## Chapter - 4 Quadratic Equations

Q: 1 Which of these is a QUADRATIC equation having one of its roots as zero?
i) $x^{3}+x^{2}=0$
ii) $x^{2}-2 x=0$
iii) $x^{2}-9=0$

1 only i) 2 only ii) 3 only i) and ii) 4 only ii) and iii)

## Q: 2 Aman solved a quadratic equation and found its roots to be real.

Which of these could represent the graph of the equation Aman solved?
i



1. only iii)

3 only iii) and iv)
ii

iv


2 only i) and ii)
4 only i), ii) and iv)

Q: 3 Three students were asked how they would verify their solution of a quadratic
equation, $(x-2)(x-5)=0$. Shown below are their responses.
Student 1 said, "In the first bracket, $x$ must equal 2 , and in the second bracket, $x$ must equal 5. So, (2-2)(5-5) = 0."

Student 2 said, "In the first bracket, $x$ must equal 2, but in the second bracket, $x$ can have any real number value. For example, (2-2)(3-5) =0 or (2-2)(10-5) = 0."

Student 3 said, "Both brackets should always have the same $x$ value. So, $x$ is either 2 or 5 in both brackets. For example, (2-2)(2-5) = 0 and (5-2)(5-5) =0."

Whose response is correct?
1 only student 1
2 only student 3
3 only students 1 and 2
4 all students - 1, 2 and 3

Q: 4
$3(3)^{2 m}+11(3)^{m}=4$

Use the substitution $3^{m}=x$ to solve for $m$. Show your steps.

Q: $5(2 x+1)^{3}=8 x\left(x^{2}+1\right)+3$
Write the zeroes of the above equation in the form ( $x, y$ ). Show your steps.

Q: 6 Shown below is the graph of a quadratic polynomial, $y(x)=p x^{2}+q x+r$.


Find the nature of the roots of the quadratic equation, $p x^{2}+q x+r=0$. Give a reason for your answer.

Q: 7 The graph below represents the path of a ball thrown by Ankush. The maximum height, $h$, the ball reaches with respect to time, $t$, is represented by the polynomial $h$ $(t)=-t^{2}+\frac{19}{4} t+\frac{5}{4}$.

(Note: The figure is not to scale.)
How long will it take for the ball to reach the ground? Show your steps.

Q: 8 Check whether the following equation is quadratic.
i) $(x-4)^{3}+20=x^{3}+2 x$
ii) $\frac{2}{x^{2}}+4 x+3=0$

Justify your answer in each case.

Q: 9 Arpit was asked to represent the following statements in the form of a quadratic equation:
"The sum of the squares of two positive integers is 225 . The square of the larger number is $\mathbf{1 6}$ times the smaller number."

If Arpit wrote the equation correctly, what could he have written?

Q: 10 Find the solution(s) of the following equation.

$$
(y-1)(y-3)\left(\frac{1}{y-3}+\frac{2 y}{y-1}\right)=2 ; y \neq 1,3
$$

Show your steps and give valid reasons.

Q: 11 One of the solutions of the following equation is $\mathbf{- 7}$ where $k$ is a constant.
$z^{2}-k z-28=0$
i) Find the value of $\boldsymbol{k}$.
ii) Find the other solution.

Show your steps.

Q: 12 When a marble is dropped from an initial height, $d$ metres, with an initial speed, $v \mathbf{m} / \mathrm{s}$, [2] the height of the marble at time $t$ is represented by $h(t)=v t-2 t^{2}+d$.

A marble is dropped from a height of 48 m with an initial speed, $\mathbf{1 0} \mathrm{m} / \mathrm{s}$. How long does it take for the marble to hit the ground? Show your steps and give valid reasons.

Q: 13 The roots of the equation below are real numbers.
$2 x^{2}-20 x+c=0$
For what value of $c$, the roots will be equal? Show your steps and give valid reasons.

Q: 14 Find all the possible value(s) of $\boldsymbol{x}$ for the following equation to be true.
$\sqrt{(15-2 x)}-x=0$

Show your steps and give valid reasons.

Q: 15 Shown below is the graph of a quadratic equation $y=\left(x^{2}+k x+12\right)$.


Without finding the value of $\boldsymbol{k}$, find the two roots of the given quadratic equation. Show your steps.

Math

Q: 16 A quilt maker has a rectangular quilt measuring 12 units by 8 units. He wants to add a [3] border to it as shown in the figure below. He has $\mathbf{6 4} \mathbf{~ s q ~ u n i t s ~ o f ~ f a b r i c ~ f o r ~ t h e ~ b o r d e r . ~}$

(Note: The figure is not to scale.)
i) If $\boldsymbol{x}$ and $2 \boldsymbol{x}$ are the widths of the border as shown, frame a quadratic equation using the total area enclosed by the new quilt (with the border).
ii) Find the measures of the new quilt (with the border).

Show your work along with valid reasons.

Q: 17 Ayush used the quadratic formula to solve a quadratic equation in $\boldsymbol{y}$ to get:

$$
y=\frac{7 \pm \sqrt{169}}{10}
$$

Write a quadratic equation Ayush could have been solving. Show your steps.

Study the given information and answer the questions that follow.
Bangalore city corporation is building parks in residential areas across the city. Shown below is one such park. The rectangular park consists of various components like walking track, kids play area, open gym, pond etc.


Length
(Note: The image is not to scale.)

Q: 18 Gate 3 has been placed exactly opposite to gate 1 on the boundary of the park. The distance between gate $\mathbf{3}$ and gate $\mathbf{2}$ is $\mathbf{1} \mathbf{~ m}$ more than the distance between gate $\mathbf{3}$ and gate 2.

The shortest distance between gates $\mathbf{1}$ and $\mathbf{2}$ is $\mathbf{2 9} \mathbf{~ m}$, find the width of the park. Show your work.

Q: 19 The caretaker of the park is attempting to plant saplings in the form of a square. That [2] is, number of rows of saplings is the same as the number of columns of saplings. On arranging the saplings, he found that 24 saplings were still left with him. When he increased the number of rows and columns by $\mathbf{1}$, he found that he was short of $\mathbf{2 5}$ saplings.

Find the number of saplings available with him. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

| Q.No | Correct Answers |
| :--- | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 | 2 |


| Q.No | Teacher should award marks if students have done the following: | Marks |
| :---: | :---: | :---: |
| 4 | Substitutes $\mathbf{3 m}=x$ and writes the given equation as: $3 x^{2}+11 x-4=0$ | 0.5 |
|  | Simplifies the above equation as: $(3 x-1)(x+4)=0$ | 0.5 |
|  | Finds the roots of the above equation as $\frac{1}{3}$ and -4. | 0.5 |
|  | Equates $3 \boldsymbol{m}$ to $\frac{1}{3}$ and finds the value of $\boldsymbol{m}$ as $\mathbf{- 1}$ | 0.75 |
|  | Equates $3 \boldsymbol{m}$ to -4 and writes that it is not possible as 3 to any variable power cannot result in a negative number. | 0.75 |
| 5 | Simplifies the given equation as: $8 x^{3}+12 x^{2}+6 x+1=8 x^{3}+8 x+3$ | 1 |
|  | Simplifies the above equation by combining like terms and taking out common factors as: $6 x^{2}-x-1=0$ | 0.5 |
|  | Factorises the above equation as: $(3 x+1)(2 x-1)=0$ | 1 |
|  | Writes the zeroes of the above equation in the form ( $x, y$ ) as: $\left(-\frac{1}{3}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$ | 0.5 |
| 6 | Writes that the quadratic equation, $p x^{2}+q x+r=0$, has no real roots since the given graph does not intersect the $x$-axis. | 1 |
| 7 | Writes the equation for the height when it reaches the ground as: $-4 t^{2}+19 t+5=0$ | 0.5 |


| Q.No | Teacher should award marks if students have done the following: | Marks |
| :---: | :---: | :---: |
|  | Re-writes the above equation as: $(4 t+1)(-t+5)=0$ | 1 |
|  | Finds the roots of the above equation as $-\frac{1}{4}, 5$ and writes that the ball reaches the ground after 5 seconds. | 0.5 |
| 8 | i) Writes that the equation is of degree 2 or writes that the simplified form is $\mathbf{- 1 2} \mathbf{x}^{\mathbf{2}}$ $+46 x-44=0$, and hence it is quadratic. | 0.5 |
|  | ii) Writes that the equation is not of degree 2 and hence it is not quadratic. | 0.5 |
| 9 | Writes $x^{2}+16 x-225=0$, where $x$ is the smaller number. <br> (Award full marks for any equivalent equation.) | 1 |
| 10 | Simplifies the given equation as $2 y^{2}-5 y-3=0$. | 0.5 |
|  | Factorises the above equation as: $(y-3)(2 y+1)=0$ | 1 |
|  | Concludes that the solution of the original equation is $\boldsymbol{y}=\frac{-1}{2}$ as $\mathbf{y} \neq 3$. | 0.5 |
| 11 | i) Substitutes $z=(-7)$ in the given equation and finds the value of $\boldsymbol{k}$ as -3. | 1 |
|  | ii) Substitutes $\boldsymbol{k}=\mathbf{- 3}$ in the given equation and factorises the LHS as $(z+7)(z-4)$. | 0.5 |
|  | Concludes that the other solution is $z=4$. <br> (Award the full 2 marks if the relationship between the roots and the coefficients of the quadratic polynomial is used to find the other solution and the value of $\boldsymbol{k}$.) | 0.5 |
| 12 | Identifies $d$ as $48 \mathrm{~m}, v$ as $10 \mathrm{~m} / \mathrm{s}$ and writes that when the marble hits the ground, $h(t)=0$. | 0.5 |

Math
Chapter 4 - Quadratic Equations CLASS 10

| Q.No | Teacher should award marks if students have done the following: | Marks |
| :---: | :---: | :---: |
|  | Substitutes the above values in the given equation, factorises it and finds the roots as $t=8$ and $t=-3$. | 1 |
|  | Mentions that $\mathbf{t} \neq \mathbf{- 3}$ as time cannot be negative. <br> Concludes that it takes 8 seconds for the marble to hit the ground. | 0.5 |
| 13 | Writes that the roots will be equal when the value of the discriminant is zero. i.e., $(-20)^{2}-4(2) c=0$. | 0.5 |
|  | Finds the value of $c$ as $\frac{400}{8}=50$. | 0.5 |
| 14 | Brings down the given equation to the standard form as: $x^{2}+2 x-15=0$ | 1 |
|  | Factorises the above equation as: $(x+5)(x-3)=0$ | 0.5 |
|  | Solves the above equation to get $x=(-5)$ and $x=3$. | 0.5 |
|  | Substitutes $x=(-5)$ in the original equation and writes that LHS $\neq$ RHS. | 0.5 |
|  | Substitutes $x=3$ in the original equation and writes that LHS = RHS. Concludes that $x=3$. | 0.5 |
| 15 | Writes the equation for the product of the roots as (-4-d)(-4+d)=12. | 1 |
|  | Solves the above equation and finds the value of $d$ as (-2) or 2 . | 0.5 |
|  | Uses $\boldsymbol{d}$ as 2 since it is the distance and finds the two roots of the given quadratic equation as (-2) and (-6). | 0.5 |
| 16 | i) Finds the area of the original quilt as $12 \times 8=96$ sq units. <br> Finds the area of the quilt with the border as $96+64=160$ sq units. | 0.5 |


| Q.No | Teacher should award marks if students have done the following: | Marks |
| :---: | :---: | :---: |
|  | Frames a quadratic equation using the total area of the quilt as: $\begin{aligned} & (4 x+12)(2 x+8)=160 \\ & \Rightarrow 8 x^{2}+56 x-64=0 \end{aligned}$ | 0.5 |
|  | ii) Solves the quadratic equation obtained in the previous step and finds the roots as (-8) and 1. | 1 |
|  | Mentions that $x \neq(-8)$ as length cannot be negative. <br> Concludes that the value of $\boldsymbol{x}$ is 1 unit. | 0.5 |
|  | Finds the length and breadth of the new quilt as $\mathbf{1 6}$ units and 10 units respectively. | 0.5 |
| 17 | Finds the value of $\boldsymbol{c}$ as $\frac{169-49}{-4 \times 5}=-6$. | 0.5 |
|  | Writes a quadratic equation Ayush could have been solving. For example, $5 y^{2}-7 y-$ $6=0$. | 0.5 |
| 18 | Takes the distance between gates 1 and 3 as ' $x$ ' $m$. Writes that the distance between gates 2 and 3 is $(x+1) m$. <br> Applies Pythagoras theorem to the triangle formed by gates 1, 2 and 3 and frames a quadratic equation as: $x^{2}+(x+1)^{2}=29^{2} \text { or } x^{2}+x-420=0$ | 1 |
|  | Solves the above quadratic equation and finds the value of $\boldsymbol{x}$ as $\mathbf{2 0 m}$. Concludes that the width of the park is 20 m . | 1 |
| 19 | Takes the former number of rows and columns of saplings as $x$ and finds the number of saplings available as $x^{2}+24$. | 0.5 |
|  | Takes the latter number of rows and columns of saplings as $(x+1)$ and finds the number of saplings available as $(x+1)^{2}-25$. | 0.5 |

Math

| Q.No | Teacher should award marks if students have done the following: | Marks |
| :--- | :--- | :---: |
| Equates the above two quadratic expressions and solves for $x$ as:  <br> $x^{2}+24=x^{2}+1+2 x-25$ <br> $=>x=24$ 0.5 <br>  Finds the number of saplings available with the caretaker as $24^{2}+24=600$. 00.5 |  |  |

