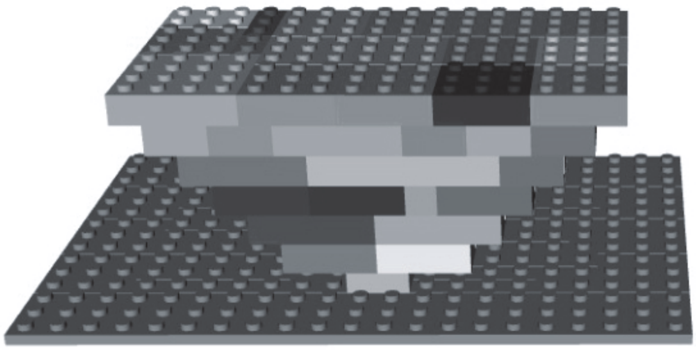
Chapter - 5
Arithmetic Progressions

Q: 1 **The cylindrical bumps on top of lego blocks are called studs.**

Pragun has built a solid inverted lego pyramid as shown below. The number of studs in successive floors forms an arithmetic progression. Pragun figures out that the sum of the number of studs used in the first p floors is given by ($6p^2 - 2p$).



(Note: The figure is only for visual representation.)

How many studs are there in the 5th floor?

1 140 2 88 3 64 4 52

Q: 2 **4 groups in a class were asked to come up with an arithmetic progression (AP). Shown below are their responses:**

Group	Arithmetic progression
M	4, 2, 0, -2, ...
N	41, 38.5, 36, 33.5, ...
O	-19, -21, -23, -25, ...
P	-3, -3, -3, -3, ...

Which of these groups correctly came up with an AP?

1 only groups M and O
2 only groups N and O
3 only groups M, N and O
4 all groups - M, N, O and P

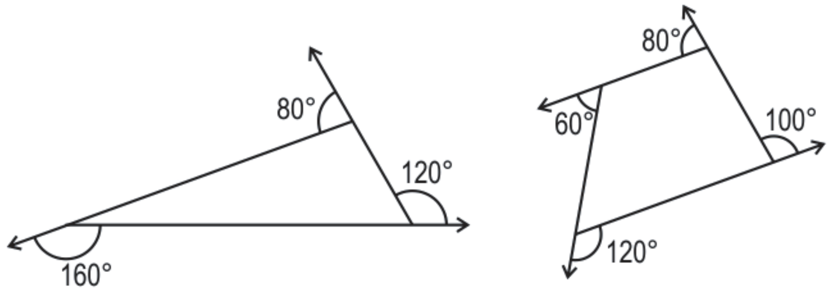
Q: 3 **Priya is preparing for the Bicycle Marathon. Her racing bicycle has a device to calculate the number of kilometres she cycled. She decides to increase the distance she cycles everyday by a fixed number of kilometres.** **[5]**
(i) On the first day Priya cycled 8 km. In 10 days she cycled a total of 170 km. How many kilometres did she cycle on the 3rd day?

(ii) Priya plans to go on a cycle tour from Bangalore to Mangalore covering 425 Km. She travels 20 km on day 1 and increases the distance covered each day by 5km. In how many days will she reach her destination?

Q: 4 **$\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}$ ...** **[1]**

Is the above pattern in AP? Justify your answer.

Q: 5 **The exterior angles marked in each of the polygons below are in arithmetic progression.** **[3]**



Minal drew one such polygon with *n* sides. The smallest exterior angle is 8° and each subsequent angle is 4° more than the previous angle.

Find the number of sides of the polygon that Minal had drawn. Show your steps.

Q: 6 **Consider the list of numbers below.** **[1]**

$\frac{1}{2} + k, \frac{2}{3} + k, \frac{3}{4} + k, \frac{4}{5} + k$, **... where *k* is an integer.**

Is the above list of numbers an arithmetic progression? Justify your answer.

Q: 7 **Sana decided to start practicing for an upcoming marathon. She decided to gradually increase the duration. She ran for 10 mins on day 1 and increased the duration by 5 minutes every day.** **[3]**

From which day onwards will she be running for 2.5 hours or more? Show your work.

Q: 8 **Two arithmetic progressions have the same first term. The common difference of one progression is 4 more than the other progression. 124th term of the first arithmetic progression is the same as 42nd term of the second.** **[3]**

Find one set of possible values of the common differences. Show your work.

Q: 9 **The sum of the first two terms of an arithmetic progression is the same as the sum of the first seven terms of the same arithmetic progression.** **[2]**

Can such an arithmetic progression exist? Justify your answer.

Q: 10 **Animation is a method in which a sequence of images are manipulated to appear as moving objects. An animation specialist wants to show the growth of a sapling into a tree through animation. She follows the steps below:** **[2]**

♦ She develops the first image by designing a sapling containing a certain number of leaves.
♦ She develops the second image by adding 15 leaves to the first image.
♦ She develops the third image by adding 22 leaves to the second image.
♦ Then the fourth image by adding 29 leaves to the third image and so on.

If she continues the process in the same manner, how many leaves will she be adding to the 25th image to develop the 26th image? Show your work.

Q: 11 **14, 21, 28, 35,... and 26, 39, 52, 65,... are two arithmetic progressions such that the *p* th term of the first arithmetic progression is the same as the *q* th term of the second arithmetic progression.** **[2]**

Derive a relationship between *p* and *q*. Show your work.

Q: 12 **A stone is thrown into still water and the figure below represents the concentric circular phenomenon known as ripple effect. The radius of the first circle is 3 cm.** **[2]**

(Note: The figure is not to scale.)

If the radius of each subsequent circle is 4 cm more than the previous, which circle has a radius of 43 cm? Show your steps.

Q: 13 **Ramit is moving towards a stationary source of sound at a constant speed. At his initial position, he hears a pitch of 5 hertz.** **[3]**

If the pitch from the source increases by 4 hertz for every 5 m Ramit travels, find the distance he would have travelled towards the source when the pitch is exactly 149 hertz. Show your steps.

Q: 14 **Given below are the details of an experiment using a bucket and a mug to understand water consumption.** **[3]**

The bucket's volume is 30 litres and the mug's volume is $\frac{1}{20}$ of the bucket. The bucket has 1 litre of water before the tap is turned on. The tap is filling the bucket at a constant rate of 0.1 litres per second. Every 30 seconds, he takes a mug full of water from the bucket.

i) Write an arithmetic progression for the volume of water in the bucket every 30 seconds.
ii) Find the volume of water in the bucket after exactly 7.5 minutes. Show your work.

(Note: Assume no spillage of water.)

Q: 15 **Kevin is baking a tall layered wedding cake as shown below. The customer has ordered a 111 kg cake and 12 layers.** **[2]**

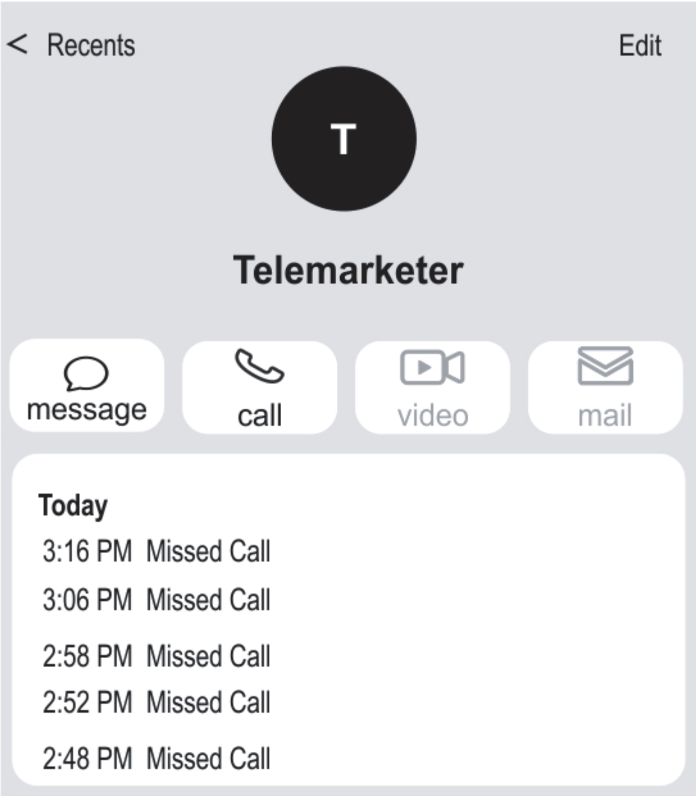
(Note: The image is for visual representation only.)

For the cake to stand properly, he makes the bottom-most cake of 17.5 kg and reduced the weight of each layer such that the difference in the weights of the consecutive layers is the SAME.

i) By what weight does he reduce each subsequent layer?
ii) What is the weight of the lightest cake layer?

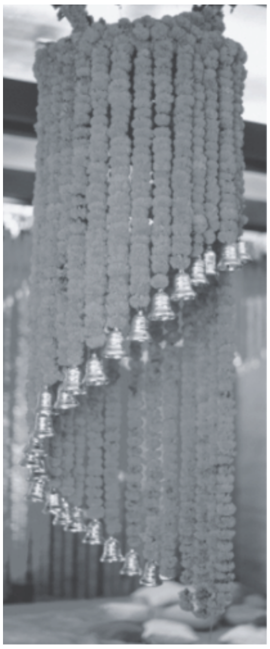
Show your work.

Q: 16 **Parth was receiving spam calls from a telemarketing centre. He got the first call at 2:48 pm as shown below.** **[3]**



If the telemarketer continues to call using the same pattern, at what time will Parth receive the 13th call? Show your work.

Q: 17 **Shivam bought a large quantity of marigold flowers to decorate his house for a family function. He used 630 flowers to recreate the pattern shown below. He used 7 flowers in the shortest garland and 35 flowers in the longest garland.** **[1]**



(Note: The figure is for visual representation only.)

If he kept the difference between two consecutive garlands the same, how many garlands did he make? Show your steps.

Q: 18 **Huner said, "The value of the 20th term of ANY arithmetic progression is double that of the 10th term."** **[2]**

Is Huner's statement correct? Justify your answer.

Q: 19 **The average of an Arithmetic Progression with 151 terms is zero. One of its terms is zero.** **[3]**

Which term of the Arithmetic Progression is zero? Show your steps.

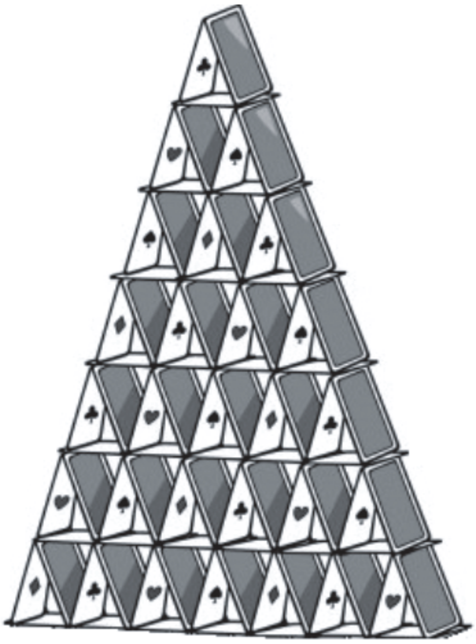
Q: 20 **The ratio of the sum of the first 11 terms of an arithmetic progression to the sum of its first 21 terms is given by 1:4.** **[3]**

i) Show that $23a + 10d = 0$, where a is the first term and d is the common difference of the arithmetic progression.
ii) Write an expression for the n th term of the above arithmetic progression only in terms of a and n.

Show your work.

Answer the questions based on the given information.

Shown below is a house of cards, a structure created by stacking playing cards on top of each other in the shape of a pyramid. Each small triangle is made using 3 cards and each layer has 1 less triangle than the layer below it.



Ankit and his friends were having a sleepover and wanted to do something fun. One of the friends suggested that they could make a house of cards.

Q: 21 **Ankit and his friends want to use 3 cards in the top layer and 18 in the bottom layer.** **[1]**

Form an AP showing the number of cards in each layer starting from the top layer.

Q: 22 **Ankit is planning to make a pyramid with the top and bottom layer containing 15 and 138 cards respectively.** **[2]**

How many layers will such a pyramid have? Show your work.

Q: 23 **They have a total of 360 cards with them.** **[2]**

Find the maximum number of layers that Ankit and his friends can make using the cards they have, if they want to have 1 triangle (3 cards) at the top layer. Show your work.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	4

Q.No	Teacher should award marks if students have done the following:	Marks
3	(i) Applies the formula for the sum of n terms correctly and finds the value of the common difference d as 2.	1
	Either applies the formula for the nth term or generates the pattern with a=8 and d=2 to find the distance cycled on the 3rd day as 12 km.	1
	(ii) Applies the formula for the sum of n terms of AP and represents the given scenario mathematically as $n^2 + 7n - 170 = 0$.	2
	Solves the above quadratic equation correctly and finds n as 10 days.	1
4	Answers that the pattern is in AP.	0.5
	Mentions that the series is in AP because the common difference is the same which is $2\sqrt{2}$.	0.5
5	Writes the equation for the sum of the arithmetic progression as: $\frac{n}{2}[16 + 4(n - 1)] = 360$	1
	Simplifies the above equation as: $n^2 + 3n - 180 = 0$	0.5
	Finds the roots of the above equation as 12, -15.	1
	Finds the number of sides of the polygon that Minal had drawn as 12.	0.5
6	Finds the difference between the consecutive terms of the given list of numbers as $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, etc and writes that the difference between the consecutive terms is not a constant.	0.5
	Concludes that the given list of numbers is not an arithmetic progression.	0.5
7	Converts 2.5 hours to minutes as $2.5 \times 60 = 150$ minutes.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Identifies that the increase in the running duration follows an arithmetic progression given by 10, 15, 20, 25,... and assumes the day corresponding to 150 mins as *n*. Finds the value of *n* as follows: $10 + (n - 1) \times 5 = 150$ $\Rightarrow n = 29$ (Award 0.5 marks if just the formula for nth term is written.)	2
	Concludes that Sana will be running for 2.5 hours or more from 29th day onwards.	0.5
8	Considers $d_1 = d_2 + 4$ or $d_2 = d_1 + 4$ where d_1 & d_2 are the common differences of the two arithmetic progressions.	0.5
	Writes $a_{124} = b_{42}$ $\Rightarrow a_1 + 123d_1 = b_1 + 41d_2$ where a_1, a_{124}, b_1 & b_{42} are the 1st, 124th, 1st and 42nd terms of the two arithmetic progressions respectively.	1
	Solves the above two equations and finds the value of d_1 as -2 and d_2 as -6 OR d_1 as 2 and d_2 as 6. $a_{124} = b_{42}$ $\Rightarrow a_1 + 123(4 + d_2) = b_1 + 41d_2$ $\Rightarrow 492 + 123d_2 = 41d_2$ $\Rightarrow d_2 = -6$ (One way of simplification is shown here. Award full marks for any other appropriate method used.)	1.5
9	Writes that an arithmetic progression having the sum of the first two terms same as the sum of the first seven terms can exist.	0.5
	Justifies by writing that such an arithmetic progression exists if the sum of the first term and four times the common difference is zero. That is, $a + 4d = 0$, where *a* is the first term and *d* is the common difference. (Award full marks if an example like -4, -3, -2, -1, 0, 1, 2, ... is written instead of an algebraic justification.)	1.5

Q.No	Teacher should award marks if students have done the following:	Marks
10	Identifies that the number of leaves being added at each step follows an arithmetic progression and writes the AP as: 15, 22, 29, ...	0.5
	Identifies the first term of the AP, a as 15 and the common difference, d as 7.	0.5
	Finds the 25th term of the above AP as: 15 + (25 - 1) × 7 = 183 and concludes that the animation specialist will be adding 183 leaves to the 25th image to develop the 26th image.	1
11	Equates the p th term of the first arithmetic progression, a_p, to the q th term of the second arithmetic progression, b_q, as $a_p = b_q$.	0.5
	Uses the expression for the general term of an arithmetic progression and rewrites the above equation as: $a + (p - 1) \times d_1 = b + (q - 1) \times d_2$ $14 + (p - 1) \times 7 = 26 + (q - 1) \times 13$ where a, b are the first terms and d_1, d_2 are the common differences of the given arithmetic progressions respectively. (Award 0.5 marks if only the formula for the general term of an arithmetic progression is written.)	1
	Simplifies the above equation to obtain the relationship between p and q as $7p - 13q = 6$.	0.5
12	Writes the equation for the n th term of an arithmetic progression as: $3 + 4(n - 1) = 43$ (Award 0.5 marks if only the formula for the n th term of an AP is written correctly.)	1
	Solves the above equation for n and finds the circle which has a radius of 43 cm as the 11th circle.	1
13	Writes an arithmetic progressions (AP) for the increase in pitch every 5 m as: 5 hertz, 9 hertz, 13 hertz, 17 hertz, ...	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Identifies the first term of the AP, a as 5 and the common difference, d as 4.	0.5
	Finds the n th term of the AP when the pitch is exactly 149 hertz as: $5 + 4(n - 1) = 149$ $\Rightarrow n = 37$ (Award 0.5 marks if only the formula for the n th term of an AP is written correctly.)	1
	Finds the distance travelled by Ramit as $36 \times 5 = 180$ m.	1
14	i) Finds the volume of water from the tap in 30 seconds as $0.1 \times 30 = 3$ litres.	0.5
	Writes an arithmetic progression (AP) for the volume of water in the bucket every 30 seconds as: 1, 2.5, 4, 5.5, ...	0.5
	ii) Identifies the first term of the AP, a as 1 and the common difference, d as 1.5.	0.5
	Finds the term corresponding to 7.5 minutes in the AP as $\frac{7.5}{0.5} + 1 = 16$.	0.5
	Finds the volume of water in the bucket exactly after 7.5 minutes as: $1 + 1.5(16 - 1) = 23.5$ litres. (Award 0.5 marks if only the formula for the n th term of an AP is written correctly.)	1
15	i) Writes the equation for the common difference, d, of the given arithmetic progression (AP) as: $111 = \frac{12}{2}[(2 \times 17.5) + (12 - 1)d]$	0.5
	Solves the above equation and finds the value of d as (-1.5) and writes that Kevin reduces the weight of each subsequent layer by 1.5 kg.	0.5
	ii) Writes the equation for the weight of the lightest layer of cake, l, as: $111 = \frac{12}{2}(17.5 + l)$	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Solves the above equation and finds the weight of the lightest layer of cake as 1 kg.	0.5
16	Identifies that the interval between the calls follows an arithmetic progression (AP) and writes the AP as: 4 minutes, 6 minutes, 8 minutes, 10 minutes, ...	1
	Identifies the first term of the AP, *a* as 4 and the common difference, *d* as 2.	0.5
	Finds the sum of 12 terms of the above AP as: $\frac{12}{2}$ [(2 × 4) + 2(12 - 1)] = 180 minutes or 3 hours. (Award 0.5 marks if only the formula for the sum of *n* terms of an AP is written correctly.)	1
	Uses the above step to conclude that Parth will receive the 13th call at 2:48 pm + 3 hours = 5:48 pm.	0.5
17	Assumes the total number of garlands made by Shivam as *n* and writes: 630 = $\frac{n}{2}$ (35 + 7)	0.5
	Solves the above equation to find the value of *n* as 30.	0.5
18	Writes that Huner statement is not correct.	0.5
	Writes that Huner's statement is correct only when the first term of an arithmetic progression (AP), *a* and common difference, *d* are equal but not for any AP.	1.5
19	Writes $\frac{S_n}{151}$ = 0 to find the sum of the given Arithmetic Progression as S_n = 0.	0.5
	Writes the equation for the sum of an Arithmetic Progression as: $\frac{n}{2}[2a + (n - 1)d] = 0$	0.5
	Simplifies the above equation as: $a + 75d = 0$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Assumes the x th term to be zero and writes the equation for the term as: $a + (x - 1)d = 0$	**0.5**
	Uses $a = -75d$ in the above equation to write: $-75d + xd - d = 0$	**0.5**
	Solves the above equation and finds the value of x as 76.	**0.5**
20	**i) Uses the given information and writes the following equation:** $4 \times \frac{11}{2}[2a + 10d] = 1 \times \frac{21}{2}[2a + 20d]$ **(Award 0.5 marks if just the formula for the sum of the first n terms of an arithmetic progression is written correctly.)**	**1**
	Solves the above equation to show that $23a + 10d = 0$.	**1**
	ii) Uses the above relation and expresses d in terms of a as: $d = \frac{-23a}{10}$ **Uses the formula to find the n th term of an arithmetic progression and arrives at an expression for the general term of the given arithmetic progression only in terms of a and n as:** $a_n = \frac{a(33-23n)}{10}$ **(Award full marks for any equivalent version of the above expression.)** **(Award 0.5 marks if just the formula for the n th term of an arithmetic progression is written correctly.)**	**1**
21	**Identifies the first term of the arithmetic progression (AP) as 3, the common difference as 3 and writes the AP as:** **3, 6, 9, 12, 15, 18.**	**1**

Q.No	Teacher should award marks if students have done the following:	Marks
22	**Assumes the total number of layers as *n* and writes:** $15 + (n - 1) \times 3 = 138$ **(Award 0.5 marks if only the formula for the *n* th term of an AP is written correctly.)**	1
	Solves the above equation for *n* and finds the number of layers that such a pyramid will have as 42.	1
23	**Assumes the total number of layers that can be made as *n* and writes:** $\frac{n}{2}[2 \times 3 + (n - 1) \times 3] = 360$	0.5
	Simplifies the above equation as: $n^2 + n - 240 = 0$	0.5
	Finds the roots of the above equation as 15, -16.	0.5
	Concludes that the maximum number of layers that Ankit and his friends can make is 15.	0.5