C. Imran.

D. Cannot be determined.

2 marks