Model Question Paper

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

1) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*
2) *Part A has 10 Multiple choice questions, 5 Fill in the blanks and 5 Very Short Answer questions of 1 mark each.*
3) *Part A should be answered continuously at one or two pages of Answer sheet and Only first answer is considered for the marks in subsection I and II of Part A.*
4) *Use the graph sheet for question on linear programming in PART E.*

PART A

I. Answer ALL the Multiple Choice Questions **10 ×1 = 10**

1. The identity element for the binary operation $*$ if $a * b = \frac{ab}{4}$,$\forall$ a, b $\in$Q

   (A) 0 (B) 4 (C) 1 (D) not exist.

2. If $\cot^{-1} x = y$, then

   (A) $0 \le y \le \pi$ (B) $0 < y < \pi$ (C) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

3. For 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$ then A is equal to

   A) $\begin{bmatrix} 2 & 3 \\ \frac{1}{2} & \frac{9}{2} \end{bmatrix}$ B) $\begin{bmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{bmatrix}$ C) $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

4. If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ then the value of x is equal to

   (A) 2 (B) 4 (C) 8 (D)$\pm 2\sqrt{2}$.

5. Left hand derivative of $f(x) = |x|$ at x = 0 is.

   (A) 1 (B) -1 (C) 0 (D)does not exist.

6. $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$=

   (A) $e^x + c$ (B) $\frac{e^x}{x} + c$ (C) $\frac{e^x}{x^2} + c$ (D) $\frac{-e^x}{x} + c$

7. The *projection* of the vector $\overrightarrow{AB}$ on the directed line l, if angle $\theta = \pi$ will be.

   (A) *Zero* vector. (B) $\overrightarrow{AB}$ (C) $\overrightarrow{BA}$ (D) *Unit* vector.

8. The equation of xy- plane is

   (A) $x = 0$ (B) $y = 0$ (C) $x = 0$ and $y = 0$ (D) $z = 0$

9. The corner points of the feasible region determined by the following system of linear inequalities: $2x + y \le 10$, $x + 3y \le 15$, $x, y \ge 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = ax + by$, where $a, b > 0$. Condition on a and b so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $a = b$ (B) $a = 2b$ (C) $a = 3b$ (D) $b = 3a$

10.If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is

(A) 0 (B) $\frac{1}{2}$ (C) not defined (D) 1

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket.
$\left(\frac{5}{2}, \frac{1}{36}, \frac{1}{3}, 0, 2\right)$ **5×1 = 5**

11. For a square matrix A in matrix equation AX = B. If $|A| = 0$ and $(adj\,A)\,B \neq 0$, then there exists …… solution.

12. The order of the differential equation. $2x^2\left(\frac{d^2y}{dx^2}\right) - 3\left(\frac{dy}{dx}\right) + y$ *is* … ….

13. Sum of the intercepts cut off by the plane $2x + y - z = 5$ is ………

14. The slope of the normal to the curve $y = 2x^2 - 3\sin x$ at $x = 0$ is ……

15. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is…..

III. Answer all the following questions **5×1 = 5**

16. Define a bijective function.

17. Find the derivative of the function $\sec(\tan\sqrt{x})$ with respect to x.

18. Define feasible solutions in a linear programming problem.

19. Find $\int \frac{1-sinx}{cos^2x}\,dx$.

20. Define Negative of a Vector.

PART B

Answer any NINE questions: **9× 2 = 18**

21. Find gof and fog, if $f : R \to R$ and $g : R \to R$ are given by $g(x) = x^{\frac{1}{3}}$ and $f(x) = 8x^3$.

22. Prove that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in R$.

23. If $\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1}x\right\} = 1$, find x.

24. Find the area of the triangle whose vertices are $(2,7),(1,1)$ and $(10,8)$ using determinants.

25. Find $\frac{dy}{dx}$, if $2x+3y=\sin x$.

26. If $y=x^{\sin x}$, $x>0$, find $\frac{dy}{dx}$.

27. Find the local maximum value of the function $g(x)=x^3-3x$.

28. Evaluate $\int \sin 3x \cos 4x\, dx$.

29. Evaluate $\int_0^{\pi/2} \cos 2x\, dx$.

30. Form the differential equation representing the family of curves $y = mx$, where, m is arbitrary constant.

31. Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

32. Find a vector in the direction of the $\vec{a}=\hat{i}-2\hat{j}$ that has magnitude 7 units.

33. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$.

34. Find the probability distribution of number of heads in two tosses of a coin.

PART C

Answer any NINE questions: **9×3 = 27**

35. Show that the relation R in R defined as $R=\{(a,b):a\le b\}$, is reflexive and transitive but not symmetric.

36. Solve: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

37. Express $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrices.

38. If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0<x<1$ then find $\frac{dy}{dx}$.

39. Verify Rolle's theorem for the function $f(x)=x^2+2x-8$, $x\in[-4,2]$.

40. Find the intervals in which the function f given by $f(x)=4x^3-6x^2-72x+30$ is
    (i) strictly increasing; (ii) strictly decreasing.

41. Find $\int x\cos x\, dx$.

42. Find $\int \frac{x}{(x+1)\,(x+2)}dx$.

43. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis in the first quadrant.

44. Find the equation of the curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

45. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ prove that $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{d}\right] = 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

46. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

47. Find the vector equation of the plane passing through the intersection of the planes 3x - y + 2z – 4 = 0 and x + y + z -3 = 0 and the point (2, 2, 1).

48. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART D

Answer any FIVE questions: **5× 5 = 25**

49. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of function f, is invertible. Find the inverse of f.

50. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, calculate AB, AC and A(B + C). Verify that $A(B+C) = AB + AC$.

51. Solve the following system of equations by matrix method:
$x - y + z = 4$; $x + y + z = 2$ and $2x + y - 3z = 0$.

52. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y_2 + x y_1 + y = 0$.

53. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

54. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x and evaluate $\int \frac{dx}{\sqrt{7 - 6x - x^2}}$.

55. Find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

56. Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 4x \cdot \operatorname{cosec} x$, $x \neq 0$,

given that $y = 0$ when $x = \frac{\pi}{2}\cdot$

57. Derive the equation of the line in space passing through two given points both in vector and Cartesian form.

58. If a fair coin is tossed 10 times, find the probability of

(i) exactly six heads (ii) at least six heads.

PART E

Answer the following questions:

59. Maximize; $z = 4x + y$ subject to constraints $x + y \leq 50,\ 3x + y \leq 90,\ x \geq 0,\ y \geq 0$ by graphical method.

OR

Prove that $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$ and hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\, dx$. **6**

60. Find the value of k so that the function $f(x) = \begin{cases} kx+1, & if\ \ x \leq 5 \\ 3x-5, & if\ \ x > 5 \end{cases}$, at $x = 5$ is a continuous function.

OR

Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. **4**