Chapter - 8
Introduction to Trigonometry

Q: 1 **If cos y = 0, then what is the value of $\frac{1}{2}\cos\frac{y}{2}$?**

1. 0
2. $\frac{1}{2}$
3. $\frac{1}{\sqrt{2}}$
4. $\frac{1}{2\sqrt{2}}$

Q: 2 **P and Q are acute angles such that P > Q.**

Which of the following is DEFINITELY true?

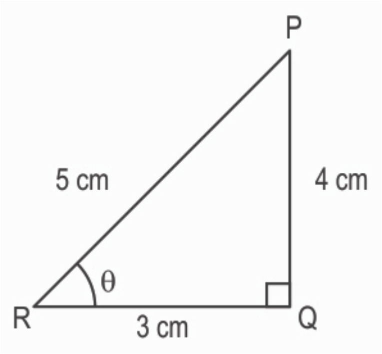
1. sin P < sin Q
2. tan P > tan Q
3. cos P > cos Q
4. cos P > sin Q

Q: 3 **In a right-angled triangle PQR, ∠Q = 90°.**

Which of these is ALWAYS 0?

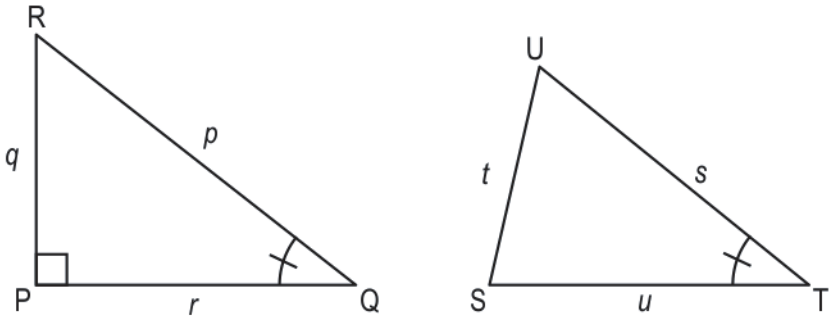
1. cos P - sec R
2. tan P - cot R
3. sin P - cosec R
4. (cannot be known without knowing the value of P)

Q: 4 **[1]**



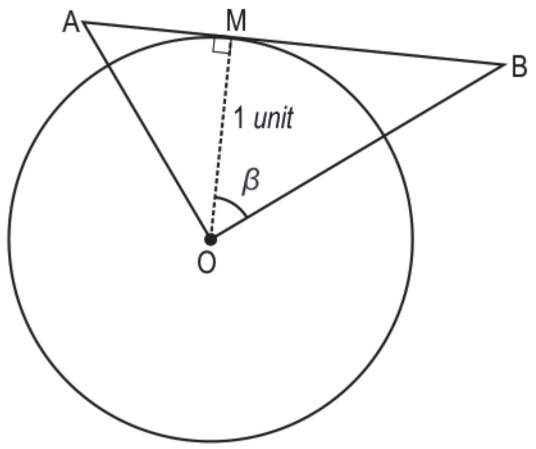
Show that sin θ = cos (90 - θ) is true using the definition of trigonometric ratios.

Q: 5 **In the triangles shown below, ∠Q = ∠T.** **[1]**



Write an expression each for cos Q and sin T.

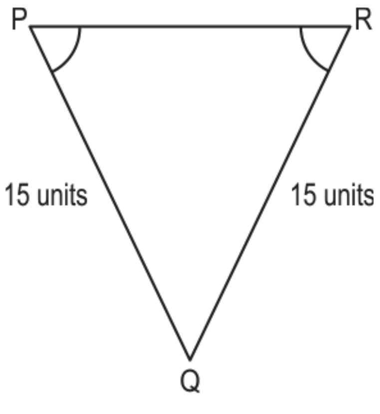
Q: 6 **A unit circle is shown below with centre O. A tangent AB is drawn to the circle at point M such that ∠MOB = β.** **[2]**



(Note: The figure is not to scale.)

If OA ⊥ OB, write the expressions that represent the lengths of
i) OB
ii) OA
iii) AB

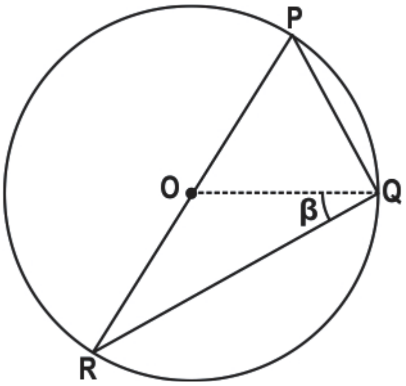
Q: 7 **In the figure below, 5sin P = 4.** **[2]**



(Note: The figure is not to scale.)

What is the length of PR? Draw a diagram and show your steps.

Q: 8 **▲PQR is inscribed in a circle with a centre O and radius *r* units.** **[3]**



If PR is the diameter of the circle and ∠RQO = β,

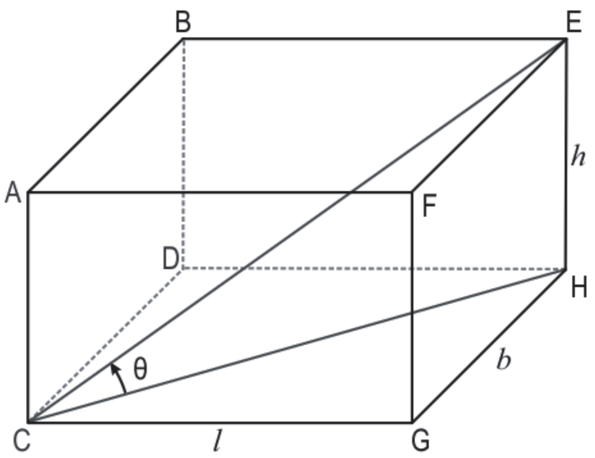
Express (QR^4 - PQ^4), in terms of *r* and β, to the simplest form.
Show your steps and give valid reasons.

Q: 9 **Prove the following.** **[3]**

i) $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{\cot\theta}{\cos\theta} = \cot\theta$

ii) $\dfrac{\tan 18^\circ}{\cos 72^\circ} - \dfrac{1}{\operatorname{cosec} 72^\circ + \tan 18^\circ} = \cot 72^\circ$

Q: 10 **Shown below is a cuboid. Its length is *l* units, breadth *b* units and height *h* units.** **[3]**



i) Express cos θ in terms of *l*, *b*, and *h*.

ii) If the figure was a cube, what would be the value of cos θ ?

Show your work.

Q: 11 **Prove that:** **[2]**

$$\frac{\operatorname{cosec}^2 x - \sin^2 x \cot^2 x - \cot^2 x}{\sin^2 x} = 1$$

Q: 12 **[5]**

If $\dfrac{1}{\sin\theta - \cos\theta} = \dfrac{\operatorname{cosec}\theta}{\sqrt{2}}$, prove that $\left(\dfrac{1}{\sin\theta + \cos\theta}\right)^2 = \dfrac{\sec^2\theta}{2}$.

Q: 13 **Solve:** **[3]**

$$\left(\frac{4\tan 53^\circ}{\cot 37^\circ}\right)^2 - \frac{\sec 34^\circ \sin 56^\circ \cos 17^\circ}{\sec 6^\circ \sin 73^\circ \sin 84^\circ}$$

Show your steps.

Q: 14 **During a math lesson, Mr. Kumar wrote the expression given below on the board and asked the students to simplify it.** **[2]**

$$\frac{\cos A}{1 - \sin A} + \frac{1 - \sin A}{\cos A}$$

Salma solved it in her notebook as follows:

$$\frac{\cos A}{1 - \sin A} + \frac{1 - \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 - \sin A)^2}{(1 - \sin A) \times \cos A} \quad \text{...(step 1)}$$

$$= \frac{\cos^2 A + \cos^2 A}{(1 - \sin A) \times \cos A} \quad \text{...(step 2)}$$

$$= \frac{2\cos^2 A}{(1 - \sin A) \times \cos A} \quad \text{...(step 3)}$$

$$= \frac{2\cos A}{1 - \sin A} \quad \text{...(step 4)}$$

Examine if Salma has made any error(s) and rectify them to find the correct answer.

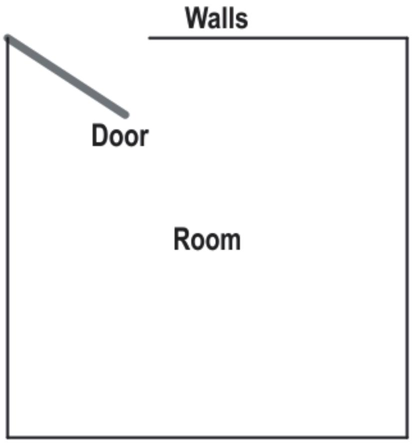
Q: 15 **The teacher asked the students to correctly complete the following sentence about the rhombus.** **[3]**

"A rhombus has a side length of *l* units and one of its angles is equal to θ. The ratio of the lengths of the two diagonals is dependent on ______."

Ashima: only *l*.
Bilal: only θ.
Chris: both *l* and θ.
Duleep: neither *l* nor θ.

Who answered the question correctly? Show your work and give valid reasons.

Q: 16 **A 90 cm wide door opens on one side of the room at a maximum angle of 90°. Due to shortage of space, a 40 cm by 80 cm table is kept behind the door along the wall such that it obstructs its path.** **[1]**



(Note: The figure is not to scale.)

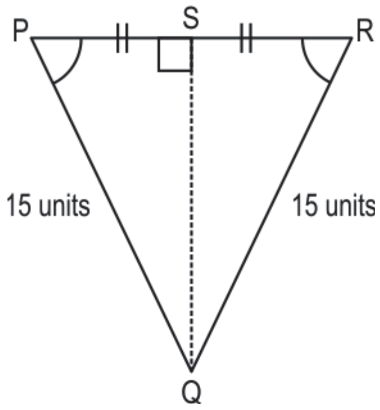
At what distance from the hinge should the table be kept such that the door opens for a maximum angle of 60°. Show your work.

(Note: Use $\sqrt{2} = 1.41$, $\sqrt{3} = 1.73$)

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	4
2	2
3	2

Q.No	Teacher should award marks if students have done the following:	Marks
4	**Marks the 3rd angle as 90 - θ and verifies the given statement using the ratio definition.**	1
5	**Writes cos Q =** $\frac{r}{p}$ **.**	0.5
	Writes sin T = sin Q = $\frac{q}{p}$ **.**	0.5
6	**i) Applies trigonometric ratios in ΔOMB to write:** $\cos(\beta) = \frac{OM}{OB} = \frac{1}{OB}$ $\Rightarrow OB = \sec\beta$	0.5
	ii) Applies trigonometric ratio in ΔOMA to write: $\cos(90° - \beta) = \frac{OM}{OA} = \frac{1}{OA}$ $\Rightarrow OA = \text{cosec}\,\beta$	0.5
	iii) Uses above steps along with Pythagoras' theorem to write: $AB^2 = OA^2 + OB^2$ $\Rightarrow AB^2 = \text{cosec}^2\beta + \sec^2\beta$ $\Rightarrow AB^2 = \frac{\cos^2\beta + \sin^2\beta}{\sin^2\beta\cos^2\beta}$ $\Rightarrow AB = \sqrt{\frac{1}{\sin^2\beta\cos^2\beta}}$ $\Rightarrow AB = \frac{1}{\sin\beta\cos\beta}$ **(Award full marks for any other variation of the correct answer.)**	1

Q.No	Teacher should award marks if students have done the following:	Marks
7	Writes that, in an isosceles triangle, the perpendicular bisects the base and draws a diagram. The diagram may look as follows:  *(Note: The figure is not to scale.)*	0.5
	Uses the value of sin P to find the length of SQ as 15sin P = $15 \times \frac{4}{5}$ = 12 units.	0.5
	Uses the Pythagoras theorem to find the length of PS as $\sqrt{(15^2 - 12^2)}$ = 9 units.	0.5
	Finds the length of PR as 2 × 9 = 18 units.	0.5
8	Identifies that ΔRQO is isosceles since OQ = OR = r and finds the measure of ∠ORQ = ∠OQR = β.	0.5
	Identifies that ΔPQR is right-angled at Q and finds the length of QR as PR × cos β = 2 r (cos β).	0.5
	Identifies that ΔPQR is right-angled at Q and finds the length of PQ as PR × sin β = 2 r (sin β).	0.5
	Uses steps 2 and 3 to express $(QR^4 - PQ^4)$ as $16 r^4 (\cos^4 \beta - \sin^4 \beta)$.	0.5
	Factorises the above expression for $(QR^4 - PQ^4)$ as $16 r^4 (\cos^2 \beta - \sin^2 \beta)(\cos^2 \beta + \sin^2 \beta)$.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
	Simplifies and factorises the above expression for (QR^4 - PQ^4) as 16 r^4 (cos β - sin β) (cos β + sin β). **(Award full marks if the student simplifies to any other variation of this equation.)**	**0.5**
9	**i) Simplifies the given LHS by rationalizing the first term as:** $\frac{cosec\,\theta + cot\,\theta}{cosec^2\theta - cot^2\theta} - \frac{cot\,\theta}{cos\,\theta}$	**0.5**
	Simplifies the above expression as: **cosec θ + cot θ - cosec θ = cot θ** **Concludes that LHS = RHS.**	**1**
	ii) Simplifies the given LHS as: $\frac{sin\,18°}{cos\,18°} \times \frac{1}{sin\,18°} - \frac{1}{cosec\,72° + cot\,72°}$	**0.5**
	Simplifies the above expression by rationalizing the second term as: $sec\,18° - \frac{cosec\,72° - cot\,72°}{cosec^2 72° - cot^2 72°}$	**0.5**
	Simplifies the above expression as: **sec 18° - sec 18° + cot 72° = cot 72°** **Concludes that LHS = RHS.**	**0.5**
10	**i) Finds the length of CH using the Pythagoras' theorem in ΔCGH as:** **CH = $\sqrt{(CG^2 + GH^2)} = \sqrt{(l^2 + b^2)}$ units**	**0.5**

Q.No	Teacher should award marks if students have done the following:	Marks
	Finds the length of CE using the Pythagoras' theorem in ΔCHE as: **CE = $\sqrt{CH^2 + EH^2} = \sqrt{l^2 + b^2 + h^2}$ units**	0.5
	Finds cos θ as: $\cos\theta = \frac{CH}{CE} = \frac{\sqrt{l^2+b^2}}{\sqrt{l^2+b^2+h^2}}$ $\Rightarrow \cos\theta = \sqrt{\frac{l^2+b^2}{l^2+b^2+h^2}}$ **(Award 0.5 marks if only the ratio for cos θ is correctly written.)**	1
	ii) Applies *l* = *b* = *h* for a cube and solves for cos θ as: $\cos\theta = \sqrt{\frac{l^2+b^2}{l^2+b^2+h^2}} = \sqrt{\frac{2}{3}}$	1
11	**Simplifies the given LHS as:** $\frac{1 - \sin^2 x \cot^2 x}{\sin^2 x}$	1
	Simplifies the above expression as: **$\operatorname{cosec}^2 x - \cot^2 x$**	0.5
	Simplifies the above expression as 1 and concludes that LHS = RHS.	0.5

Q.No	Teacher should award marks if students have done the following:	Marks
12	**Squares both sides of the given equation as:** $$\frac{1}{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta} = \frac{\text{cosec}^2\theta}{2}$$	0.5
	Simplifies the above equation as: $$\frac{2}{\text{cosec}^2\theta} = 1 - 2\sin\theta\cos\theta$$	1
	Simplifies the above equation as: **$2\sin\theta\cos\theta = 1 - 2\sin^2\theta$**	1
	Squares the LHS of the equation to be proved as: $$\frac{1}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}$$	0.5
	Uses step 3 and simplifies the above expression as: $$\frac{1}{2 - 2\sin^2\theta}$$	1
	Simplifies the above expression as: $$\frac{1}{2\cos^2\theta} = \frac{\sec^2\theta}{2}$$ **Concludes that LHS = RHS.**	1

Q.No	Teacher should award marks if students have done the following:	Marks
13	Simplifies the given expression as: $$\left(\frac{4\cot(90°-53°)}{\cot 37°}\right)^2-\frac{\operatorname{cosec}(90°-34°)\times\frac{1}{\operatorname{cosec}56°}\times\sin(90°-17°)}{\operatorname{cosec}(90°-6°)\times\sin 73°\times\frac{1}{\operatorname{cosec}84°}}$$	1.5
	Simplifies the given expression as: $$\left(\frac{4\cot 37°}{\cot 37°}\right)^2-\frac{\operatorname{cosec}56°\times\frac{1}{\operatorname{cosec}56°}\times\sin 73°}{\operatorname{cosec}84°\times\sin 73°\times\frac{1}{\operatorname{cosec}84°}}$$	1
	Simplifies the above expression as: $4^2 - 1 = 15$	0.5
14	Identifies that step (2) has an error.	0.5
	For step (2), identifies that incorrect identity is used and writes the correct identity as: $(1 - \sin A)^2 = 1 + \sin^2 A - 2\sin A$	0.5
	Writes the step by step solution to get the correct simplified form as 2sec A or $\frac{2}{\cos A}$.	1
15	Draws a rhombus, say ABCD, and connects diagonals AC and BD bisecting at a point, say E.	0.5
	In ΔEAD, applies the properties of the rhombus to get i) $\angle AED = 90°$ ii) $AE = \frac{AC}{2}$ iii) $DE = \frac{BD}{2}$ iv) $\angle EAD = \frac{\theta}{2}$	1

Q.No	Teacher should award marks if students have done the following:	Marks
	Applies trigonometric ratio to get **tan** $\frac{\theta}{2} = \frac{AE}{DE} = \frac{AC}{BD}$	**0.5**
	Writes that the ratio of the diagonals $\frac{AC}{BD}$ **is only dependent on θ and not *l*.** **Writes that Bilal answered it correctly.**	**1**
16	**Assumes the required distance as *x* cm and writes the ratio as:** $\tan 30° = \frac{40}{x}$	**0.5**
	Solves the above equation to find the value of *x* as 40 × 1.73 = 69.2 cm.	**0.5**