Institute of Actuaries of India

ACET September 2019

Mathematics

1. The sum of 1.23425001 and 2.03049912, when rounded to 4 decimal places, is

   A. 3.2646
   B. 3.2648
   C. 3.2647
   D. 3.26474913 1 mark

2. The rank of B, where

$$B = \begin{pmatrix} b & 1 & 0 \\ 3 & b-2 & 1 \\ 3(b+1) & 0 & b+1 \end{pmatrix}$$

   with $b \neq -1,0,2$, is
   A. 3
   B. 2
   C. 1
   D. 0 3 marks

3. The system $6x + 9y = 4,\ 2x + 3y = 5$ has

   A. a unique solution
   B. more than two solutions
   C. no solution
   D. exactly two solutions 1 mark

4. If $g: R \to R$ is a differentiable function and $g\left(\frac{1}{2}\right) = 2,$ then the value of

$$\lim_{x \to \frac{1}{2}} \int_2^{g(x)} \frac{2t}{x - \frac{1}{2}} dt$$

   is

   A. $4\left[g'\left(\frac{1}{2}\right)\right]^2$
   B. $4g'\left(\frac{1}{2}\right)$
   C. $2g'\left(\frac{1}{2}\right)$
   D. 4 2 marks

5. The value of the limit $\lim_{x\to\infty}\left(\frac{x^2-2}{x^2+1}\right)^{x^2}$ equals to

   A. e
   B. e^2
   C. e^3
   D. $\frac{1}{e^3}$ 2 marks

6. If $f(x) = [x]$, the greatest integer function, then $f(x)$ is

   A. continuous everywhere
   B. continuous nowhere
   C. continuous where x is integer
   D. continuous where x is not integer 1 mark

7. If $h(x) = f(x) + f\left(\frac{1}{x}\right)$ and $f(x) = \int_1^x \frac{\log_e y}{y+1}\,dy$, then the value of $h(e)$ is equal to

   A. 1
   B. 0
   C. $\frac{1}{2}$
   D. $e + 1$ 3 marks

8. Let $f: [0,\infty) \to (-\infty,\infty)$ be defined by $f(x) = x$, and $g: (-\infty,\infty) \to (-\infty,\infty)$ be defined by $g(x) = |x|$. Then

   A. $g \circ f$ and g both are one-to-one
   B. g is one-to-one, but not $g \circ f$
   C. $g \circ f$ is one-to-one, but not g
   D. neither of $g \circ f$ and g is one-to-one 2 marks

9. The remainder when $1! + 2! + 3! + \cdots + 99!$ is divided by 8 is

   A. 4
   B. 3
   C. 2
   D. 1 2 marks

10. The shortest interval that contains all values of the sequence $\{x_n\}$, where $x_n = \frac{(-1)^n(2n-1)}{n}, n = 1,2,\ldots$, is

    A. $(-1,1)$
    B. $(-2,2)$
    C. $\left(-\frac{1}{2},\frac{1}{2}\right)$
    D. $(-\infty,\infty)$ 1 mark

11. If

$$f(x) = \left(1 - |x|^{\frac{2}{3}}\right)^{\frac{3}{2}}, \qquad -1 < x < 1,$$

then at $x = 0$

A. $f'(x)$ exists and $f(x)$ is maximum
B. $f'(x)$ exists and $f(x)$ is minimum
C. $f'(x)$ does not exist and $f(x)$ is maximum
D. $f'(x)$ does not exist and $f(x)$ is minimum 2 marks

12. If

$$\lim_{x \to 0} \frac{\sin 2x + \tau \sin x}{x^2}$$

is finite, then the value of τ is

A. -2
B. -1
C. 0
D. 2 1 mark

13. The function $f(x) = |\sin x|$ is differentiable at

A. $x = n\pi,\ n = 0, \pm 1, \pm 2, \ldots$
B. $x = \frac{n\pi}{2},\ n = 0, \pm 1, \pm 2, \ldots$
C. At any real value of x except $x = n\pi,\ n = 0, \pm 1, \pm 2, \ldots$
D. any real value of x 1 mark

14. For a continuous function $f(x), f(0) = 1, f(1) = 3, f(2) = 9$ and $f(4) = 81$, the value of $f(3)$, by the most suitable linear interpolation, is

A. 45
B. 55
C. 23.5
D. 15 1 mark

15. The equation $a\cos x - b\sin x = d$ admits a solution for x, if and only if

A. $d \geq \min\{a, b\}$
B. $d \leq \max\{a, b\}$
C. $-\sqrt{a^2 + b^2} \leq d \leq \sqrt{a^2 + b^2}$
D. $\min\{|a|, |b|\} \leq d \leq \max\{|a|, |b|\}$ 2 marks

16. Suppose $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\left(\vec{a} \times \vec{b}\right) = 2(\vec{a} \times \vec{c})$, $|\vec{a}| = |\vec{c}| = 2$ and $\left|\vec{b}\right| = 4$. Then $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$ is equal to

A. 16
B. 12
C. 8
D. 0 1 mark

17. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & c-4 \\ 1 & -2 & c+1 \end{bmatrix}$ is equal to 1 if the value of c is

A. 4
B. -6
C. 0
D. 14 1 mark

18. If $0.272727 \ldots, x, 0.727272 \ldots$ are in harmonic progression, then x must be

A. a rational number lying between 0 and 1
B. an integer
C. an irrational
D. a non-integer rational number larger than 1 2 marks

19. If $\log_a b = \log_b c$, where a, b, c are positive and none equals to unity, then

A. $\log_{10} a$, $\log_{10} b$ and $\log_{10} c$ are in A.P.
B. $\log_{10} a$, $\log_{10} b$ and $\log_{10} c$ are in G.P.
C. $\log_{10} a$, $\log_{10} b$ and $\log_{10} c$ are in H.P.
D. None of these 1 mark

Statistics

20. If a polygon has 65 diagonals, its number of sides is

A. 12
B. 14
C. 16
D. 13 1 mark

21. The number of permutations of six letters chosen from the set $\{A, B, C, D, E, F, G, H, I, J\}$, so that A, B and C are always chosen and they occur together is

A. ${}^{7}P_{3}$
B. ${}^{7}P_{3} \times 6$
C. ${}^{7}P_{3} \times 4$
D. $7!$ 2 marks

22. If there exist only four letters R, A, N, K in an English dictionary, what will be the rank of the word $'RANK'$ in that dictionary?

A. 20
B. 24
C. 23
D. 16 1 mark

23. If A and B are two mutually exclusive events, then they cannot be independent, if

A. $P(A) = 0$ and $P(B) \neq 0$
B. $P(A) = 0$ and $P(B) = 0$
C. $P(A) \neq 0$ and $P(B) \neq 0$
D. $P(A) \neq 0$ and $P(B) = 0$ 1 mark

24. If $P(A \cup B) = 0.7, P(A \cap B) = 0.3$, the value of $P(A^c) + P(B^c)$ is

A. 0
B. 0.4
C. 0.21
D. 1 1 mark

25. A bag contains $2m + 1$ coins. It is known that m of these have a tail on both sides, whereas the rest are fair. A coin is drawn at random from the bag and is tossed. If the probability that 'toss produces tail' is $\frac{31}{42}$, the value of m is

A. 9
B. 10
C. 11
D. 21 3 marks

26. If x is the median of the integers $\{11, 13, 3, 9, 7, 19, 2, 3, 21, 17, x\}$, which of the following could possibly be the value of x?

A. 8
B. 10
C. 12
D. 14

1 mark

27. Let $y_1, y_2, y_3, y_4, y_5, y_6$ be observations with standard deviation m. The standard deviation of the observations $ay_1 + b, ay_2 + b, ay_3 + b, ay_4 + b, ay_5 + b, ay_6 + b$ is

A. $|a|m + b$
B. $m/|a|$
C. m
D. $|a|m$

1 mark

28. Coefficients of variation of two distributions are 0.4 and 0.5, and their means are 25 and 20, respectively. Difference of their standard deviations is

A. 20
B. 10
C. 0
D. 22.5

1 mark

29. Let X be a discrete random variable that takes values 1, 2 and 3 with probabilities 0.5, 0.3 and 0.2, respectively. Then the expected value of $|X - 2|$ is

A. 0.7
B. 0.3
C. -0.3
D. 0.

1 mark

30. Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} cx^2 & \text{for } 0 < x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

for some positive constant c. The value of $P\left(x \leq \frac{2}{3} \middle| x > \frac{1}{3}\right)$ is

A. 3/26
B. 5/26
C. 7/26
D. 11/26

2 marks

31. The scores (out of 100) of students appearing for a competitive examination are approximately normally distributed with a mean 50 and a standard deviation of 10. How high must a student score in the examination to be in the top 5 percentile? It is given that $P(Z < 1.645) = 0.95$.

A. 66.45
B. 95.00
C. 51.645
D. 69.6

1 mark

32. In a population, 60% are smokers and 40% are female. Assuming smoking habit is independent of gender, find the probability of no female smoker in a random sample of 10 individuals.

A. 0.24^{10}
B. $1 - 0.24^{10}$
C. 0.76^{10}
D. $1 - 0.76^{10}$. 1 mark

33. The number of calls coming per minute into a customer call centre is Poisson random variable with mean 5. Assume that the number of calls arriving in two different minutes are independent. What is the probability that at least two calls will arrive in a given period of two minutes?

A. $11e^{-10}$
B. $1 - 10e^{-10}$
C. $1 - 11e^{-10}$
D. None of these 2 marks

34. If the continuous random variable X is uniformly distributed over the interval $[a, 3a]$, then $E(X^2)$ is equal to

A. $13a^2/3$
B. $13a^3/3$
C. $26a^3/3$
D. $26a^2/3$ 1 mark

35. If the correlation between $2X + 3$ and $5 - Y$ is 0.4, then the correlation between $5 - X$ and $2Y + 3$ is

A. -0.1.
B. 0.1.
C. -0.4.
D. 0.4. 1 mark

36. Based on some data, it is found that the average monthly price and standard deviation of FMCG (Fast Moving Consumer Goods) in Kolkata are Rs. 60 and Rs. 2.5, respectively, and the same in Mumbai are Rs. 80 and Rs. 3.5, respectively. The correlation coefficient between the prices in the two cities is 0.9. If in a particular month the price of FMCG in Kolkata is Rs. 85, then the expected price at Mumbai in that month, obtained through linear regression, is

A. Rs. 96.0
B. Rs. 102.4
C. Rs. 111.5
D. Rs. 113.3 2 marks

37. While shuffling through a pack of cards, four are accidentally dropped. Then, the probability that the missing cards are one from each suit is

A. ${}^{13}P_4/\binom{52}{4}$

B. $13^4/\binom{52}{4}$

C. $\binom{13}{4}/\binom{52}{4}$

D. $13^4/52^4$ 2 marks

38. Let $S = \{x | x \text{ is a prime number and } x < 30\}$. The number of different rational numbers whose numerator and denominator belongs to S is

A. 90

B. 91

C. 100

D. 180 2 marks

39. The joint distribution of the discrete random variables X and Y is as described below.

Values of $P(X = x, Y = y)$

y	x		
	1	2	3
1	$\frac{1}{3}$	a	$\frac{1}{9}$
2	b	$\frac{1}{9}$	c

In order that X and Y are independent, the values of a, b and c must be

A. $\frac{2}{9}, \frac{1}{6}, \frac{1}{18}$

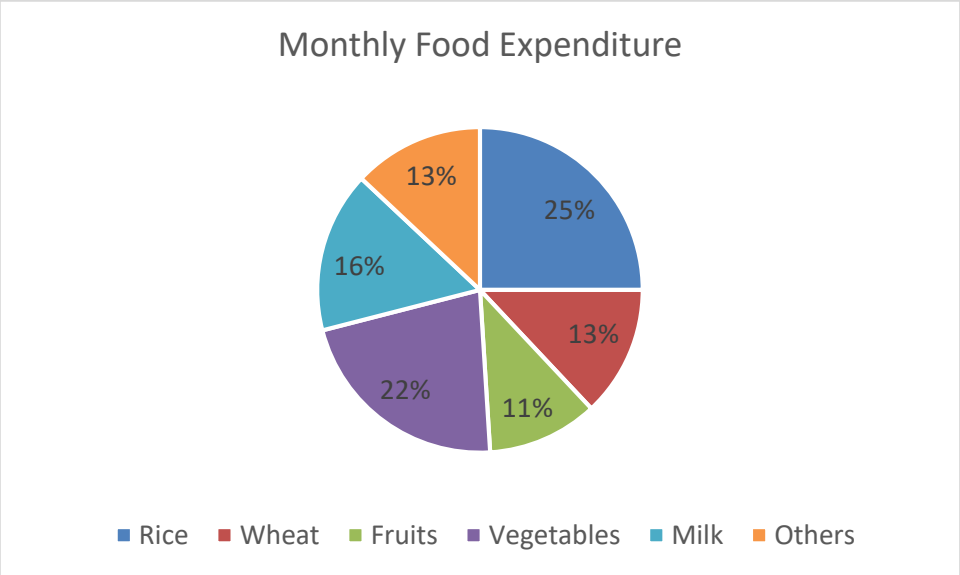
B. $\frac{8}{9}, \frac{2}{3}, \frac{8}{9}$

C. $\frac{5}{9}, \frac{4}{9}, \frac{4}{9}$

D. $\frac{1}{9}, \frac{1}{3}, \frac{1}{9}$ 3 marks

Data Interpretation

The following pie-chart shows the distribution of the monthly expenditure on various food items by a typical family. The information given is in terms of percentage. Study the graph and answer questions 40 and 41.



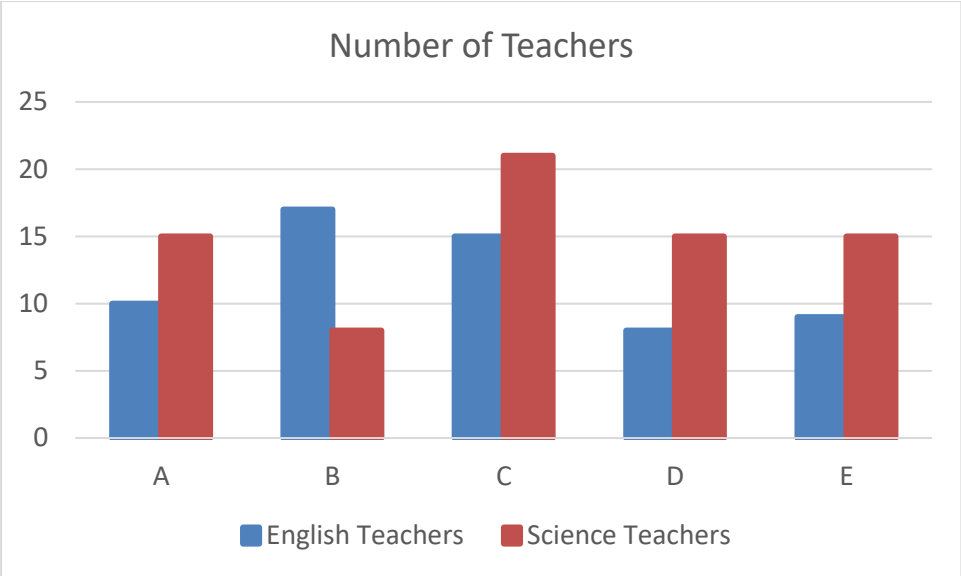
40. If the expenditure on fruits and vegetables is Rs. 2000, then the expenditure on Rice is approximately

    A. Rs. 1500
    B. Rs. 1800
    C. Rs. 2000
    D. Rs. 2500 1 mark

41. The ratio of the expenditure on Milk to the expenditure on other items is close to

    A. 1/6
    B. 1/3
    C. 1/5
    D. 2/5 1 mark

The following comparative bar chart shows the number of English and Science Teachers in various schools named as A, B, C, D and E. Study the chart and answer questions 42 and 43.



42. Which two schools have the same total number of teachers, combining both Science and English Teachers?

A. E & A
B. D & A
C. C & A
D. B & A

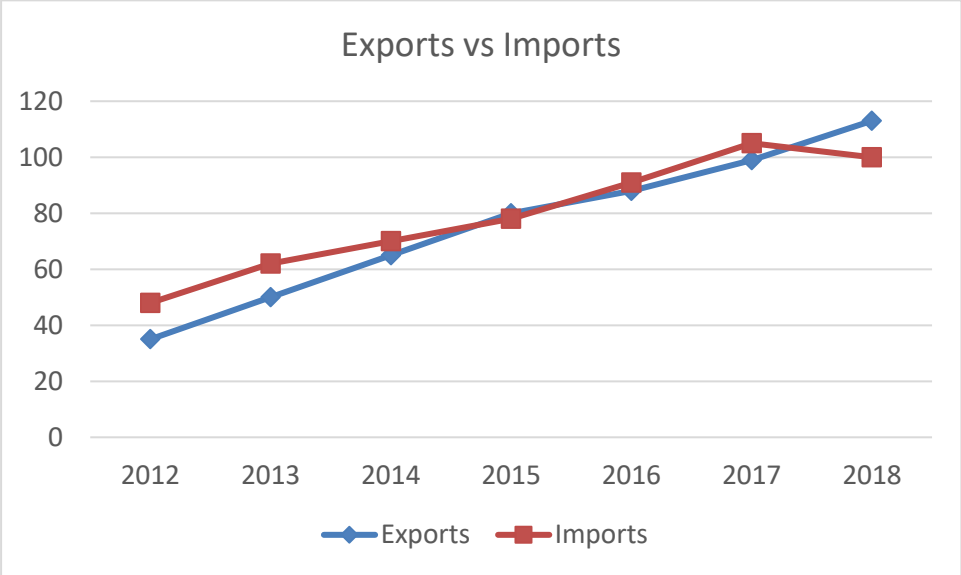
1 mark

43. Which two schools have the same difference between the number of Science Teachers and English Teachers?

A. A & C
B. A & E
C. C & D
D. C & E

1 mark

The following chart shows the value of exports and imports of a country by its currency over different years. Study the bar chart and answer questions 44 and 45.



44. Out of given years, in how many years was the value of imports higher than the value of exports, but by less than 10%?

A. 1
B. 2
C. 3
D. 4

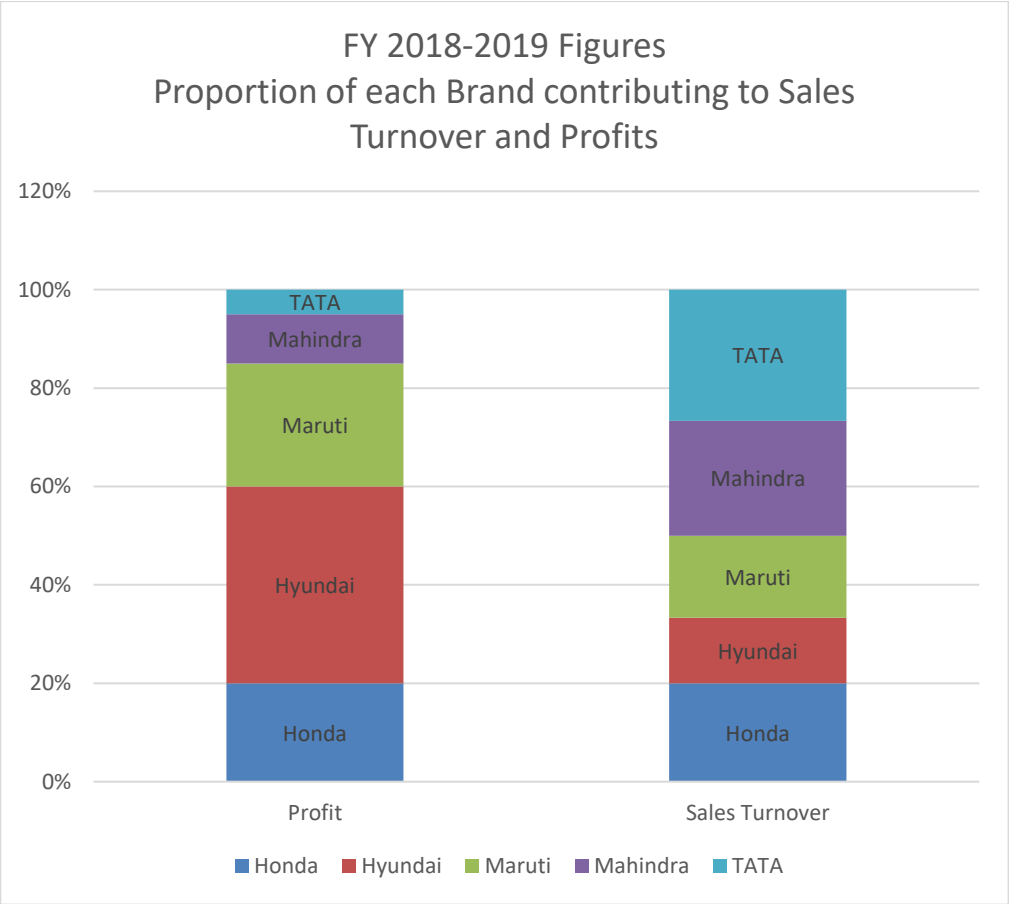
1 mark

45. From the years 2013 to 2018, in which year the growth of exports compared to the previous year was the lowest?

A. 2014
B. 2015
C. 2016
D. 2017

2 marks

The following composite bar charts depict the performance of a Multi Brand motor dealer in terms of Overall Sales Turnover and Profits in FY 2018-19. The total Sales turnover was Rs. 150 Crores and Overall Profits was Rs. 10 Crores. Study the charts and answer questions 46–48.



46. If overall profits had grown at 20% annually compounded since FY 2016-17, the approximate profits in FY 2016-17 had been

    A. Rs. 694 lakhs
    B. Rs. 674 lakhs
    C. Rs. 654 lakhs
    D. Rs. 634 lakhs 1 mark

47. The brand category with the highest profitability in FY 2018-19, where profitability is defined as the ratio of Profits and Sales Turnover, is

    A. Maruti
    B. Hyundai
    C. Honda
    D. Mahindra 2 marks

48. The ratio of sales of the Maruti Brand to Hyundai Brand in FY 2018-19 is approximately

    A. 1.35
    B. 1.30
    C. 1.25
    D. 1.20 1 mark

In an engineering college, three programming languages are taught. Each student is required to study at least one of the three languages which are C, C++ and JAVA. 40 students study all the three languages. 404 students study only Java, 222 students study only C, 500 students study C++, 114 students study both Java and C and 388 students study only C++. Answer questions 49–51.

49. How many students study exactly two programming languages including C++?

A. 36
B. 48
C. 60
D. 72 1 mark

50. What is the number of students studying exactly two languages?

A. 156
B. 146
C. 136
D. 126 1 mark

51. If the number of students studying the Java language is 568, then what is the number of students studying the C language?

A. 436
B. 448
C. 358
D. 398 2 marks

English

52. The opposite of "Dilate" is:
    A. Grow
    B. Stagnate
    C. Swell
    D. Contract 1 mark

53. Synonym of "Candid" is:
    A. Balanced
    B. Frank
    C. Beautiful
    D. Rude 1 mark

54. Synonym of "Erstwhile" is:
    A. Traditional
    B. Historic
    C. Former
    D. Modern 1 mark

55. General pardon of political offenders is called
    A. Cowardice
    B. Amnesty
    C. Autocracy
    D. Generosity 1 mark

56. A person who cannot help but steal whatever he can find is also called as:
    A. Optometrist
    B. Thanatomaniac
    C. Theomaniac
    D. Kleptomaniac 1 mark

Choose the most appropriate word to fill in the blank in Questions 57 and 58.

57. The family did not know where the thieves _____ from.
    A. had come
    B. have come
    C. come
    D. has come 1 mark

58. The dog was _____ by a car.
    A. run over
    B. run through
    C. run on
    D. run out 1 mark

59. Speed is related to Fast as Temperature is related to:
    A. Warm
    B. Thermometer
    C. Celsius
    D. Fever 1 mark

60. Select the meaning of underlined phrase in the sentence: "Do not trust a man who blows his own trumpet."

A. flatters
B. abuses others
C. praises himself
D. prides of himself 2 marks

61. Select the correct sentence:

A. It was until many years later that Gabbar Singh became a rebel against the system.
B. It was not until many years later that Gabbar Singh became a rebel against the system.
C. It was till many years later that Gabbar Singh became a rebel against the system.
D. It was not many years later that Gabbar Singh became a rebel against the system. 2 marks

Read the passage below and answer Question No. 62:

Globalization means the integration of economies around the world, particularly through trade and financial flows. There are also broader cultural, political and environmental dimensions of globalization. Due to globalization, many developed nations are facing the problem of unemployment. Different taxation norms of nations also attract the accumulation of black money. The process of diminishing interdependence and integration among economies is referred to as de-globalization. It stands in contrast to globalization. De-globalization periods lie between discrete periods of globalization. While globalization and de-globalization are contrasts, they are not mirror-images of each other.

I. Based on the above passage, select the incorrect statement:
   (i) Globalization involves enhanced trade.
   (ii) Different taxation norms of nations are good for clean economy.
   (iii) De-globalization means less dependence among economies of the world.

II. Based on the above passage, select the correct statement:
   (i) Globalization does not involve economic integration only.
   (ii) Globalization creates unemployment all over the world.
   (iii) Once globalization is set in motion, it continues.

III. Globalization
   (i) integrates the taxation norms.
   (ii) integrates the economies.
   (iii) integrates the employment.

62. The correct answers to I, II and III are:

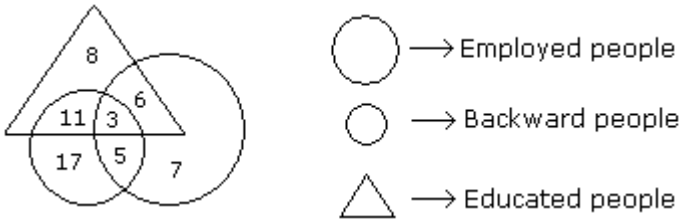
A. ii, i, ii, respectively.
B. i, ii, i, respectively.
C. i, ii, iii, respectively.
D. ii, ii, i, respectively. 3 marks

Logical Reasoning

63. A has two sons, B and C. B is husband of D. E is mother-in-law of D and F. H is son of F. How is A related to H ?

    A. Grandfather
    B. Father
    C. Brother
    D. Father-in-law 1 mark

Answer Question 64 based on the following figure.



64. How many backward people are educated?

    A. 9
    B. 20
    C. 14
    D. 11 1 mark

65. Five plays K, L, M, N and O are to be staged from Monday to Friday of a week. On each day, only one play will be staged. N or O should not be either the first or last to be staged. O should be immediately followed by M. L should be staged immediately after N. One play is staged between K and L.

    Which is the second play to be staged ?

    A. L
    B. M
    C. N
    D. O 1 mark

66. The maximum number of years between two successive leap years is

    A. 4
    B. 8
    C. 400
    D. 100 1 mark

67. Tiebreaker is an additional contest or period of play designed to establish a winner among tied contestants. Which situation below is the best example of a Tiebreaker?

    A. At halftime of a basketball match, the score is tied at 28-28. The second half of the match is about to begin.
    B. Punjab and Haryana each finished having scored no goals in a football match, and now they are battling out in a thirty minute overtime.
    C. The referee tosses a coin to decide which team will have possession of the ball first.
    D. Delhi and Bihar have each scored eight goals in a football tournament. They will now play each other in the final match of the tournament. 1 mark

68. Three sides adjacent to a single corner of a cube of side equal to 4 cm are painted red. It is then cut into 64 small cubes of equal size. How many small cubes have at least two sides painted?

    A. 9
    B. 10
    C. 11
    D. 12 1 mark

69. At what time between 9 and 10 o'clock will the hands of a watch be together?
    A. 45 minutes past 9
    B. 50 minutes past 9
    C. 49 1/11 minutes past 9
    D. 48 2/11 minutes past 9 2 marks

70. In the following, two statements are given, followed by three conclusions numbered I, II and III. You have to take the given statements to be true even if they seem to be at variance with commonly known facts. Read all the conclusions and then decide which of the given conclusions logically follows from the given statements, disregarding commonly known facts.

    Statements:

    I. All eye drops are liquids.
    II. Some eye drops are ear drops.

    Conclusions:

    I. Some eye drops are drinkable.
    II. Some ear drops are drinkable.
    III. All liquids are eye drops.

    Which of the conclusions follow logically?

    A. I and II follow.
    B. II and II follow.
    C. Only III follows.
    D. None follows. 2 marks