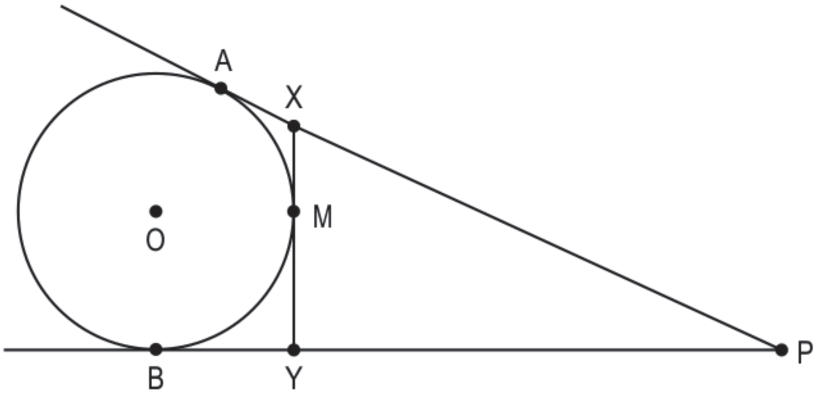
Chapter - 10
Circles

Q: 1 **In the figure below, ΔPXY is formed using three tangents to a circle centred at O.**

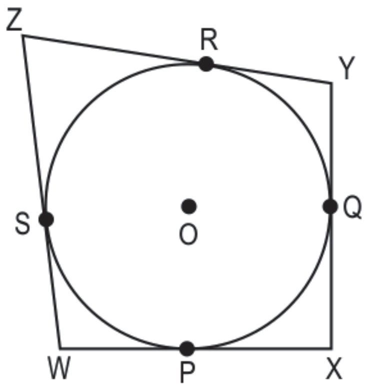


(Note: The figure is not to scale.)

Based on the construction, the sum of the tangents PA and PB is _________ the perimeter of ΔPXY.

1 lesser than
2 greater than
3 equal to
4 (cannot be answered without knowing the tangent lengths)

Q: 2 **Raghav drew the following figure on a board where a circle is inscribed in a quadrilateral.**



(Note: The figure is not to scale.)

Then he wrote the following relationships.

i) ZW + WX = XY + YZ
ii) ZY + WX = ZW + YX

Which of the above relationships is/are DEFINITELY true?

1 only i
2 only ii
3 both i and ii
4 neither i nor ii

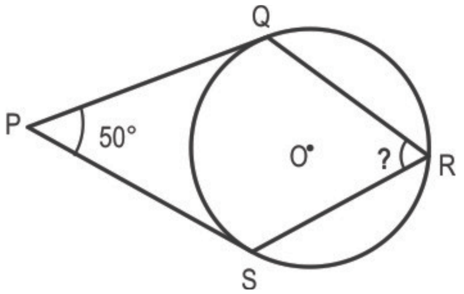
Q: 3 **A circle has a centre O and radii OQ and OR. Two tangents, PQ and PR, are drawn from an external point, P.**

In addition to the above information, which of these must also be known to conclude that the quadrilateral PQOR is a square?

i) OQ and OR are at an angle of 90°.
ii) The tangents meet at an angle of 90°.

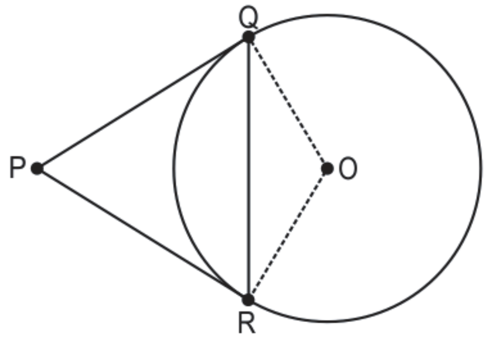
1 only i) 2 only ii) 3 either i) or ii) 4 both i) and ii)

Q: 4 **In the following figure, O is the centre of the circle and PQ and PS are tangents to the circle at points Q and S respectively.** **[1]**



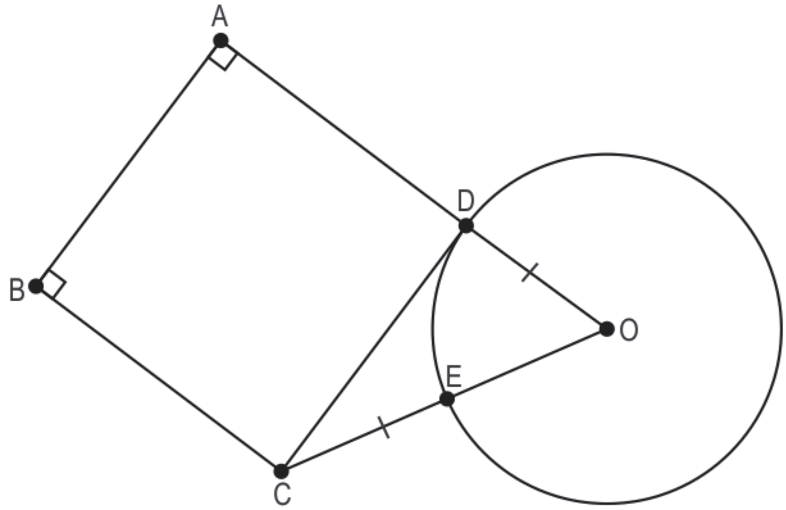
What is the measure of ∠QRS? Show your work.

Q: 5 **Shown below is a circle with centre O and tangents PQ and PR.** **[3]**



Using triangles QOR and PQR, and without doing any extra constructions, prove that the tangents PQ and PR are equal in length.

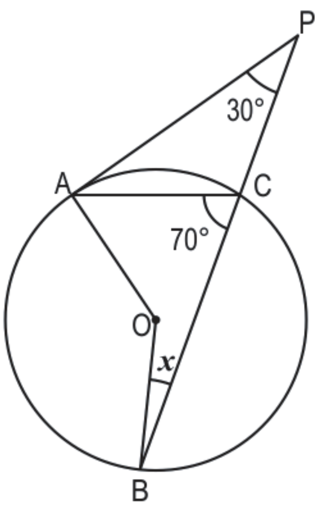
Q: 6 **ABCD is a square. CD is a tangent to the circle with centre O as shown in the figure below.** **[3]**



(Note: The figure is not to scale.)

If OD = CE, what is the ratio of the area of the circle and the area of the square? Show your steps and give valid reasons.

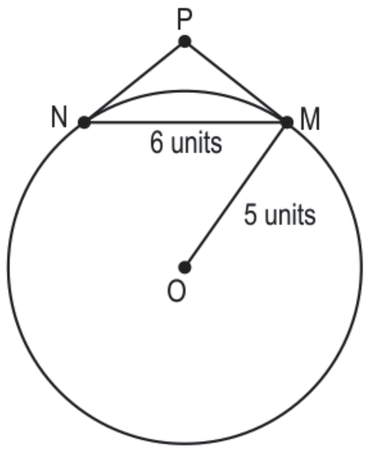
Q: 7 **In the figure given below, PA is a tangent to the circle with centre O and PCB is a straight line.** **[3]**



(Note: The figure is not to scale.)

Find the measure of∠OBC. Show your steps and give valid reasons.

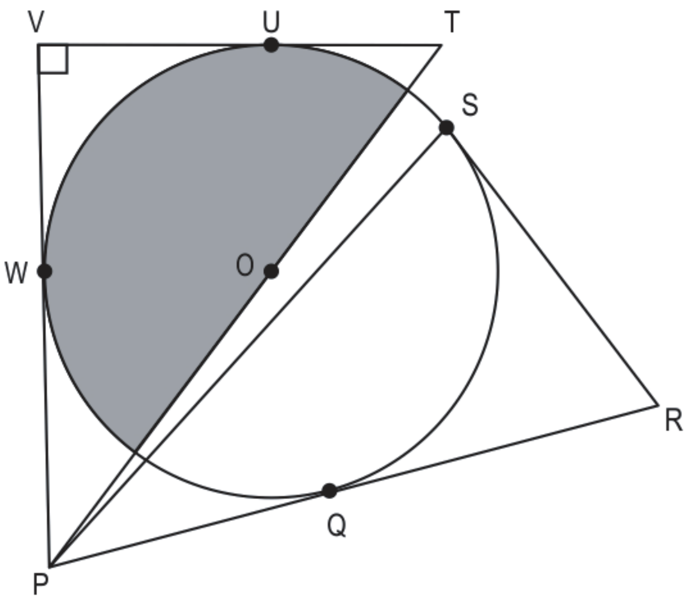
Q: 8 **Shown below is a circle with centre O and radius 5 units. PM and PN are tangents and the length of chord MN is 6 units.** **[5]**



(Note: The figure is not to scale.)

Find the length of (PM + PN). Show your work.

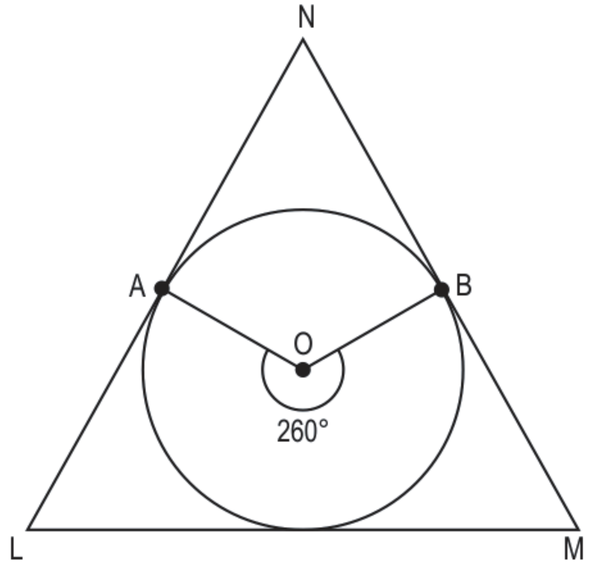
Q: 9 **Shown below is a circle with centre O. VP = 34 cm, PR = 36 cm and RS = 17 cm. U, W, Q and S are the points of tangency.** **[5]**



(Note: The figure is not to scale.)

Find the area of the shaded region in terms of π. Show your steps and give valid reasons.

Q: 10 **In the figure below, a circle with centre O is inscribed inside ΔLMN. A and B are the points of tangency.** **[1]**



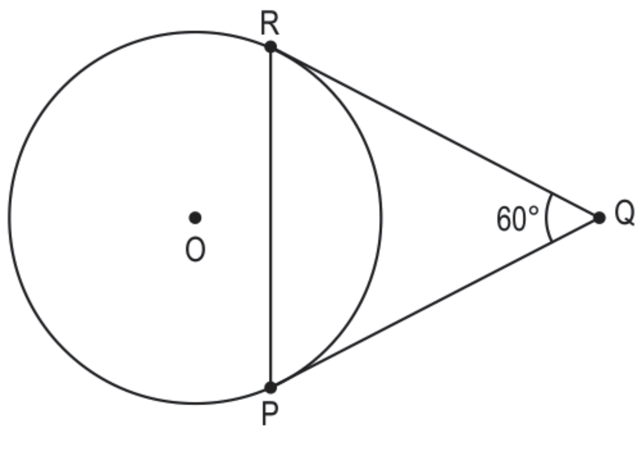
(Note: The figure is not to scale.)

Find ∠ANB. Show your steps.

Q: 11 **A point is 25 cm from the centre of a circle of radius 15 cm.** **[1]**

Find the length of the tangent from the point to the circle. Show your steps.

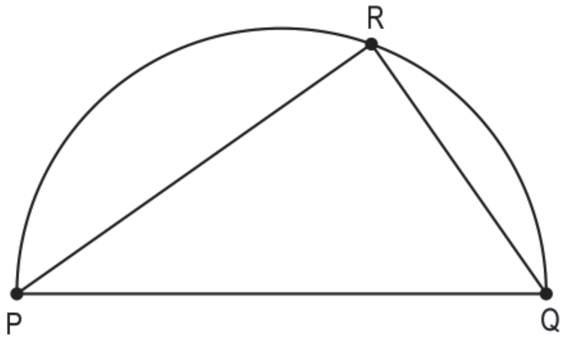
Q: 12 **In the figure below, PQ and RQ are tangents to the circle with centre O and radius $6\sqrt{3}$ cm.** **[5]**



(Note: The figure is not to scale.)

i) Prove that ΔPQR is an equilateral triangle.
ii) Find the length of RP. Show your steps along with a diagram and give valid reasons.

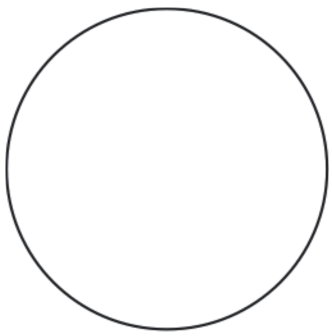
Q: 13 **Shown below is a ΔPQR inscribed in a semicircle.** **[2]**



A circle is drawn such that QR is a tangent to it at the point R.

How many such circles can be drawn? Justify your answer.

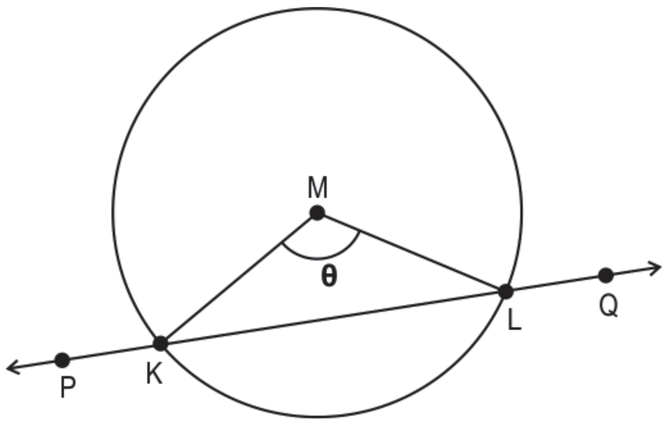
Q: 14 **Shown below is a circle whose centre is unknown.** **[3]**



State true or false for the statements below and give valid reasons.

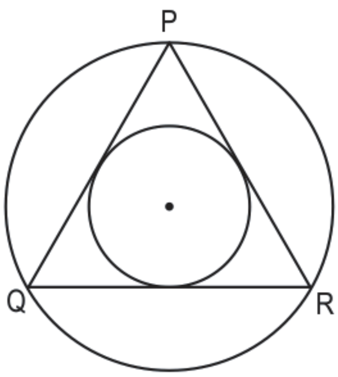
i) The centre of the circle can be found using any 2 tangents.
ii) The centre of the circle can be found using any 2 chords.

Q: 15 **Shown below is a circle with centre M. PQ is a secant and ∠KML = θ.** **[2]**



i) Show that, when θ = 0°, PQ becomes a tangent to the circle.
ii) What is the point of contact of the tangent in part i) with the circle?

Q: 16 **In the figure below, O is the centre of two concentric circles. ΔPQR is an equilateral triangle such that its vertices and sides touch the bigger and smaller circles respectively. The difference between the area of the bigger circle and the smaller circle is 616 cm^2.** **[5]**



(Note: The figure is not to scale.)

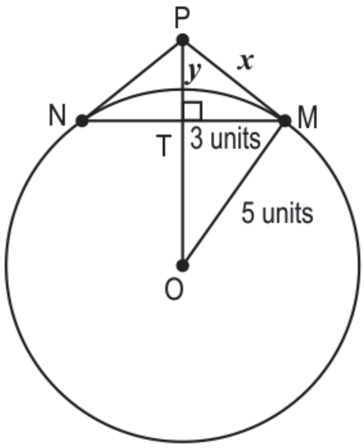
Find the perimeter of ΔPQR. Draw a rough diagram, show your work and give reasons.

(Note: Take π as $\frac{22}{7}$.)

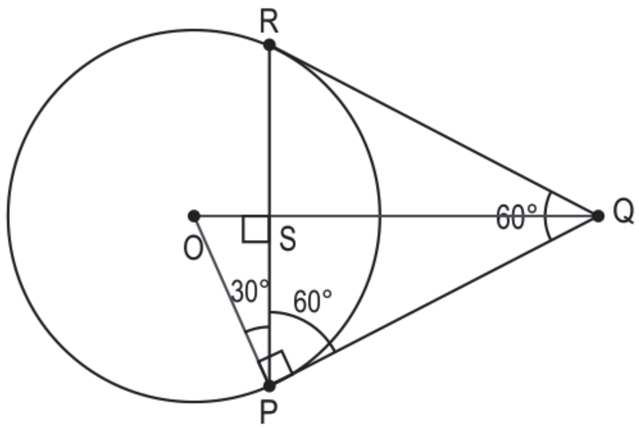
The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	2
3	3

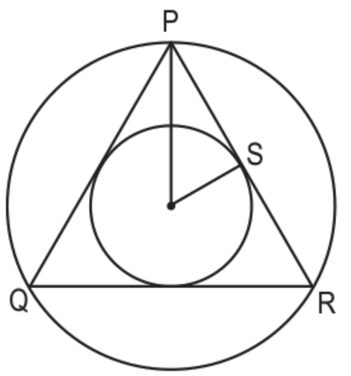
Q.No	Teacher should award marks if students have done the following:	Marks
4	Finds the angle as 65°.	0.5
	Explains that ∠QOS is 130° and ∠QRS=130/2 = 65°.	0.5
5	Assumes the base angles of ΔQOR as θ each as angles opposite equal sides are equal.	1
	Writes that ∠PRQ and ∠PQR are (90 - θ)° each as radius of a circle is perpendicular to the tangents.	1
	Writes that in ΔQPR, PQ and PR are equal as sides opposite to equal angles in a triangle are equal.	1
6	Writes that OE = OD (radii of the same circle) and CE = OD (given). Finds the length of OC as 2OD.	0.5
	Writes that as CD is tangent to the circle, OD ⊥ CD and applies Pythagoras' theorem in ΔODC to find the length of CD as: $OC^2 = CD^2 + OD^2$ $\Rightarrow CD^2 = OC^2 - OD^2 = 4\times OD^2 - OD^2$ $\Rightarrow CD^2 = 3 \times OD^2$	1
	Finds the area of the circle as $\pi\times OD^2$ sq units.	0.5
	Finds the area of the square as $3\times OD^2$ sq units, using step 2. (Award 1.5 marks if the student has combined steps 2 and 4 together).	0.5
	Finds the ratio of the areas as Area(circle):Area(square) = π:3	0.5
7	Writes that sum of angles on a straight line is 180° and finds the measure of ∠ACP as 110°.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that sum of angles in a triangle is 180° and finds the measure of ∠CAP as 40°.	0.5
	Writes that ∠OAP = 90° since PA is tangent to the circle and finds the measure of ∠OAC as 50°.	0.5
	Joins OC. Writes that ΔOAC is isosceles and finds the measure of ∠OCA as 50°.	0.5
	Finds the measure of ∠OCB as 20°.	0.5
	Writes that ΔOCB is isosceles and finds the measure of ∠OBC as 20°.	0.5
8	Connects P to O intersecting MN at T. Applies chord properties to conclude that i) PO ⊥ MN ii) MT = NT = 3 units.  	0.5
	Applies Pythagoras' theorem in ΔOTM to find OT as 4 units.	1
	Assumes PM = x units and PT = y units and applies Pythagoras' theorem to find two equations in x and y as From ΔPTM, $x^2 = y^2 + 3^2$ .... (1) From ΔPMO, $(y + 4)^2 = x^2 + 5^2$ .... (2)	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Solves the pair of equations in the previous step to get *y* (PT) as $\frac{9}{4}$ or 2.25 units.	1
	Inputs the value of *y* in one of the equations from step 3 to find the value of *x* (PM) as $\frac{15}{4}$ or 3.75 units.	1
	Applies tangent property from external point to conclude that PM = PN and finds PM + PN as $\frac{30}{4}$ or 7.5 units.	0.5
9	Writes that RQ = RS = 17 cm and gives the reason that the lengths of tangents drawn from an external point to a circle are equal.	0.5
	Finds the length of PQ = 36 - 17 = 19 cm.	0.5
	Writes that PQ = PW = 19 cm and gives the reason that the lengths of tangents drawn from an external point to a circle are equal.	0.5
	Finds the length of VW = 34 - 19 = 15 cm.	0.5
	Writes that ∠VWO = ∠VUO = 90° and gives the reason that the tangent at any point of a circle is perpendicular to the radius through the point of contact.	1
	Uses above step and ∠WVO = 90° (given), to conclude that, VWOU is a square.	0.5
	Concludes that the radius of the circle, UO = VW = 15 cm.	0.5
	Finds the area of the shaded region, the semi circle as $\frac{1}{2} \times \pi \times 15 \times 15 = 112.5\pi$ cm^2.	1
10	Finds minor ∠AOB as 360° - 260° = 100°.	0.5
	Finds ∠ANB as 360° - (90° + 90° + 100°) = 80°.	0.5
11	Finds the length of the tangent as $\sqrt{(25^2 - 15^2)}$ = 20 cm.	1
12	i) Writes that PQ = RQ and gives the reason that the lengths of tangents drawn from an external point to a circle are equal.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the above step and the angle sum property in ΔPQR to find ∠RPQ = ∠PRQ = 60°. Concludes that ΔPQR is an equilateral triangle as all the angles are equal to 60°.	0.5
	ii) Joins OP and OQ and writes that ∠OPQ = 90°. The figure may look as follows:  *(Note: The figure is not to scale.)*	1
	Finds ∠OPS as ∠OPQ - ∠SPQ = 90° - 60° = 30°.	0.5
	Writes that ∠OQP = 30° and gives the reason that the centre of a circle lies on the bisector of the angle between the two tangents.	0.5
	Uses the angle sum property in ΔOPQ to find ∠POQ as 60°. Uses the angle sum property in ΔOPS to find ∠OSP as 90°.	1
	Uses the sine ratio in ΔOPS to find the length of PS as: sin 60° = $\frac{PS}{PO}$ ⇒ PS = 9 cm Writes that RS = PS = 9 cm and gives the reason that the perpendicular from the centre of a circle to a chord bisects the chord.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the length of RP as RS + PS = 9 + 9 = 18 cm.	0.5
13	Writes that infinite such circles can be drawn.	0.5
	Justifies the answer. For example, writes that ∠PRQ = 90° because it is the angle in a semicircle. Also, writes that the radius is perpendicular to the tangent at the point of contact. Concludes that infinite circles with their radii lying on extended PR and R being a point on the circumference of the circle can be drawn.	1.5
14	i) Writes true.	0.5
	Gives a reason. For example, drawing 90° angles at the points of contact of the 2 tangents and extending perpendicular lines to meet at a point gives the centre of the circle.	1
	ii) Writes true.	0.5
	Gives a reason. For example, drawing the perpendicular bisectors of the 2 chords and extending them to meet at a point gives the centre of the circle.	1
15	i) Writes that ΔKLM is an isosceles triangle and finds the measures of ∠KLM = ∠LKM = $\frac{180°-\theta}{2}$ = 90° - $\frac{\theta}{2}$.	0.5
	Writes that angles on a straight line are supplementary and finds the measures of ∠PKM = ∠QLM = 180° - (90° - $\frac{\theta}{2}$) = 90° + $\frac{\theta}{2}$.	0.5
	Writes that, when θ = 0°, KM and LM coincide and ∠PKM = ∠QLM = 90°. Hence, concludes that PQ becomes a tangent to the circle.	0.5
	ii) Writes the point of contact of the tangent in part i) with the circle as K or L.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
16	**Joins OP and OS to make ΔOPS. The figure may look as follows:**  ***(Note: The figure is not to scale.)***	1
	Writes that, since OP and OS are the radii of the bigger and smaller circles respectively, $\pi OP^2 - \pi OS^2 = 616\ cm^2$.	0.5
	Finds the value of $(OP^2 - OS^2)$ as $196\ cm^2$.	0.5
	Writes that ΔOPS is a right-angled triangle because the tangent at any point on a circle is perpendicular to the radius at the point of contact.	0.5
	Writes that, in right-angled ΔOPS, $PS^2 = OP^2 - OS^2 = 196\ cm^2$	0.5
	Finds the length of PS as 14 cm.	0.5
	Writes that, since PR is the chord of the bigger circle and OS ⊥ PR, PR = 2× PS because the perpendicular from the centre of a circle to a chord bisects the chord and finds the measure of PR as 2 × 14 = 28 cm.	1
	Finds the perimeter of ΔPQR as 3 × 28 = 84 cm.	0.5