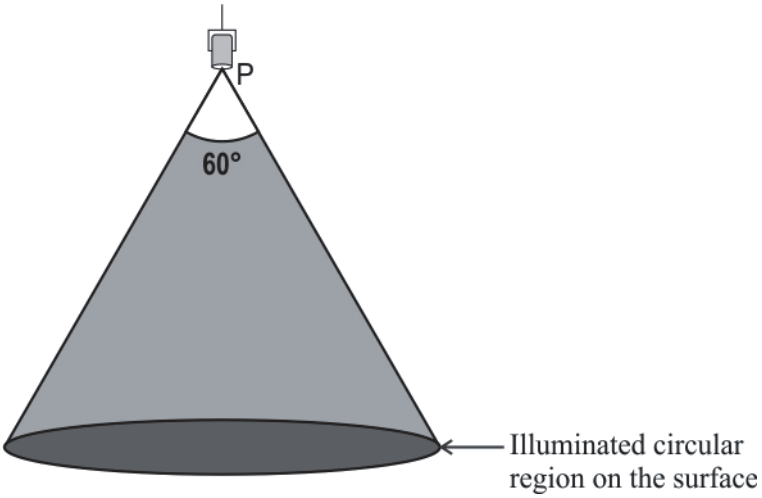
# Chapter - 16
# Multiple Concepts

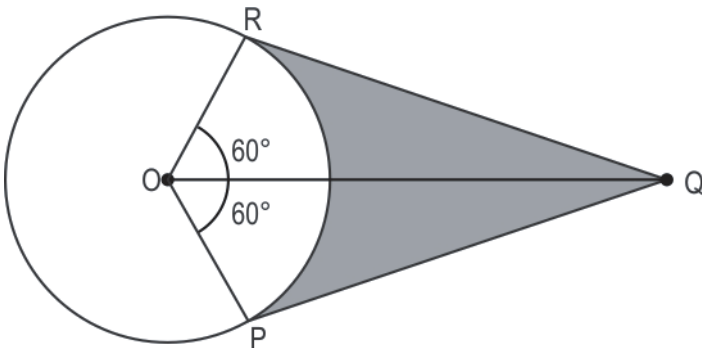
**Q: 1** **A spotlight, P, is at a height of 12 m from the surface. The light from P, forms a right circular cone and illuminates a circular region on the surface as shown below.** **[3]**




***(Note: The figure is not to scale.)***

**Find the area of the illuminated circular region on the surface, in terms of π. Show your steps along with a rough diagram with all the relevant measures marked.**

**Q: 2** **Shown below is a circle with centre O and radius 10 cm. Two tangents QR and QP are drawn to it from an external point Q and OQ = 20 cm.** **[3]**




***(Note: The figure is not to scale.)***

**Find the area, in terms of π, occupied by the shaded region. Show your work.**

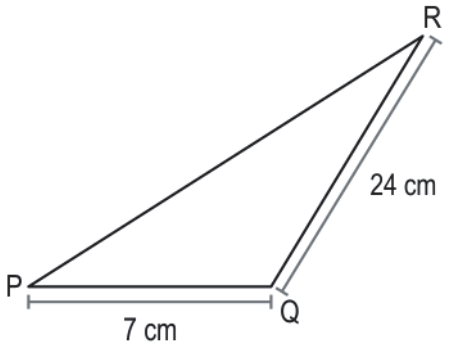
**Q: 3** **The line *m* is a tangent to the circle C at the point (9, 12). The circle is centred at the origin.** **[2]**

**Check if the line *m* passes through the point (15, 0). Show your work.**

**Q: 4** **If $3\cot\theta + \tan\theta = 5\sec\theta$; $0° \leq \theta \leq 90°$, find the value of θ.** **[3]**

**Show your steps and give valid reasons.**

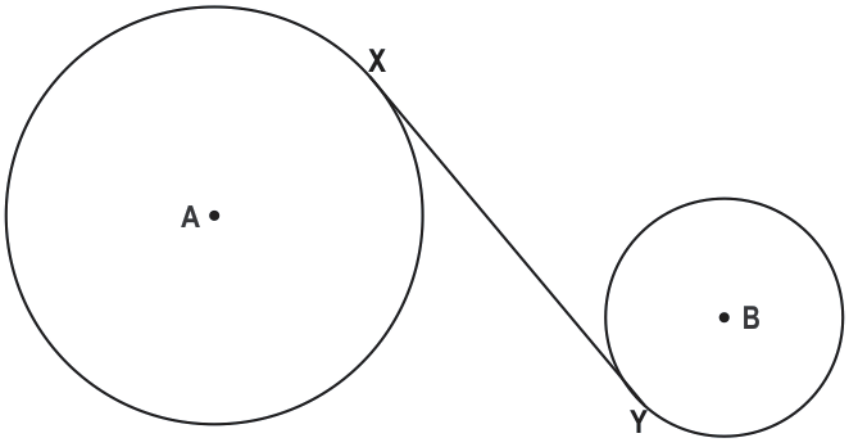
**Q: 5** **Rehan has a set of 40 cards numbered 1 - 40 without repeating any number. Rehan wants to construct ▲PQR shown below by choosing the length of PR, in cm, from the set of the cards.** **[3]**




***(Note: The figure is not to scale.)***

**What is the probability that a card chosen at random makes ▲PQR an OBTUSE angled triangle with Q as obtuse angle? Show your work.**

**Q: 6** **Kiran is making a toy by using two circular disks of radii 12 cm and 6 cm respectively, connected by a taut wire. The wire, XY, is the internal tangent to both disks as shown below.** **[5]**




***(Note: The figure is not to scale.)***

**If the centres of the circular disks, A and B are 30 cm apart, what is the length of the wire XY? Show your steps with a figure and give valid reasons.**

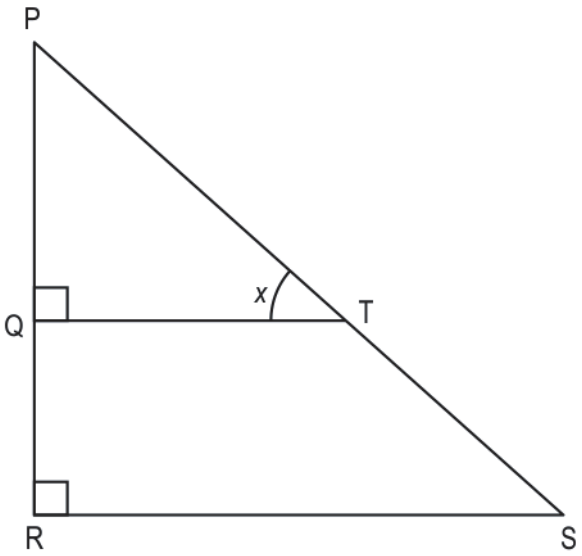
**Q: 7** **Shown below are two paths from Madrid to New York. The dotted line is the shortest distance, approximately 6000 km, on a 2D map. The other solid circular arc is the shortest distance on a 3D globe and the actual path taken by a flight. The arc subtends an angle of 60° at the centre of the circular arc.** **[3]**


**(Note: The figure is not to scale.)**

**What is the actual distance covered by the fight? Draw a rough diagram and show your work.**

***(Note: Take π = 3.14, if needed.)***

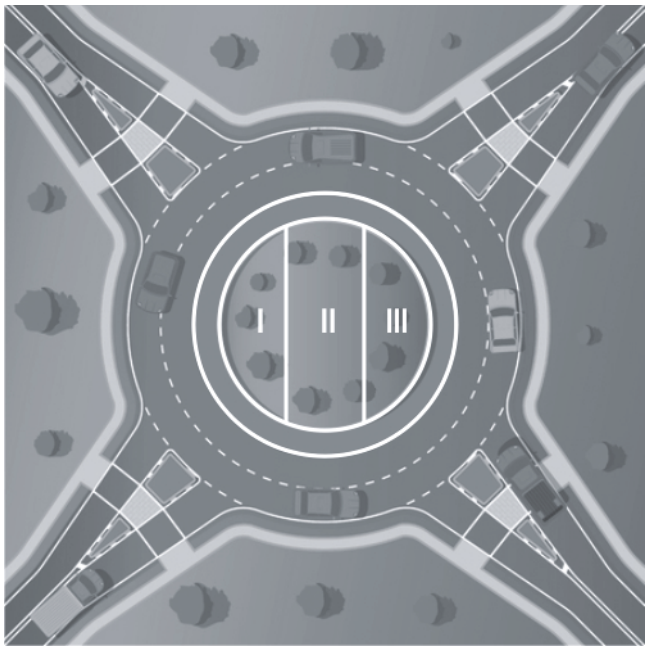
**Q: 8** **In the figure below, tan $x = \frac{3}{4}$, PT = 10 cm and QR = 3 cm.** **[3]**




***(Note: The figure is not to scale.)***

**Find the area of quadrilateral RSTQ. Show your steps and give valid reasons.**

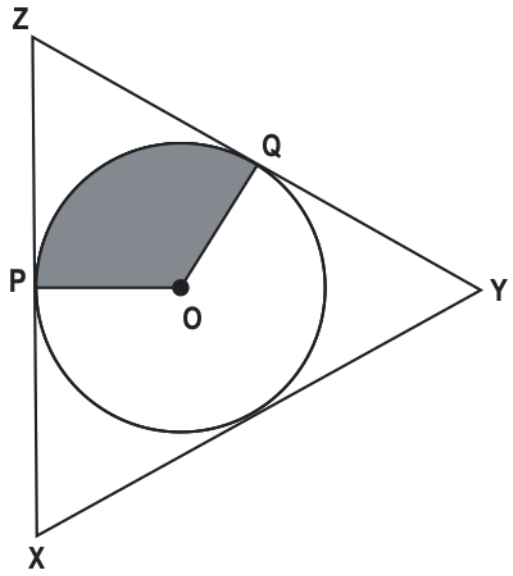
**Q: 9** **At a certain roundabout, the radius of the circular region is 50 ft. The circular region includes a 2 ft wide circular concrete footpath and a flowering bed. The flowering bed is divided into three parts using two dividers of length 48√3 ft each as shown below.** **[5]**




**(Note: The figure is not to scale.)**

**Find the area of regions (I + III), in terms of π. Draw a rough diagram and show your steps.**

**Q: 10** **In the figure below, a circle with centre O is inscribed in an equilateral triangle ΔXYZ. The area of the shaded region of the circle is $2\pi$ cm$^2$.** **[2]**



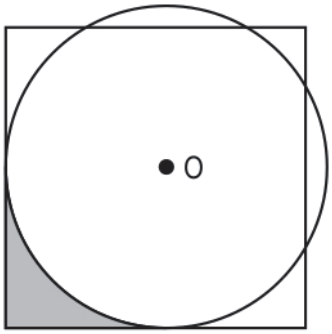

***(Note: The figure is not to scale.)***

**What is the radius of the circle? Show your steps and give valid reasons.**

**Q: 11** **A point P lies on a circle. Two friends, Priya and Sumedha, drew tangents to the circle at point P.** **[1]**

**What is the probability that their tangents coincide? Give a valid reason for your answer.**

**Q: 12** **Shown below is a circle and a square. The circle with centre O has a radius of 4 cm.** **[3]**




**(Note: The figure is not to scale.)**

**Find the area of the shaded region. Show your work and give valid reasons.**

***(Note: If needed, take π as 3.14.)***

**Q: 13** **Given below are two equations.** **[2]**

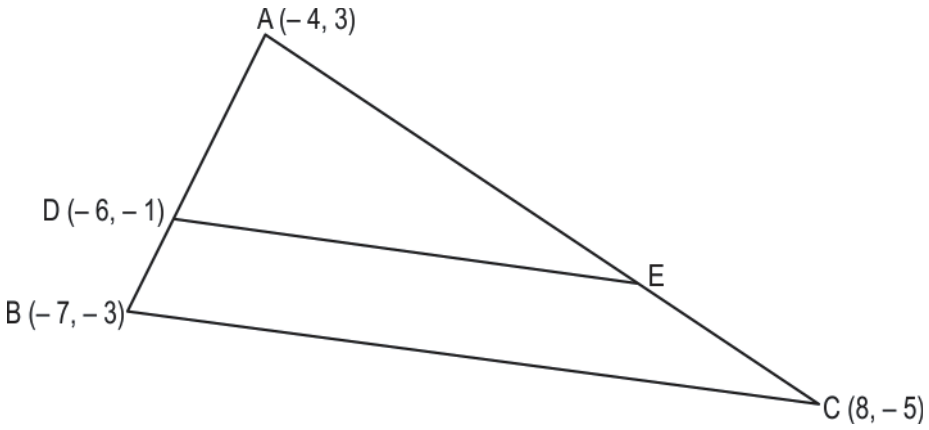
$$49^{(x + 3y)} = 7$$
$$7^{(4x + 12y)} = 49^4$$

**i) Frame a pair of linear equations in two variables by simplifying the given equations.**
**ii) Do these linear equations form a pair of coincident lines? Justify your answer.**

**Q: 14** **A rhombus PQRS has a side length of 5.8 cm and a diagonal QR of length 8.4 cm.** **[2]**

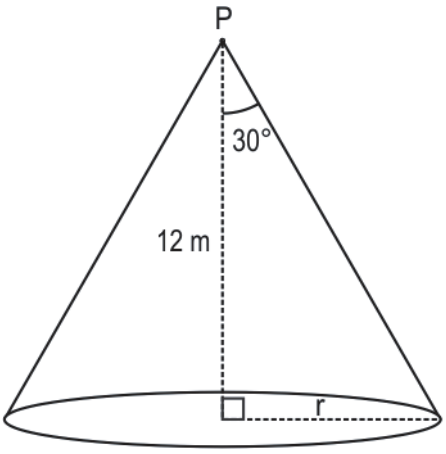
**Use a ruler and compass to draw the rhombus PQRS.**

**Q: 15** **In the figure shown below, DE || BC.** **[3]**

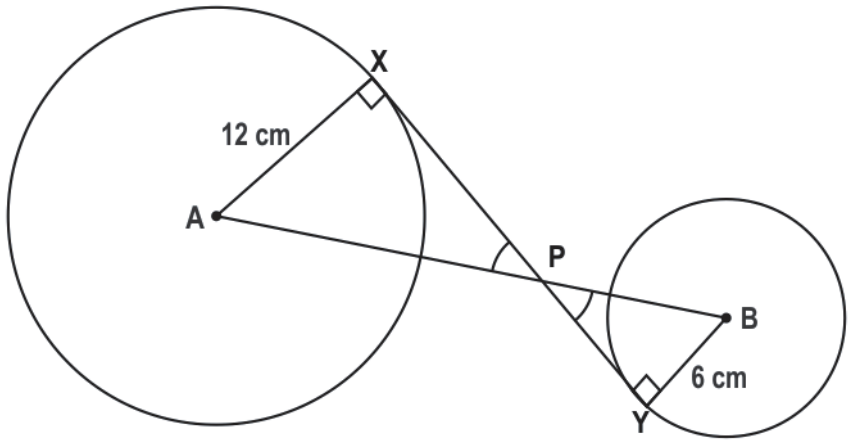


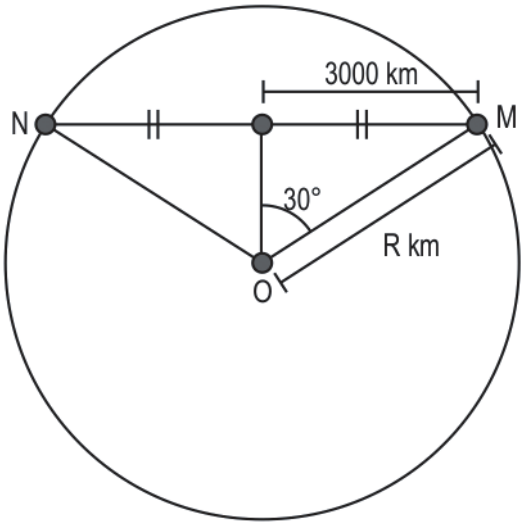

***(Note: The figure is not to scale.)***

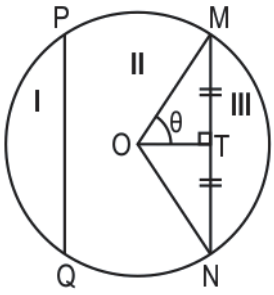
**Find the coordinates of point E. Show your work and give valid reasons.**

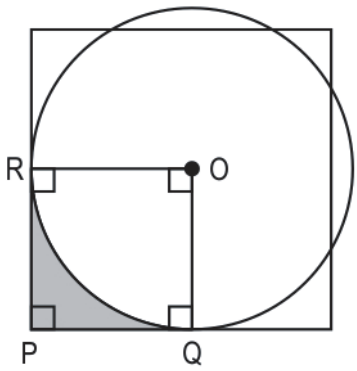
| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| **1** | **Draws a rough figure representing the given information. It may look as follows:**<br>  <br> | **0.5** |
| | **Assumes the radius of the circular region to be *r* and finds the value of *r* as 12(tan30°) = 4√3 m.** | **1.5** |
| | **Finds the area of the illuminated circular region as π × $(4\sqrt{3})^2$ = 48π $m^2$.** | **1** |
| **2** | **Finds the lengths of the tangents RQ and PQ as 10√3 cm each, either by applying the Pythagoras theorem as √($20^2$ - $10^2$) or by considering RQ and PQ as 10tan60°.** | **0.5** |
| | **Finds the area of ΔORQ as $\frac{1}{2}$ × 10 × 10√3 = 50√3 $cm^2$.**<br>**Similarly, finds the area of ΔOPQ as 50√3 $cm^2$.** | **0.5** |
| | **Finds the area of the quadrilateral PQRO as 2 × 50√3 = 100√3 $cm^2$.** | **0.5** |
| | **Finds the area of the minor sector OPR as:**<br>$\frac{120°}{360°} \times \pi \times (10)^2$<br>$= \frac{100\pi}{3}\ cm^2$<br>**(Award 0.5 marks if only the formula for area of a sector is correctly written.)** | **1** |

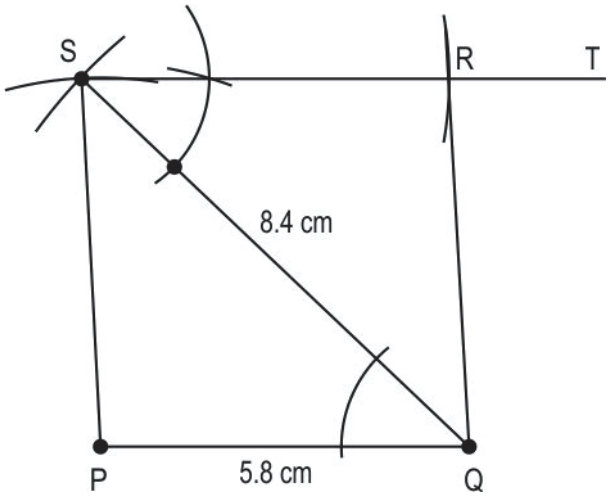
| Q.No | Teacher should award marks if students have done the following: | Marks |
|---|---|---|
| | **Finds the area occupied by the shaded region as:**<br><br>**(Area of the quadrilateral PQRO) - (Area of the minor sector OPR)**<br><br>**$= 100\sqrt{3}\ cm^2 - \frac{100\pi}{3}\ cm^2$**<br><br>**(Award full marks if the correct answer is obtained by an alternate method.)** | **0.5** |
| **3** | **Finds the radius of the circle using the distance formula between (0, 0) and (9, 12) as 15 units.** | **1** |
| | **Concludes that (15, 0) is a point on the circle and the line *m* being a tangent touches the circle at only one point, the line cannot pass through the point (15, 0).**<br><br>**(Award full marks if graphical representation is used to conclude the answer.)** | **1** |
| **4** | **Simplifies the given equation as:**<br><br>**$3\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = 5\frac{1}{\cos\theta}$**<br><br>**$\Rightarrow 3\cos^2\theta + \sin^2\theta - 5\sin\theta = 0$** | **0.5** |
| | **Simplifies the above equation as:**<br><br>**$2\sin^2\theta + 5\sin\theta - 3 = 0$** | **0.5** |
| | **Factorises the above equation to get $\sin\theta = \frac{1}{2}$ or (-3).** | **1** |
| | **Writes that $\sin\theta$ cannot be equal to (-3) as $0 \le \sin\theta \le 1$ and hence concludes that $\sin\theta = \frac{1}{2}$.** | **0.5** |
| | **Uses the above step to find the value of $\theta$ as 30° since $\sin 30° = \frac{1}{2}$.** | **0.5** |
| **5** | **Identifies that at PR = 25 cm, ΔPQR is a right-angled triangle. Hence, for obtuse-angled triangle, PR > 25 cm.** | **1** |
| | **Writes that PR < (24 + 7) cm or PR < 31 cm for PQR to be a triangle.** | **1** |

| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| | **Uses the above steps and finds the probability that a card chosen at random makes ΔPQR an OBTUSE angled triangle at Q as $\frac{5}{40}$ .** | **1** |
| **6** | **Draws the figure by joining AB, AX and YB. The figure may look as follows:**<br>  <br><br>***(Note: The figure is not to scale.)*** | **0.5** |
| | **Writes that in ΔAXP and ΔBYP:**<br>**∠AXY = ∠XYB = 90° (as tangent is ⊥ to radius)**<br>**∠APX = ∠BPY (vertically opposite angles)**<br>**Conclude that ΔAXP ~ ΔBYP by using the AA similarity criterion.** | **1.5** |
| | **Uses the above step and writes:**<br>$\frac{AP}{PB} = \frac{XP}{PY} = \frac{AX}{BY} = \frac{12}{6} = 2$<br>**=> AP = 2PB.**<br>**Uses AP + PB = 30 (given) and AP = 2PB to get PB as 10 cm and AP as 20 cm.** | **1.5** |
| | **Uses the Pythagoras theorem for ΔBYP and finds PY as 8 cm.**<br>**Finds XP as 16 cm by using the ratio XP = 2PY.**<br>**(Award full marks if the Pythagoras theorem is used for ΔAXP and ΔBYP to find the lengths of XP and PY respectively.)** | **1** |

| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| | **Finds the length of the wire, XY as 16 + 8 = 24 cm.** | **0.5** |
| **7** | **Draws a rough diagram representing the given situation. The diagram may look as follows:**<br>  <br><br>***(Note: The figure is not drawn to scale.)*** | **1** |
| | **Finds the radius of the circle, R as $\frac{3000}{\sin 30^\circ}$ = 6000 km.** | **1** |
| | **Finds the actual distance covered by the flight as $\frac{60^\circ}{360^\circ} \times 2 \times 3.14 \times 6000 = 6280$ km.**<br>**(Award 0.5 marks if only the formula for the arc length is correctly written.)** | **1** |
| **8** | **Uses tan $x = \frac{3}{4}$ for ΔPQT and PT = 10 cm to find the length of PQ and QT as 6 cm and 8 cm respectively.** | **1** |
| | **Writes that in ΔPQT and ΔPRS:**<br>**∠PQT = ∠PRS = 90°**<br>**∠QPT = ∠RPS (common angle)**<br>**Concludes that ΔPQT ~ ΔPRS by using the AA similarity criterion.** | **0.5** |

| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| | **Uses the above two steps and writes the relation as:**<br>$\frac{PQ}{PR} = \frac{QT}{RS}$<br>$\Rightarrow \frac{6}{9} = \frac{8}{RS}$<br>$\Rightarrow$ **RS = 12 cm** | **1** |
| | **Finds the area of quadrilateral RSTQ as** $\frac{1}{2} \times (8 + 12) \times 3 = 30$ **sq cm.** | **0.5** |
| **9** | **Draws a rough diagram. The diagram may look as follows:**<br>  <br> | **1** |
| | **Writes that sin θ =** $\frac{24\sqrt{3}}{48} = \frac{\sqrt{3}}{2}$ **and finds the angle subtended by the chord at the centre as 2 × 60° = 120°.** | **1** |
| | **Finds the area of the sector with central angle of 120° as** $\frac{120°}{360°} \times \pi \times (48)^2 = 768\pi$ **ft**$^2$ **.**<br>**(Award 0.5 marks if only the formula for the area of a sector is correctly written.)** | **1** |
| | **Finds the height OT of ΔMON as:**<br>$\cos 60° = \frac{OT}{48}$<br>$\Rightarrow$ **OT = 24 ft**<br>**(Award full marks if Pythagoras' theorem is correctly used to find the height.)** | **1** |
| | **Finds the area of ΔMON as** $\frac{1}{2} \times 48\sqrt{3} \times 24 = 576\sqrt{3}$ **ft**$^2$**.** | **0.5** |

| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| | **Finds the area of the segments (I + III) as $2(768\pi - 576\sqrt{3})$ or $384(4\pi - 3\sqrt{3})$ ft$^2$.** | **0.5** |
| **10** | **Writes that ∠PZQ = 60° and gives the reason that ΔXYZ is an equilateral triangle.**<br><br>**Writes that ∠ZPO = ∠ZQO = 90° and gives the reason that the tangent at any point of a circle is perpendicular to the radius through the point of contact.**<br><br>**Finds ∠POQ as 360° - 90° - 90° - 60° = 120° and gives the reason that the sum of internal angles of a quadrilateral is 360°.** | **1** |
| | **Uses the area of the shaded region of the circle and finds the radius of the circle, *r*, as:**<br><br>**$2\pi = \frac{120°}{360°} \times \pi r^2$**<br><br>**=> $r = \sqrt{6}$ cm** | **1** |
| **11** | **Writes that the probability of their tangents coinciding is 1 and gives a reason. For example, writes that there is only one tangent at any point on the circumference of the circle.** | **1** |
| **12** | **Writes that radius of a circle is perpendicular to the tangent at the point of contact and draws two radii. The figure may look as follows:**<br><br>  <br><br><br>**(Award 1 mark if an equivalent explanation is written without a figure.)** | **1** |
| | **Writes that PQOR is a square and finds its area as 4 × 4 = 16 cm$^2$.** | **0.5** |

| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| | **Writes that QOR is a quadrant and finds its area as $\frac{90}{360} \times 3.14 \times 4 \times 4 = 12.56\ cm^2$.** | **1** |
| | **Finds the area of the shaded region as $16 - 12.56 = 3.44\ cm^2$.** | **0.5** |
| **13** | **i) Frames the pair of linear equations in two variables using the given information as:**<br><br>**$2x + 6y = 1$**<br>**$4x + 12y = 8$**<br><br>**(Award 0.5 marks for each correct linear equation in two variables.)** | **1** |
| | **ii) Writes that these linear equations do not form a pair of coincident lines.** | **0.5** |
| | **Justifies the answer. For example, writes that $\frac{2}{4} = \frac{6}{12} \neq \frac{1}{8}$.** | **0.5** |
| **14** | **Draws the base PQ of length 5.8 cm, diagonal QS of length 8.4 cm and PS of length 5.8 cm.** | **0.5** |
| | **Draws a ray ST parallel to PQ by making ∠PQS equal to ∠TSQ.** | **1** |
| | **Cuts a length of 5.8 cm from S and joins SR and QR to get the rhombus PQRS.** | **0.5** |
| | **The construction may look as follows:**<br><br>  <br> | **0** |

| Q.No | **Teacher should award marks if students have done the following:** | Marks |
|---|---|---|
| **15** | **Assumes that D divides AB in ratio *m* : *n* and uses section formula to write:**<br>**$(-6, -1) = \left(\frac{-7m-4n}{m+n}, \frac{-3m+3n}{m+m}\right)$** | **0.5** |
| | **Finds *m* : *n* as 2:1.** | **1** |
| | **Writes that in ΔABC, according to the Basic Proportionality Theorem (BPT) point E will divide AC in ratio 2:1 since DE \|\| BC and D divides AB in ratio 2:1.** | **0.5** |
| | **Uses the section formula to find the coordinates of point E as:**<br>**$\left(\frac{8(2)+(-4)(1)}{2+1}, \frac{(-5)(2)+3(1)}{2+1}\right)$** | **0.5** |
| | **Evaluates the above expressions to find the coordinates of point E as $(4, \frac{-7}{3})$.** | **0.5** |