Chapter - 6
Triangles

Q: 1 **Which of the following are DEFINITELY similar to each other?**

1. any two rhombuses
2. any two right triangles
3. any two regular pentagons
4. any two isosceles triangles

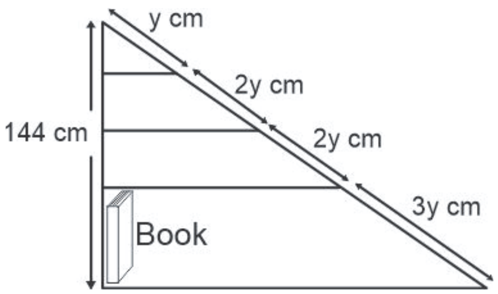
Q: 2 **ΔABC and ΔPQR are similar triangles. A_1 and A_2 are the areas, P_1 and P_2 are the perimeters of ΔABC and ΔPQR respectively.**

Which of these is the same as the ratio of the height of ΔABC to that of ΔPQR?

$\frac{A_1}{A_2}$ (i) $\quad$ $\frac{P_1}{P_2}$ (ii) $\quad$ $\sqrt{\frac{A_1}{A_2}}$ (iii) $\quad$ $\sqrt{\frac{P_1}{P_2}}$ (iv)

1. only (i) and (ii)
2. only (ii) and (iii)
3. only (iii) and (iv)
4. (Cannot be concluded from the given information.)

Q: 3 **Leela has a triangular cabinet that fits under his staircase. There are four parallel shelves as shown below.**

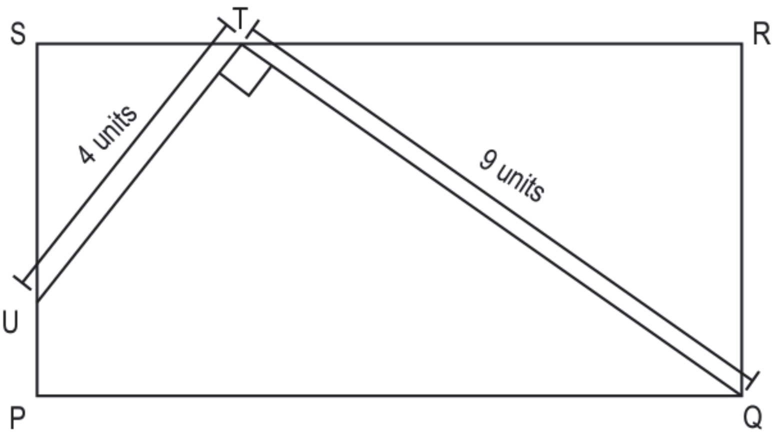


(Note: The figure is not to scale.)

The total height of the cabinet is 144 cm. What is the maximum height of a book that can stand upright on the bottom-most shelf?

1. 18 cm
2. 36 cm
3. 54 cm
4. 86.4 cm

Q: 4 **Shown below is a rectangle PQRS. ∠UTQ is a right angle.**



(Note: The figure is not to scale.)

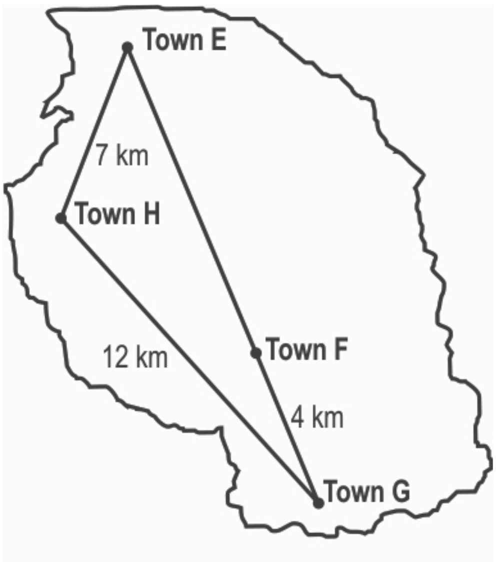
Which of these is TRUE about ΔUST and ΔTRQ?

1. The perimeters are in the ratio 16:81.
2. The ratio of ∠SUT:∠RTQ is 4:9.
3. The ratio of US:TR is 4:9.
4. The ratio of US:QR is 4:9.

Q: 5 **The areas of two similar triangles are 64 cm^2 and 121 cm^2.** **[2]**

If the length of a side of the larger triangle is 55 cm, find the length of the corresponding side of the smaller triangle. Show your work.

Q: 6 **[1]**

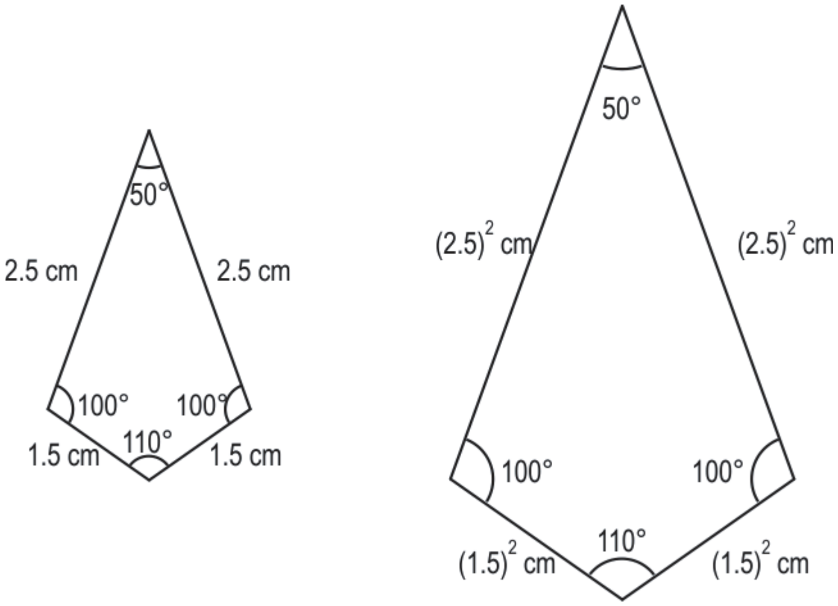


(Note: Map is not to scale.)

Looking at the above figure, Hari said that the shortest distance between Town E and Town F is 15 km.

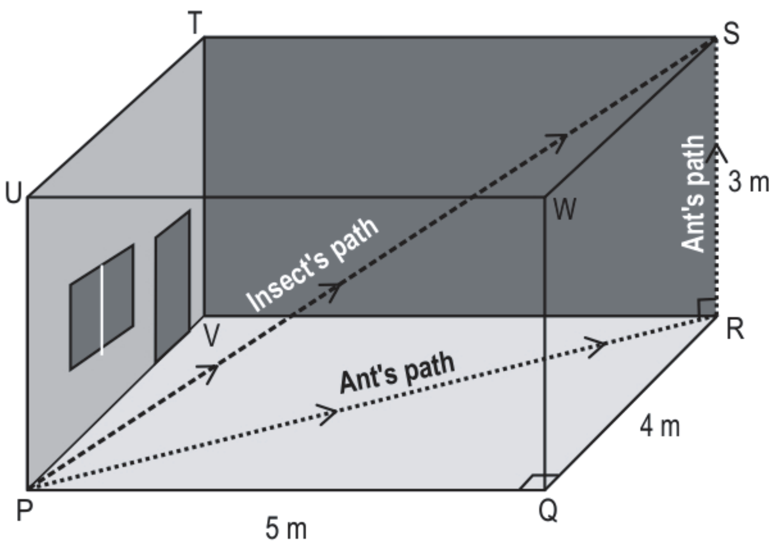
Is the statement true or false? Justify your answer using the relevant properties.

Q: 7 **Are the two quadrilaterals shown below similar? Give a reason for your answer.** **[1]**



(Note: The figure is not to scale.)

Q: 8 **An insect sitting at corner P of a room flies along the dotted line PS and reaches corner S. Whereas, an ant sitting at corner P, reaches corner S by crawling along the path PR, followed by RS. Both the paths are shown below.** **[3]**



(Note: The figure is not to scale.)

i) Find the length of the path taken by the ant.
ii) Find the length of the path of the insect's flight.

Show your steps.

Q: 9 **Panchami is standing on the ground and flying a kite at a vertical height of 22 m from the ground. The length of the taut string to which the kite is connected, is 29 m. Panchami is holding the string roller 1 m above the ground.** **[2]**

i) Draw a figure representing the above scenario.
ii) Find the horizontal distance between the kite and Panchami. Show your work.

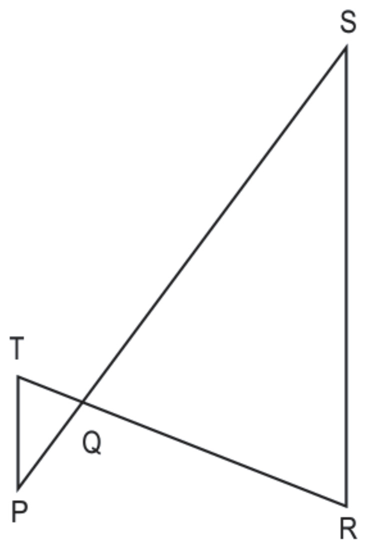
Q: 10 **During a mathematics class, a teacher wrote the following three algebraic expressions on the board:** **[3]**

$(m^2 - n^2)$, $(2mn)$ and $(m^2 + n^2)$, where m and n are positive integers with $n < m$

One of the students, Kaivalya, claimed that the above set of expressions ALWAYS represent the sides of a right-angled triangle.

Is Kaivalya's claim correct? Justify your answer.

Q: 11 **In a mathematics class, a teacher drew the following figure where $\frac{TQ}{QR} = \frac{1}{3}$. She then asked, "What is the sufficient condition required to prove that ΔTQP ~ ΔRQS?"** **[5]**



(Note: The figure is not to scale.)

♦ **Darsh said that it is sufficient if it is given that $\frac{TP}{SR} = \frac{1}{3}$.**

♦ **Bhargav said that it is sufficient if it is given that ∠P = ∠S.**

♦ **Tanvi said that it is sufficient if it is given that $\frac{PQ}{QS} = \frac{1}{3}$.**

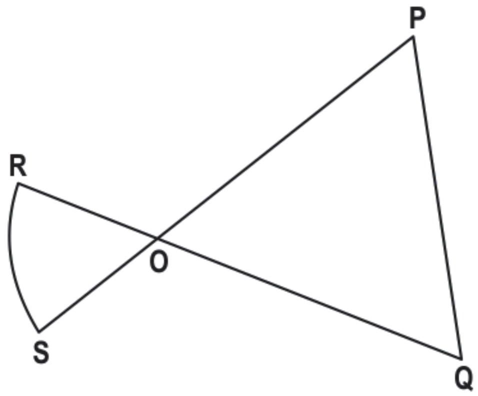
Examine whether each of their responses is correct or incorrect. Give reasons.

Q: 12 **Is it possible to have an isosceles right-angled triangle, such that the length of each of its sides is an integer? Give a reason to support your answer.** **[2]**

Q: 13 **ΔABC is similar to ΔPQR. The ratio of the perimeter of ΔABC to the perimeter of ΔPQR is 4:9.** **[3]**

i) Sara said $\frac{AB}{PQ} = \frac{4}{9}$. Is it true? Justify your answer.
ii) Find the ratio of the area of ΔABC to the area of ΔPQR.

Q: 14 **In the figure below, OPQ is a triangle with OP = OQ. RS is an arc of a circle with centre O.** **[2]**

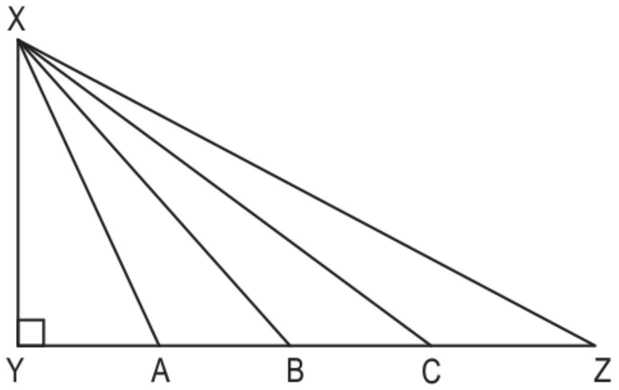


(Note: The figure is not to scale.)

Triangle OSR is similar to triangle OPQ.

Is the above statement true or false? Justify your reason.

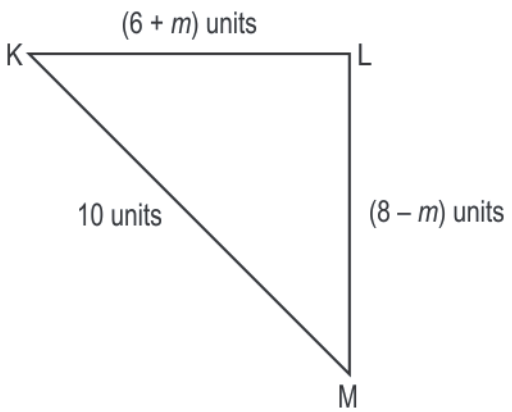
Q: 15 **In the figure below, XYZ is a right-angled triangle. A, B and C are the three points on YZ such that they divide YZ into 4 equal parts.** **[2]**



(Note: The figure is not to scale.)

Prove that $3XA^2 + XB^2 + XC^2 - XZ^2 = 4XY^2$.

Q: 16 **A teacher drew the below figure on the board and asked her students, "Is there any value of *m* such that ΔKLM becomes a right triangle?".** **[2]**



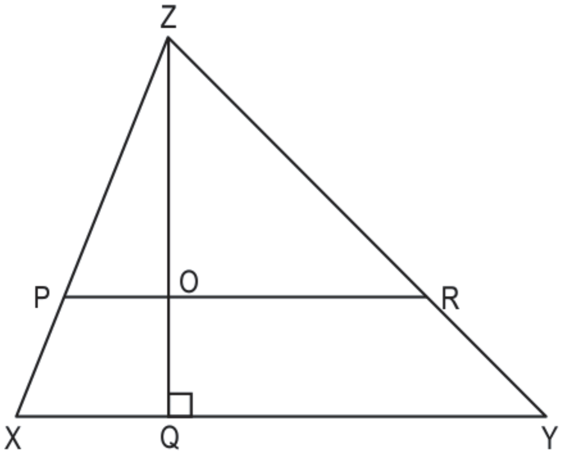
(Note: The figure is not to scale.)

Lata said, "The value of *m* can be 0".

Hina said, "The value of *m* can be 1".

Write if the above statements are true or false and justify your answer.

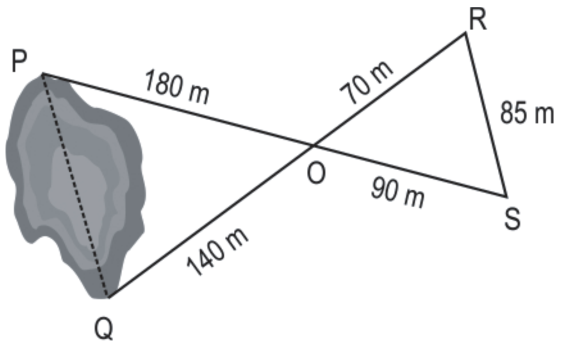
Q: 17 **In the figure below, P and R are points on the sides XZ and YZ respectively. PR || XY and PR bisects the area of ΔXYZ.** **[3]**



(Note: The figure is not to scale.)

If ZQ = 10 cm, find the length of OQ. Show your steps and give valid reasons.

Q: 18 **A geologist asked his assistant Annie, if the length of the lake, PQ, can be found from the information shown below.** **[3]**

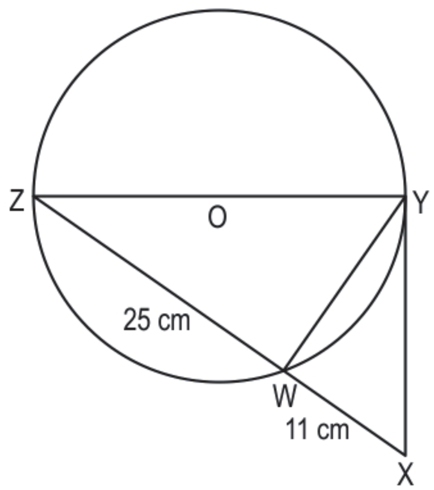


(Note: The figure is not to scale.)

Annie said, "it is possible to find the length of the lake, PQ."

Is Annie's statement correct? Justify your answer with valid reasons.

Q: 19 **Shown below is a circle with centre O. YX is the tangent to the circle at Y.** **[3]**

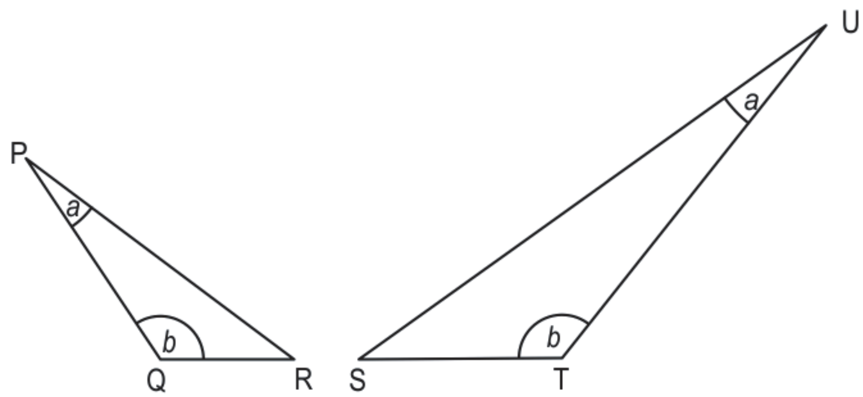


(Note: The figure is not to scale.)

i) Prove that ΔZWY ~ ΔZYX.
ii) Using part i), find the length of ZY.

Show your steps and give valid reasons.

Q: 20 **In the below figure, QR = 4 cm, RP = 8 cm and ST = 6 cm.** **[2]**



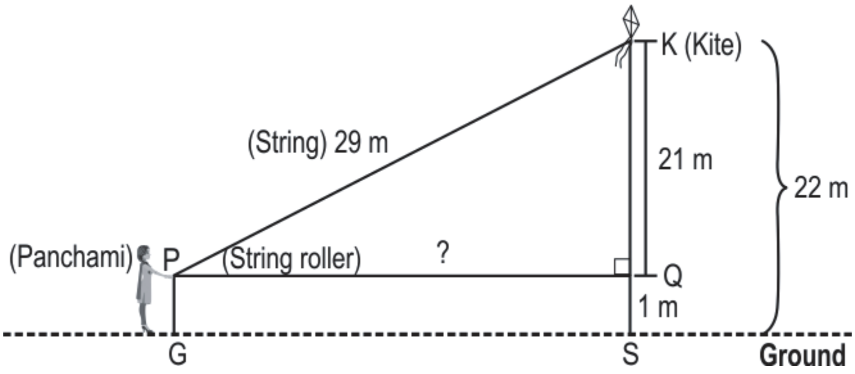
(Note: The figure is not to scale.)

If the perimeter of ΔSTU is 27 cm, find the length of PQ. Show your steps.

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	3
2	2
3	3
4	3

Q.No	Teacher should award marks if students have done the following:	Marks
5	Assumes the length of the corresponding side of the smaller triangle as *x* and uses the theorem, "the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides" to write, $\frac{64}{121} = (\frac{x}{55})^2$	1
	Solves the equation in the above step and finds the length of the corresponding side of the smaller triangle as 40 cm.	1
6	Mentions that the statement is false.	0.5
	Explains that it can't be 15 using triangular inequality theorem.	0.5
7	Writes that the two quadrilaterals are not similar.	0.5
	Writes the reason that the corresponding sides of the quadrilateral are not in the same ratio.	0.5
8	i) Finds the length of PR using the Pythagoras theorem in ΔPQR as: $PR^2 = PQ^2 + QR^2$ $\Rightarrow PR^2 = 5^2 + 4^2$ $\Rightarrow PR^2 = 25 + 16$ $\Rightarrow PR = \sqrt{41}$	1
	Finds the length of the path taken by the ant as ($\sqrt{41}$ + 3) m.	1
	ii) Finds the length of PS using the value of PR obtained in step 1 and the Pythagoras theorem in ΔPRS as: $PS^2 = PR^2 + RS^2$ $\Rightarrow PS^2 = 41 + 3^2$ $\Rightarrow PS^2 = 41 + 9$ $\Rightarrow PS = \sqrt{50}$ Concludes that the length of the path of the insect's flight is $\sqrt{50}$ m or $5\sqrt{2}$ m. (Award full marks if the correct answer is obtained using the formula for the diagonal of a cuboid instead of the Pythagoras theorem.)	1

Q.No	Teacher should award marks if students have done the following:	Marks
9	i) Draws a figure representing the given scenario. The figure may look as follows: 	1
	ii) Applies Pythagoras' theorem and finds the horizontal distance between the kite and Panchami, PQ, as $\sqrt{(29^2 - 21^2)}$ m = 20 m. (Award 0.5 marks if only the expression to find the the horizontal distance using Pythagoras' theorem is written correctly.)	1
10	Expands $(m^2 - n^2)^2$ as $m^4 + n^4 - 2m^2n^2$.	0.5
	Expands $(2mn)^2$ as $4m^2n^2$.	0.5
	Expands $(m^2 + n^2)^2$ as $m^4 + n^4 + 2m^2n^2$.	0.5
	Uses the above steps and writes: $(m^4 + n^4 - 2m^2n^2) + (4m^2n^2) = (m^4 + n^4 + 2m^2n^2)$ $\Rightarrow (m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$	1
	Concludes that, by the converse of Pythagoras' theorem, Kaivalya's claim is correct.	0.5
11	Writes that Darsh's answer is incorrect.	0.5
	Writes that in order to apply any of the similarity criterion, atleast one more piece of information is necessary.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that Bhargav's answer is correct.	0.5
	Gives the following reason: ∠TQP = ∠RQS [Vertically opposite angles] ∠P = ∠S [Given] ∠T = ∠R [Angle sum property] Hence, concludes that by AAA criterion of similarity of triangles, ΔTQP ~ ΔRQS.	1.5
	Writes that Tanvi's answer is correct.	0.5
	Gives the following reason: $\frac{TQ}{QR} = \frac{1}{3}$ [Given] ∠TQP = ∠RQS [Vertically opposite angles] $\frac{PQ}{QS} = \frac{1}{3}$ [Given] Hence, concludes that by SAS criterion of similarity of triangles, ΔTQP ~ ΔRQS.	1.5
12	Assumes that such a triangle exists. Takes the length of the equal sides to be k units, where k is an integer.	0.5
	Uses Pythagoras's theorem and finds the length of the third side (hypotenuse) as: $\sqrt{k^2 + k^2} = \sqrt{2}\,k$ units. Writes that the length of the third side (hypotenuse) is not an integer.	1
	Concludes that there cannot exist an isosceles right-angled triangle, such that the length of each of its sides is an integer.	0.5
13	i) Writes that Sara's claim is true.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the given information, ΔABC is similar to ΔPQR to write: AB = k PQ, BC = k QR, CA = k RP, where k is a constant.	0.5
	Uses the ratio of the perimeters to write: $\frac{AB+BC+CA}{PQ+QR+RP} = \frac{4}{9}$	0.5
	Uses steps 1 and 2 to solve for k and finds the ratio $\frac{AB}{PQ}$ as $\frac{4}{9}$.	0.5
	ii) Finds the ratio of the areas of the two triangles as: $\frac{\text{Area-of-}\Delta ABC}{\text{Area-of-}\Delta PQR} = (\frac{4}{9})^2 = \frac{16}{81}$	1
14	Writes True(T).	0.5
	Writes that in ΔOSR and ΔOPQ: ∠ROS = ∠POQ = x ° (vertically opposite angles) ∠ORS = ∠OSR= ∠OPQ = ∠OQP = $\frac{180°-x°}{2}$ (angle sum property of isoceles triangles)	1
	Uses the above step to justify that ΔOSR ~ ΔOPQ by using the AAA similarity criterion.	0.5
15	Assumes YA = AB = BC = CZ = d and writes YB = 2 d, YC = 3d and YZ = 4 d.	0.5
	Uses the Pythagoras theorems for ΔXYA, ΔXYB, ΔXYC and ΔXYZ and writes: $XA^2 = XY^2 + d^2$ $XB^2 = XY^2 + 4d^2$ $XC^2 = XY^2 + 9d^2$ $XZ^2 = XY^2 + 16d^2$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Substitutes the above relations in $3XA^2 + XB^2 + XC^2 - XZ^2$ as: $3(XY^2 + d^2) + (XY^2 + 4d^2) + (XY^2 + 9d^2) - (XY^2 + 16d^2) = 4XY^2 + 16d^2 - 16d^2 = 4XY^2$	0.5
16	**Writes that Lata's statement is true.**	0.5
	Justifies the above answer. For example: When $m = 0$, $10^2 = 100$ and $6^2 + 8^2 = 100$, which satisfies the Pythagoras theorem. Hence, one of the angles should be 90°.	0.5
	Writes that Hina's statement is false.	0.5
	Justifies the above answer. For example: When $m = 1$, $10^2 = 100$ and $7^2 + 7^2 = 98 \neq 100$, which does not satisfy the Pythagoras theorem. Hence, ΔKLM is not a right triangle at $m = 1$. (Award full 2 marks if the quadratic equation is framed using the Pythagoras theorem to find the value of *m* and if right conclusions are made.)	0.5
17	**Proves ΔPRZ ~ ΔXYZ by giving appropriate reasons. For example:** In ΔPRZ and ΔXYZ: ∠ZPR = ∠ZXY (corresponding angles) ∠ZRP = ∠ZYX (corresponding angles) Hence, ΔPRZ ~ ΔXYZ using the AA similarity criterion.	1.5
	Applies the similarity and writes: $\frac{ZO}{ZQ} = \frac{\sqrt{\text{Area-of-}\Delta ZPR}}{\sqrt{\text{Area-of-}\Delta ZXY}} = \frac{1}{\sqrt{2}}$. Finds that $ZO = \frac{10}{\sqrt{2}}$ cm and $OQ = ZQ - ZO = 10 - \frac{10}{\sqrt{2}}$ or $10 - 5\sqrt{2}$ cm.	1.5
18	**Writes that Annie is right.**	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Writes that in ΔPOQ and ΔSOR: $\frac{PO}{SO} = \frac{QO}{OR} = 2$ **(given)** **∠POQ = ∠SOR (vertically opposite angles)** **Concludes that ΔPOQ ~ ΔSOR using SAS similarity criterion.**	**1.5**
	Uses the above step and finds PQ = 2 × RS = 2 × 85 = 170 m. **Hence, justifies that it is possible to find the length of the lake, PQ.**	**1**
19	**i) Writes that ∠ZYX = 90° and gives the reason that radius is always perpendicular to the tangent at the point of contact.**	**0.5**
	Writes that ∠ZWY = 90° and gives the reason that angle in a semicircle is always 90°.	**0.5**
	Writes that, in ΔZWY and ΔZYX: **∠Z is common.** **∠ZWY = ∠ZYX = 90° (using step 1 and 2)** **Concludes that ΔZWY ~ ΔZYX by using the AA similarity criterion.**	**1**
	ii) Uses above step and writes the relation as: $\frac{ZY}{ZX} = \frac{ZW}{ZY}$ $=> \frac{ZY}{25+11} = \frac{25}{ZY}$	**0.5**
	Finds the length of ZY as 30 cm.	**0.5**
20	**Writes that ΔRQP and ΔSTU are similar by the AA similarity criterion.**	**0.5**
	Finds the length of SU as: $\frac{QR}{ST} = \frac{RP}{SU}$ $=> \frac{4}{6} = \frac{8}{SU}$ **=> SU = 12 cm**	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Uses the given value of the perimeter of triangle STU to find the length of UT as 9 cm.	0.5
	Finds the length of PQ as: $\frac{QR}{ST} = \frac{PQ}{UT}$ $=> \frac{4}{6} = \frac{PQ}{9}$ => PQ = 6 cm	0.5