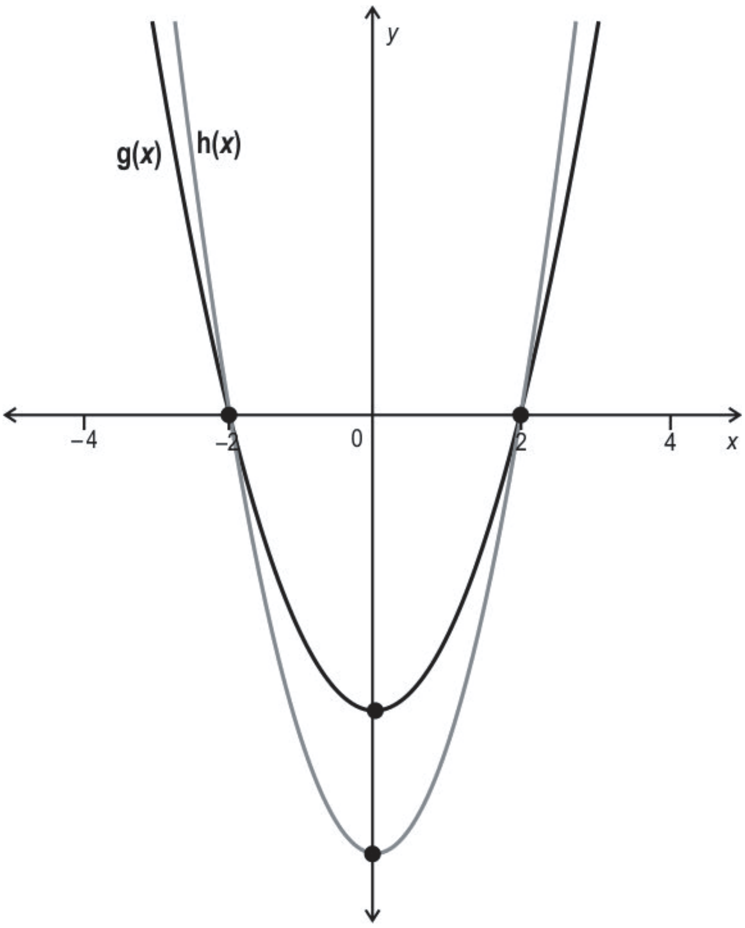
Chapter - 2
Polynomials

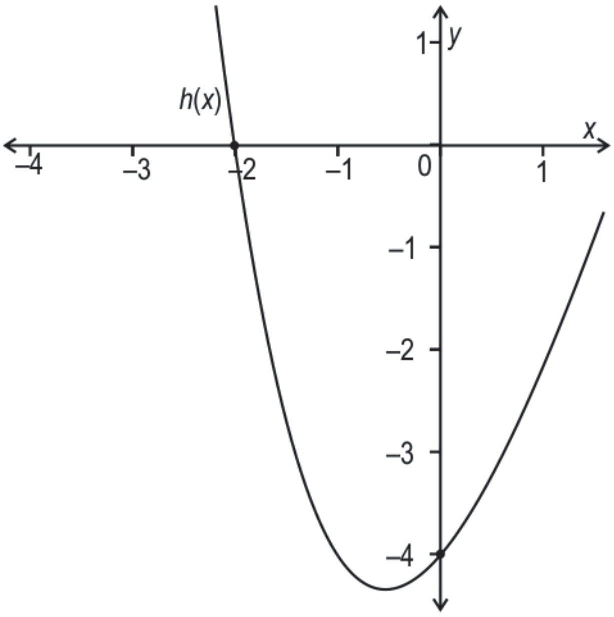
Q: 1 **Shown below are the parts of graphs of two polynomials, *g* (*x*) and *h* (*x*). When *h* (*x*) is divided by (*x* - 3), the remainder is *k*.**



Which of these is true for the remainder when *g* (*x*) is divided by (*x* - 3)?

1. It is less than *k*.
2. It is equal to *k*.
3. It is more than *k*.
4. (cannot conclude without knowing the polynomials)

Q: 2 **Shown below is a part of the graph of a polynomial *h* (*x*).**

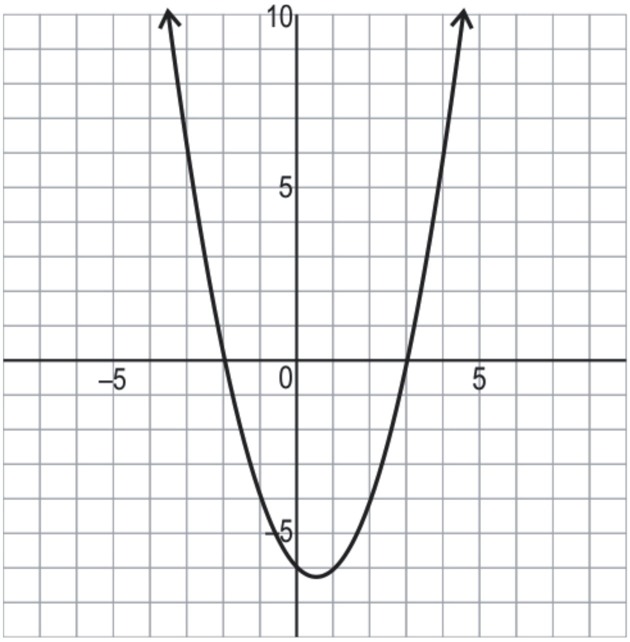


On dividing *h* (*x*) by which of the following will the remainder be zero?

i) (*x* - 2)
ii) (*x* + 2)
iii) (x - 4)
iv) (x + 4)

1 only ii)
2 only i) and iii)
3 only ii) and iv)
4 (cannot be determined without knowing the polynomial *h* (*x*))

Q: 3 **[2]**



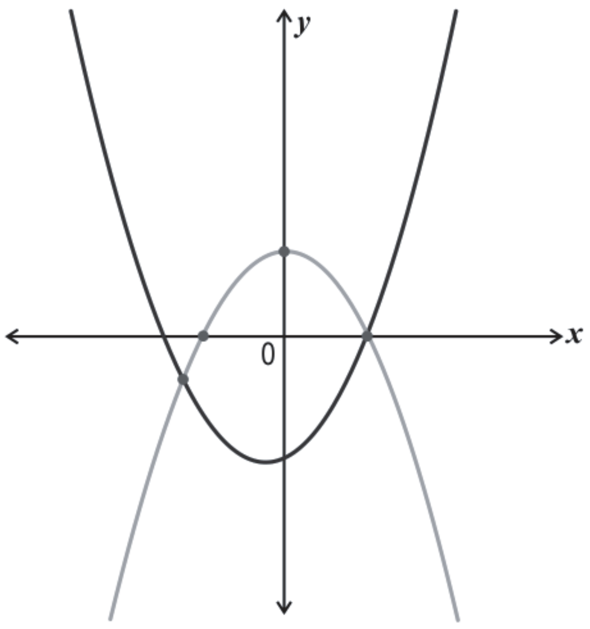
Write a quadratic polynomial whose sum of zeros is less than that of the polynomial shown in the graph above.

Q: 4 **[2]**

$$\frac{x^2 - 3\sqrt{2}x + 4}{x - \sqrt{2}} \; ; x \neq \sqrt{2}$$

At how many points does the graph of the above expression intersect the *x* -axis? Show your work.

Q: 5 Two polynomials are shown in the graph below. **[1]**



Find the number of zeroes that are common to both the polynomials. Explain your answer.

Q: 6 p and q are zeroes of the polynomial $2x^2 + 5x - 4$. **[2]**

Without finding the actual values of p and q, evaluate $(1 - p)(1 - q)$. Show your steps.

Q: 7 A polynomial is given by $q(x) = x^3 - 2x^2 - 9x + k$, where k is a constant. **[3]**

The sum of two zeroes of $q(x)$ is zero.

Using the relationship between the zeroes and coefficients of a polynomial, find the:

i) zeroes of $q(x)$.
ii) value of k.

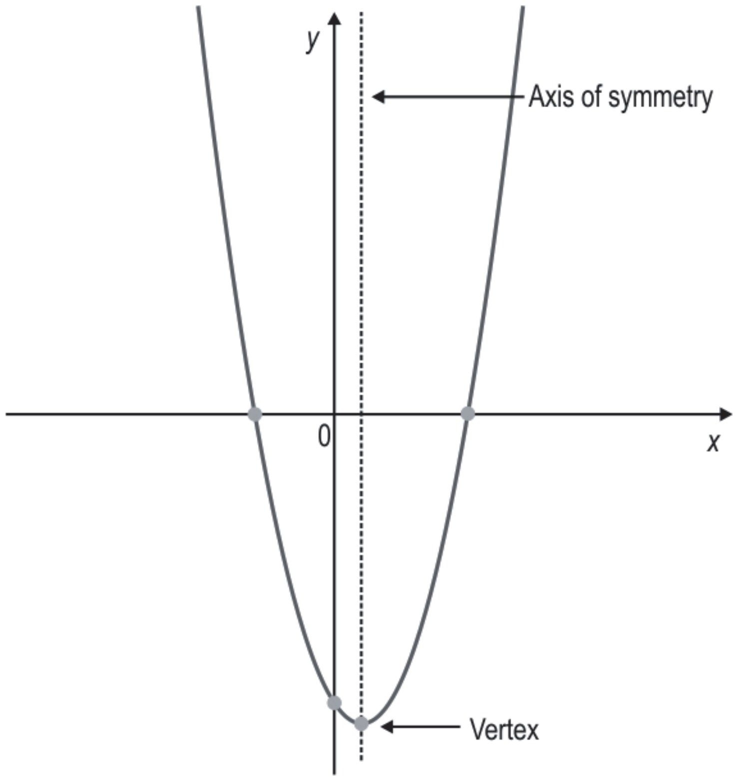
Show your steps.

Q: 8 $p(x) = ax^2 - 8x + 3$, where a is a non-zero real number. One zero of $p(x)$ is 3 times the other zero. **[3]**

i) Find the value of a. Show your work.
ii) What is the shape of the graph of $p(x)$? Give a reason for your answer.

Q: 9 $f(x) = 2x^2 - 4x + k$, where k is a non-zero real number. When $f(x)$ is divided by $(x + 2)$, it leaves a remainder of (-14). **[5]**

i) Find the zeroes of $f(x)$.
ii) Shown below is the graph of $f(x)$. The vertex is the minimum value of $f(x)$ and the dotted line drawn through the vertex is the axis of symmetry of the graph.



At what point does the axis of symmetry intersect the x-axis? Find the minimum value of $f(x)$.

Show your steps.

Q: 10 $p(x) = 2x^2 - 6x - 3$. The two zeroes are of the form: **[2]**

$$\frac{3 \pm \sqrt{k}}{2}$$; Where k is a real number

Use the relationship between the zeroes and coefficients of a polynomial to find the value of k. Show your steps.

Q: 11 Find the distance between the zeroes of the polynomial $f(x) = 2x^2 - x - 6$. Show your steps. **[2]**

Q: 12 $x^4 + ax^3 + bx^2 + 2x + 3 = (x^2 - 2)q(x) - 2x - 3$ where a, b are non-zero real constants and $q(x)$ is a non-zero polynomial. **[5]**

i) Find the values of a and b.
ii) Find the zeroes of $q(x)$.

Show your steps.

Q: 13 **[3]**

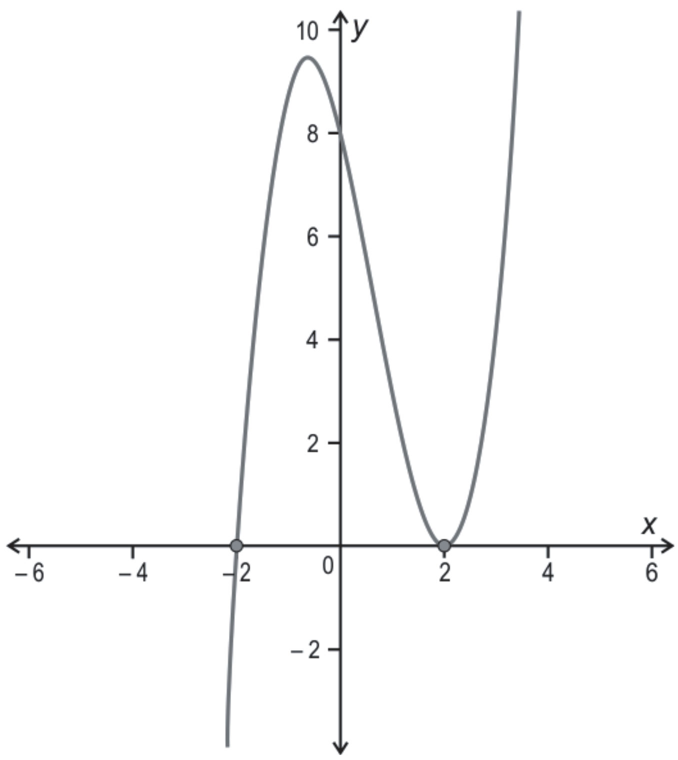
f(x) = $x^3 - ax^2 + (a - 3)x + 6$, where a is a non-zero real number. When $f(x)$ is divided by $(x + 1)$, there is no remainder.

If $f(x)$ is completely factorisable, find the zeroes of $f(x)$. Show your steps.

Q: 14 **One zero of $f(x) = x^3 - 3x^2 + 4$ is 2.** **[2]**

At how many points will the graph of $f(x)$ intersect the x-axis? Show your steps.

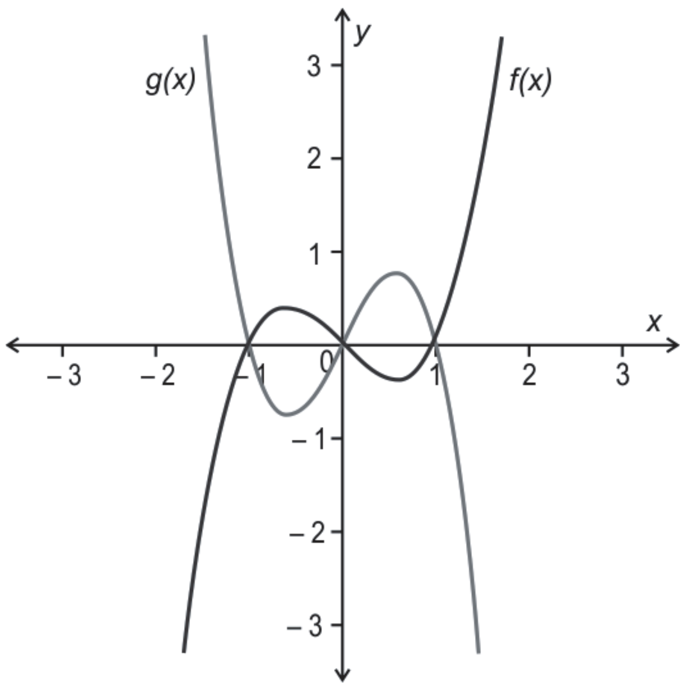
Q: 15 **Students of a class were shown the graph below.** **[1]**



Based on their answers, they were divided into two groups. Group 1 said the graph represented a quadratic polynomial whereas group 2 said the graph represented a cubic polynomial.

i) Which group was correct?
ii) Write the polynomial represented by the graph.

Q: 16 **Shown below are the graphs of two cubic polynomials, $f(x)$ and $g(x)$. Both polynomials have the zeroes (-1), 0 and 1.** **[1]**



Anya said, "Both the graphs represent the same polynomial, $f(x) = g(x) = (x + 1)(x - 0)(x - 1)$ as they have the same zeroes."

Pranit said, "Both the graphs represent two different polynomials, $f(x) = (x + 1)(x - 0)(x - 1)$ and $g(x) = -(x + 1)(x - 0)(x - 1)$ and only two such polynomials exist that can have the zeroes (-1), 0 and 1."

Aadar said, "Both the graphs represent two different polynomials and infinitely many such polynomials exist that have the zeroes (-1), 0 and 1."

Who is right? Justify your answer.

Q: 17 **$p(x) = (x + 3)^2 - 2(x - c)$; where c is a constant.** **[2]**

If $p(x)$ is divisible by x, find the value of c. Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	1
2	1

Q.No	Teacher should award marks if students have done the following:	Marks
3	Identifies the sum of the zeroes of the given polynomial as 3 - 2 = 1.	1
	Writes a quadratic polynomial whose sum of zeroes is less than 1. For example, $x^2 + 3x - 5 = 0$.	1
4	Factorises the numerator to write the given expression as: $$\frac{(x-\sqrt{2})(x-2\sqrt{2})}{x-\sqrt{2}}$$	1
	Writes that the graph of the above expression, $x - 2\sqrt{2}$, intersects the x-axis at exactly one point.	1
5	Finds the number of zeroes that are common to both the polynomials as 1.	0.5
	Explains the answer. For example, the two polynomials intersect at 2 points but only 1 of them lie on the x-axis.	0.5
6	Expands $(1 - p)(1 - q)$ to get $1 - (p + q) + pq$.	0.5
	Finds the sum of the zeroes as $\frac{-5}{2}$.	0.5
	Finds the product of the zeroes as $\frac{-4}{2} = -2$.	0.5
	Uses the above steps to find the value of $(1 - p)(1 - q)$ as $1 - (-\frac{5}{2}) - 2 = \frac{3}{2}$.	0.5
7	i) Assumes the values of zeroes of $q(x)$ as $(-\alpha)$, α and β.	0.5
	Writes the sum of zeroes as: $-\alpha + \alpha + \beta = 2$ Finds β as 2.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes the equation for the sum of the products of zeroes taken two at a time as: $-\alpha^2 - \alpha\beta + \beta\alpha = -9$ Finds α^2 as 9.	1
	Finds the 3 zeroes of $q(x)$ as (-3), 3 and 2.	0.5
	ii) Writes the equation for the product of zeroes as $(-\alpha^2\beta) = (-k)$ and finds the value of k as 18.	0.5
8	i) Assumes the roots of $p(x)$ to be α and β to write the relation $\alpha = 3\beta$.	0.5
	Writes the sum of the roots as $4\beta = \frac{8}{a}$ to get β as $\frac{2}{a}$.	0.5
	Writes the product of the roots as $3\beta^2 = \frac{3}{a}$ to get a as 4.	1
	ii) Writes that, since a is positive, the graph of $p(x)$ is an open upward parabola or open upwards like U.	1
9	i) Writes that, since remainder of $\frac{f(x)}{(x+2)}$ is -14, therefore, $f(-2) = -14$.	0.5
	Uses the above step to write the equation as: $2(-2)^2 - 4(-2) + k = -14$ Finds the value of k as -30.	1
	Factorises $f(x)$ as $(2x + 6)(x - 5)$ and finds the zeroes as -3 and 5.	1.5
	ii) Finds the point at which the axis of symmetry intersects the x-axis as the average of the two zeroes: $\frac{(-3+5)}{2} = 1$.	1
	Finds the minimum value of $f(x)$ as: $f(1) = 2(1)^2 - 4(1) - 30 = -32$.	1

Q.No	Teacher should award marks if students have done the following:	Marks
10	Writes the equation for the product of zeroes as: $\left(\frac{3+\sqrt{k}}{2}\right)\left(\frac{3-\sqrt{k}}{2}\right)=\frac{-3}{2}$	1
	Simplifies the above equation and writes: $\frac{9-k}{4}=\frac{-3}{2}$	0.5
	Solves the above equation and finds the value of k as 15.	0.5
11	Factorises $f(x)$ as $(x - 2)(2x + 3)$.	1
	Writes $f(x) = 0$ and finds the coordinates of the zeroes as (2, 0) and ($\frac{-3}{2}$, 0). (Award full marks if only the zeroes of $f(x)$ are written.)	0.5
	Finds the distance between the zeroes as $\frac{7}{2}$ units.	0.5
12	i) Writes the given equation as $x^4 + ax^3 + bx^2 + 4x + 6 = (x^2 - 2)\, q(x)$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Divides $x^4 + ax^3 + bx^2 + 4x + 6$ by $(x^2 - 2)$ to get $q(x)$ as $x^2 + ax + b + 2$. For example: Quotient: $x^2 + ax + b + 2$ $x^2 - 2 \,\big)\, x^4 + ax^3 + bx^2 + 4x + 6$ $-\;\; x^4 - 2x^2$ $ax^3 + x^2(b + 2) + 4x + 6$ $-\;\; ax^3 - 2ax$ $x^2(b + 2) + x(2a + 4) + 6$ $-\;\; x^2(b + 2) - 2b - 4$ $x(2a + 4) + 2b + 10$	2
	Equates the coefficient of x in the remainder to 0 and finds the value of a as -2.	0.5
	Equates the constant term in the remainder to 0 and finds the value of b as -5.	0.5
	ii) Uses step 3 and writes $q(x)$ as $x^2 - 2x - 3$.	0.5
	Factorises $q(x)$ as $(x + 1)(x - 3)$ and finds its zeroes as -1 and 3.	1
13	Writes that, since $f(x)$ is divisible by $(x + 1)$, $f(-1) = 0$ and finds the value of a as 4.	0.5
	Uses the above step and writes $f(x)$ as $x^3 - 4x^2 + x + 6$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Divides $f(x)$ by $(x + 1)$ and finds the quotient as $x^2 - 5x + 6$. For example: $x^2 - 5x + 6$ $x + 1 \overline{)\, x^3 - 4x^2 + x + 6}$ $-\; x^3 + x^2$ $\overline{\quad -5x^2 + x + 6}$ $-\; -5x^2 - 5x$ $\overline{\quad 6x + 6}$ $-\; 6x + 6$ $\overline{\quad 0}$	1
	Factorises the quotient as $(x - 2)(x - 3)$.	0.5
	Finds the zeroes of $f(x)$ as (-1), 2 and 3.	0.5
14	Divides $f(x)$ by $(x - 2)$ and finds the quotient as $x^2 - x - 2$. For example: $x^2 - x - 2$ $x - 2 \overline{)\, x^3 - 3x^2 + 4}$ $-\; x^3 - 2x^2$ $\overline{\quad -x^2 + 4}$ $-\; -x^2 + 2x$ $\overline{\quad -2x + 4}$ $-\; -2x + 4$ $\overline{\quad 0}$	1
	Factorises the quotient as $(x - 2)(x + 1)$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Concludes from the above step that the graph of $f(x)$ intersects the x-axis at two points.	0.5
15	i) Writes that group 2 was correct.	0.5
	ii) Writes the polynomial represented by the graph as $(x - 2)^2(x + 2)$.	0.5
16	Writes that Aadar is right and gives a justification. For example, the factored form of a cubic polynomial with the zeroes (-1), 0 and 1 can be written as $k(x + 1)(x - 0)(x - 1)$ where k is an integer.	1
17	Writes the given polynomial as: $p(x) = x^2 + 9 + 4x + 2c$	0.5
	Writes that, if $p(x)$ is divisible by x, $p(0) = 0$. OR Writes that the remainder of $\frac{p(x)}{x}$, which is $9 + 2c$, should be 0.	1
	Finds the value of c as $\frac{-9}{2}$.	0.5