Institute of Actuaries of India
ACET June 2019
Mathematics

1. The value of $\frac{\log_{16} 625}{\log_8 125}$ is

   A. 1.
   B. 2.
   C. 3.
   D. 4.   1 mark

2. The domain of $f(x) = \sqrt{\frac{x+2}{x-1}}$ as a valid function is

   A. $(-\infty, -2) \cup [1, \infty)$.
   B. $(-\infty, -2) \cup (1, \infty)$.
   C. $(-\infty, -2] \cup [1, \infty)$.
   D. $(-\infty, -2] \cup (1, \infty)$.   2 marks

3. The value of $\cos^2 5° + \cos^2 10° + \cos^2 15° + \cdots + \cos^2 85° + \cos^2 90°$ is

   A. $7\frac{1}{2}$.

   B. $8\frac{1}{2}$.

   C. $9\frac{1}{2}$.

   D. 10.   2 marks

4. In trying to find the root of the equation $x^2 - 5 = 0$, using the Newton- Raphson method and starting with the initial value 2, the solution after one iteration is

   A. 2.50.
   B. 2.25.
   C. 2.20.
   D. 1.75.   1 mark

5. The Maclaurin's series for $f(x) = \frac{1}{1+x}$, x real , is

   A. $1 - x + x^2 - x^3 + \cdots$.
   B. $1 + x + x^2 + x^3 + \cdots$.
   C. $1 + x - x^2 + x^3 - \cdots$.
   D. $1 - x + 2x - 3x + \cdots$.   1 mark

6. If $\dfrac{x^2+1}{(x-1)^2(x+3)} = \dfrac{A}{(x-1)^2} + \dfrac{B}{(x-1)} + \dfrac{C}{(x+3)}$, then $A + 4B$ is equal to

A. 0.
B. 1.
C. 2.
D. 3.   2 marks

7. If $\binom{16}{2r} = \binom{16}{3r+1}$, then the value of $\binom{4r}{3}$ is

A. 1.
B. 4.
C. 16.
D. 220.   1 mark

8. The constant term in $\left(2x^2 + \dfrac{1}{x}\right)^9$ is

A. 268.
B. 136.
C. 672.
D. 84.   1 mark

9. $\lim\limits_{x \to 4} \dfrac{\log_e x - \log_e 4}{x - 4}$ is

A. 4.
B. $\dfrac{1}{4}$.
C. 0.
D. 1.   1 mark

10. The function $f(x) = \dfrac{2x}{\log_e x}$, $x > 0$ is an increasing function of x in

A. (e, ∞).
B. $(1, e)$.
C. $(-\infty, e)$.
D. $(0, 1)$.   1 mark

11. If $x = r \cos \theta$ and $y = r \sin \theta$, then $\frac{\partial r}{\partial x}$ is equal to

    A. $\sin \theta$.
    B. $\cos \theta$.
    C. $\tan \theta$.
    D. $\cot \theta$.   1 mark

12. $\int x^3 \log_e x \, dx$ is equal to

    A. $\frac{x^4}{4}\left(\log_e x - \frac{1}{4}\right) + c$.
    B. $\frac{x^4}{4}\left(\log_e x + \frac{1}{4}\right) + c$.
    C. $x^4 \left(\log_e x - \frac{1}{4}\right) + c$.
    D. $\frac{x^3}{3}\left(\log_e x - \frac{1}{4}\right) + c$.   2 marks

13. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ is

    A. $\frac{3}{2}$.
    B. 2.
    C. $\frac{2}{3}$.
    D. 1.
                  2 marks

14. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{2} e^{-|x|} dx$ is

    A. 0.
    B. $\frac{1}{2}$.
    C. 1.
    D. e.   1 mark

15. The vectors $\vec{a} = 2\vec{i} + m\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$ are perpendicular to each other. Then the value of m is

    A. 1.
    B. 0.
    C. $\frac{2}{3}$.
    D. $\frac{3}{2}$.   1 mark

16. The projection of $\vec{a} = 2\vec{i} + 3\vec{j} - 2\vec{k}$ on $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ is

    A. $\sqrt{14}$.

    B. $\frac{2}{\sqrt{14}}$.

    C. $\frac{1}{7}$.

    D. $\frac{1}{11}$.   1 mark

17. The rank of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ is

    A. 0.
    B. 1.
    C. 2.
    D. 3.   1 mark

18. If $A(x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $[A(x)]^{-1}$ is

    A. $A(-x)$.
    B. $A(x)$.
    C. $-A(x)$.
    D. $-A(-x)$.   3 marks

Statistics

19. A shipment of 10 television sets includes four that are defective. In how many ways can a hotel purchase four of these sets and receive at most two defective sets?

   A. 195.
   B. 170.
   C. 90.
   D. 185.   1 mark

20. Let A and B be two events with $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B) = 0.1$. Then $P(A \cap B | A \cup B)$ is

   A. 1/7.
   B. 1/5.
   C. 3/5.
   D. 1/9.   1 mark

21. Two methods, A and B are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However B is more expensive and hence is used only 30% of the time. A worker is taught the skill by one of the methods but failed to learn it correctly. What was the probability that she was taught by method A?

   A. 7/13.
   B. 3/17.
   C. 14/17.
   D. 6/13.   2 marks

22. Consider the following frequency distribution of a variable

x	-3	-2	-1	0	1	2	3	Total
Frequency	3	7	12	16	12	7	3	60

   Which of the following is true?

   A. Mean $= 0$, Mode $= 0$, Standard deviation $= 0$.
   B. Mean $\neq 0$, Mode $= 0$, Standard deviation $= 0$.
   C. Mean $= 0$, Mode $= 0$, Standard deviation $\neq 0$.
   D. Mean $= 0$, Mode $\neq 0$, Standard deviation $= 0$.   2 marks

23. The median of 20 observations is 12.8. Suppose each observation is first multiplied by 4 and 10 is added after multiplication. The median of the new observations is

A. 12.8.
B. 61.2.
C. 51.2.
D. 22.8.   1 mark

24. The probability distribution of a discrete random variable X is given in the table below.

x	0	1	2	3	4	5
$P(X = x)$	0.1	0.3	0.15	0.25	0.15	0.05

Then $P(1 < X \leq 4)$ is

A. 0.55.
B. 0.85.
C. 0.70.
D. 0.40.   1 mark

25. Suppose X follows the Poisson distribution with mean 3. Then $E(40 - 2X - X^2)$ is

A. 31.
B. 22.
C. 28.
D. 25.   2 marks

26. Let X be a random variable with the probability distribution
$$P(X = x) = \binom{10}{x} \times 0.5^{10}, \ x = 0, 1, \dots, 10.$$

Then variance of $Y = X/10$ is

A. 0.25.
B. 2.5.
C. 0.05.
D. 0.025.   1 mark

27. Suppose the random variable X has a probability density function
$$f(x) = \begin{cases} kx^3 \, e^{-x/2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$
The value of k is

A. 1/96.
B. 96.
C. 8/3.
D. 1/4.   2 marks

28. An aptitude test administered to bus driver trainees requires a series of operations to be performed in quick succession. Suppose the time needed to complete the test is normally distributed with mean 50 minutes and standard deviation 12 minutes. The probability that the time required to complete a test exceeds 75 minutes is

   A. More than 0.1.
   B. Between 0.05 and 0.1.
   C. Between 0.025 and 0.05.
   D. Less than 0.025.   2 marks

29. Suppose X_1 and X_2 are independent random variables with $E(X_1) = 0$, $\text{Var}(X_1) = 0.5$ $E(X_2) = 1$ and $\text{Var}(X_2) = 0.8$. Then $\text{Var}(X_1 X_2)$ is

   A. 0.04.
   B. 0.90.
   C. 1.30.
   D. 0.72.   3 marks

30. The correlation coefficient between X and Y is 0.3. Then the correlation coefficient between $-1.5X$ and $2Y + 3$ is

   A. -0.9.
   B. -0.3.
   C. 0.3.
   D. 0.9.   1 mark

31. Suppose the regression line of y on x is $x + 3y - 5 = 0$ and regression line of x on y is $4x + 3y - 8 = 0$. Then $(\bar{x}, \bar{y})$ is

   A. (4/3, 1).
   B. (1, 4/3).
   C. (1, 1).
   D. (2, 3/5).   3 marks

Data Interpretation and Data Visualization

Answer the questions 32-34 based on the data given in the following table.

The percentiles for family income in a country in 2015 are given below.

Percentile	Income (in $)
1	0
10	15000
25	28000
50	52000
75	96000
90	140000
99	450000

32. The median family income is

    A. 50000.
    B. 62000.
    C. 52000.
    D. 40000.   1 mark

33. The interquartile range of income distribution is

    A. 68000.
    B. 450000.
    C. 52000.
    D. 62000.   1 mark

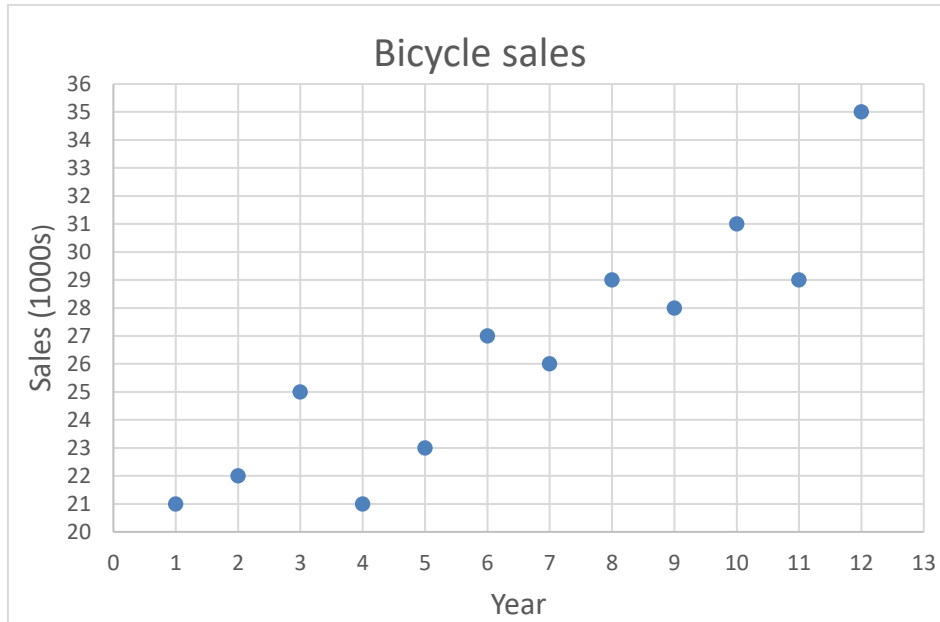34. The percentage of families with income between 52000 and 140000 is

    A. 35.
    B. 45.
    C. 50.
    D. 40.   1 mark

Answer questions 35 and 36 based on the following graph.

The graph provides data on sales of bicycles for a particular manufacturer over the past 12 years.

35. In how many years the sales decrease over the previous year's sales
    A. 5.
    B. 4.
    C. 3.
    D. 2.   1 mark

36. The percentage of sales increase over the previous year is maximum in the year
    A. 3.
    B. 6.
    C. 12.
    D. 10.   1 mark

Answer questions 37 and 38 based on the following problem.

An organization has 80 employees. Of those 80 employees, 35 employees have opted an insurance policy P_1, 45 have opted insurance policy P_2 and 15 have opted neither.

37. The number of employees who have opted both P_1 and P_2 is

    A. 65.
    B. 15.
    C. 80.
    D. 0.   2 marks

38. The number of employees who have opted only P_1

    A. 15.
    B. 35.
    C. 20.
    D. None of the above.   1 mark

English

39. The opposite of "Foremost" is:
    A: Backward.
    B: Unimportant.
    C: Afterwards.
    D: Almost.   1 mark

40. The opposite of "Coercive" is:
    A: Powerful.
    B: Opinionated.
    C: Gentle.
    D: Liberal.   1 mark

41. The opposite of "Heresy" is:
    A: Conformity.
    B: Heterodoxy.
    C: Mindful.
    D: Authentic.   1 mark

42. Synonym of "Absolute" is:
    A: Section.
    B: Partial.
    C: Complete.
    D: Half.   1 mark

43. Synonym of "Pandemonium" is
    A: Chaos.
    B: Great joy.
    C: Loud noise.
    D: Randomness.   1 mark

44. Synonym of "Aesthetic" is
    A: Well-managed.
    B: Complete.
    C: Artful.
    D: Ugly.   1 mark

45. An anarchist is a person who
    A: is against a particular religion.
    B: always suspects others.
    C: Questions the need for any form of government authority.
    D: is appointed by two parties to solve a dispute.   1 mark

46. A cosmopolitan is a person who
    A: has experience of many different parts of the world.
    B: can speak all languages.
    C: has knowledge of all topics.
    D: is urban.   1 mark

47. A person who talks in sleep is called as
    A: somniloquist.
    B: altruist.
    C: slipperwort.
    D: somnambulist.   1 mark

Choose the most appropriate word to fill in the blank in Questions 10 – 12.

48. On the basis of inadequate attendance, he was debarred _____ appearing in examination.
    A: from.
    B: to.
    C: for.
    D: of.   1 mark

49. Bananas are sold by ____ dozen.
    A: a.
    B: an.
    C: the.
    D: per.   1 mark

50. This story will make a good subject ____ a movie.
    A: of.
    B: in.
    C: for.
    D: to.   1 mark

51. Entrepreneur is related to Profit as Laborer is related to:
    A: Loan.
    B: Compulsion.
    C: Work.
    D: Wages.   1 mark

52. Caption is related to Photograph as Annotate is related to:
    A: Novel.
    B: Law.
    C: Text.
    D: Film.   1 mark

53. Venerate is related to Worship as Extol is related to:
    A: Recommend.
    B: Glorify.
    C: Compliment.
    D: Homage.   1 mark

54. Select the right meaning of the sentence: "The best of friends must part."
    A: One should not assume success prior to actually achieving it.
    B: When things seem at their worst, they may just begin to improve.
    C: It is what we do that matters and not just what we say.
    D: Nothing is permanent.   2 marks

55. Select the right meaning of the sentence: "Absence makes the heart grow fonder."
    A: Our affection increases after parting from our loved ones.
    B: Live within your means.
    C: What pleases the sight varies from one person to another.
    D: Absence of kindness in one's behavior brings restlessness.   2 marks

56. Meaning of the phrase "To Cry Wolf" is:
    A: To give false alarm.
    B: To listen eagerly.
    C: To turn pale.
    D: To cry relentlessly.   2 marks

57. Meaning of the phrase "To end in smoke" is:
    A: To disappear.
    B: To ruin something.
    C: To fail.
    D: To escape.   2 marks

58. Select the correct sentence:
        A: Sorry to have kept you. How long are you waiting?
        B: Sorry to keep you. How long have you been waiting?
        C: My apologies for the delay. How long have you been waiting?
        D: My apologies for delaying. Since how long are you waiting?   2 marks

59. Select the correct sentence:
        A: How on earth it had been better?
        B: How on earth had it been better?
        C: How had it been better on earth?
        D: How on earth had been it better?   2 marks

60. Select the correct sentence:
        A: The manager immediately ordered his dismissed.
        B: The manager immediately ordered dismissal of him.
        C: The manager ordered his immediate dismissal.
        D: The manager ordered for immediately dismissal of him.   2 marks

Read the passage below and answer Question No. 61.

A successful businessperson needs to be a strong leader as well as a manager. Leadership is about getting people to understand and believe in your vision and to work with you to achieve your goals. On the other hand, managing is more about administering and making sure the routine things are happening as they should. Honesty and integrity are crucial to getting your people to believe in you and enter the journey you are taking them on. Having a clear vision is very important for success. Vision is to know where you are, where you want to go with your team in charting a path for the future. You should be able to inspire your team to be all they can by making sure they understand their role in the bigger picture. A strong leader is not afraid to challenge the status quo, do things differently and have the courage to think out of the box. A successful manager is able to take a strategic vision and break it down into a roadmap to be followed by the team. She is able to direct day-to-day work efforts, review resources needed, anticipate needs along the way, establish work rules, processes, standards, and operating procedures, look after her people, their needs, listen to them and involve them.

I. A successful businessperson should have
   (i) more leadership qualities than managerial skills.
   (ii) more managerial skills than leadership qualities.
   (iii)leadership qualities as well as managerial skills.
II. A strong leader
   (i) listens to people.
   (ii) thinks out of the box.
   (iii)look after the people.
III. A good manager
   (i) listens to people.
   (ii) is not afraid to do the things differently.
   (iii)Both (i) and (ii).

61. The correct answers to I, II and III are:
   A: iii, iii, iii, respectively.
   B: i, ii, i, respectively.
   C: i, ii, iii, respectively.
   D: iii, ii, i, respectively.   3 marks

Read the passage below and answer Question No. 62.

A hobby can prove to be a great stress reliever. It is an activity that you do only for fun. Nowadays, many people think that their leisure activities are primarily surfing the internet and catching up on social networking sites. These cannot be considered as healthy hobbies. A healthy hobby provides a creative outlet that can help you forget your daily problems, unwind after a hard day and give you a lot of pleasure. You may think about the activities you enjoyed as a kid. If something from the past does not immediately come to your mind, you may go to a craft store, a music store, a sports shop, or a book store. Browse around and see what captures your attention. You may not be able to find a perfect activity for you immediately, but you can have a lot of fun trying out new things and exploring what's out there.

I. Based on the above passage, select the incorrect statement:
   (i) Everyone has a hobby.
   (ii) Internet surfing is not a healthy hobby.
   (iii) Everyone does not have a hobby .

II. Based on the above passage, select the correct statement:
   (i) One must adopt one's childhood hobby.
   (ii) Hobby is a fun activity.
   (iii)Both (i) and (ii).

III. A hobby
   (i) can prove to be a stress buster.
   (ii) develops competitiveness.
   (iii)both (i) and (ii)

62. The correct answers to I, II and III are:

   A: iii, iii, iii, respectively.
   B: iii, ii, i, respectively.
   C: i, ii, i, respectively.
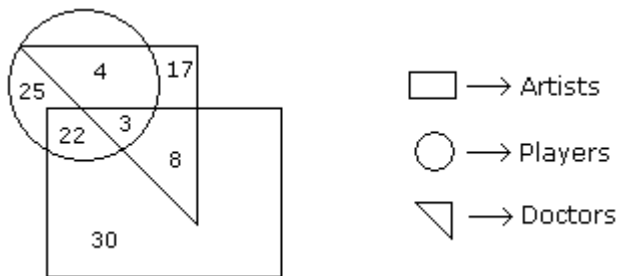   D: iii, ii, iii, respectively.   3 marks

Logical Reasoning

63. How is the son-in-law of my wife's only sister related to the husband of my wife's sister?

    A. Cousin.
    B. Uncle.
    C. Father-in-law.
    D. Son-in-law.   1 mark

64. Answer the question below, based on the following figure:

How many doctors are neither artists nor players?

    A. 17.
    B. 30.
    C. 25.
    D. 8.   1 mark

65. Arrange the words given below in a meaningful sequence.

    1. House   2. Street   3. Room   4. Town   5. District

    A. 3, 2, 1, 4, 5.
    B. 3, 1, 4, 2, 5.
    C. 3, 1, 2, 4, 5.
    D. 3, 1, 2, 5, 4.   1 mark

66. How many days are there in x weeks x days?

    A. $7x$ multiplied by x.
    B. $8x$.
    C. $14x$.
    D. $7x$.   1 mark

67. Establishing a Power of Attorney occurs when a legal document is created that gives one individual the authority to act for another. Which situation below is the best example of Establishing a Power of Attorney?

    A. K is selling his house and he hires a lawyer to review the contract.

    B. P's mother can no longer go to the bank to cash her cheques and make deposits, so she has taken legal steps to enable P to do these things for her.

    C. Q's father is elderly and Q thinks he is no longer able to make decisions for himself.

    D. At his son's urging, Y opens up a retirement account with the local bank.

                    1 mark

68. A cube of side equal to 4 cm is painted red on all its sides. It is then cut into 64 small cubes of equal size. How many small cubes have no side painted?

    A. 27.
    B. 16.
    C. 32.
    D. 8.   1 mark

69. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions?

    A. 45 minutes past 4.
    B. 40 minutes past 4.
    C. 50 4/11 minutes past 4.
    D. 54 6/11 minutes past 4.   2 marks

70. In the following, two statements numbered as I and II are given, followed by two conclusions, also numbered as I and II. You have to take the two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusion logically follows the given two statements, disregarding the known facts.

Statements
I. All skaters are good swimmers.
II. All good swimmers are runners.

Conclusions
I. Some runners are skaters.
II. Some good swimmers may not be skaters.

    A. Only conclusion I follows.
    B. Only Conclusion II follows.
    C. Both conclusions I and II follow.
    D. Neither conclusion I nor II follows.   2 marks