FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 3:00 PM to 06:00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The dimension of mutual inductance is :

(A) $[ML^2T^{-2}A^{-1}]$ (B) $[ML^2T^{-3}A^{-1}]$

(C) $[ML^2T^{-2}A^{-2}]$ (D) $[ML^2T^{-3}A^{-2}]$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. e_2 : induced emf in secondary coil

i_1 : Current in primary coil

M : Mutual inductance

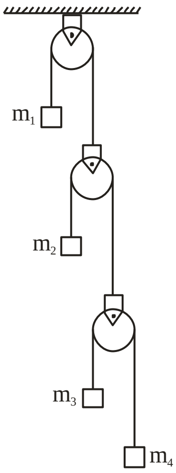
$$e_2 = -M\frac{di_1}{dt}$$

$$M = -\frac{e_2}{\frac{di_1}{dt}}$$

$$[M] = \frac{[e_2]}{\left[\frac{di_1}{dt}\right]} = \frac{\left[\frac{W}{q}\right]}{\left[\frac{di_1}{dt}\right]} = \frac{\frac{[ML^2T^{-2}]}{[AT]}}{[AT^{-1}]}$$

$= [ML^2T^{-2}A^{-2}]$

2. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the accelerations of masses m_1, m_2, m_3 and m_4 respectively. Which of the following relation is true for this arrangement?



(A) $4a_1 + 2a_2 + a_3 + a_4 = 0$

(B) $a_1 + 4a_2 + 3a_3 + a_4 = 0$

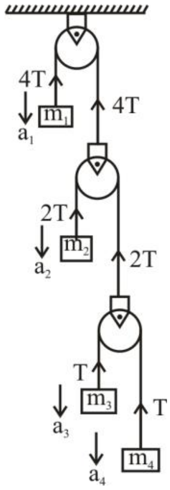
(C) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$

(D) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol.



Using costraint

$$\sum \vec{T}.\vec{a} = 0$$

$-4Ta_1 - 2Ta_2 - Ta_3 - Ta_4 = 0$

$4a_1 + 2a_2 + a_3 + a_4 = 0$

3. Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3 and W_4 are the work done corresponding to figure a, b, c and d respectively.

Figure-a

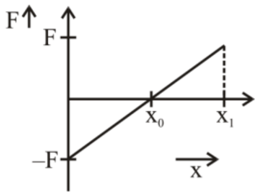


Figure-b

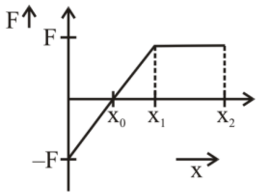


Figure-c

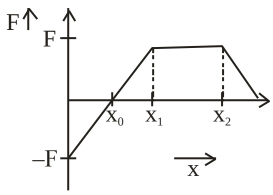
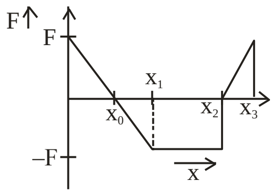


Figure-d



(A) $W_3 > W_2 > W_1 > W_4$

(B) $W_3 > W_2 > W_4 > W_1$

(C) $W_2 > W_3 > W_4 > W_1$

(D) $W_2 > W_3 > W_1 > W_4$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Work done = area under F – x curve. Area below x –axis is negative & area above x-axis is positive.

so

$W_3 > W_2 > W_1 > W_4$

4. Solid spherical ball is rolling on a frictionless horizontal plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is :-

(A) $\frac{2}{5}$ (B) $\frac{2}{7}$ (C) $\frac{1}{5}$ (D) $\frac{7}{10}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $K_{total} = K_{rotational} + K_{Translational}$

$$K_{total} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mV_{cm}^2$$

$v_{cm} = R\omega$ for pure rolling

$$I_{cm} = \frac{2}{5}mR^2$$

$$K_{Rot} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\times\frac{2}{5}mR^2\times\frac{v_{cm}^2}{R^2} = \frac{1}{5}mv_{cm}^2$$

$$K_{Total} = \frac{1}{5}mv_{cm}^2 + \frac{1}{2}mv_{cm}^2 = \frac{7}{10}mv_{cm}^2$$

$$\frac{K_{Rot}}{K_{Total}} = \frac{\frac{1}{5}mv_{cm}^2}{\frac{7}{10}mv_{cm}^2} = \frac{2}{7}$$

5. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R : At equator, the direction of acceleration due to the gravity is towards the center of earth.

In the light of above statements, choose the correct answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

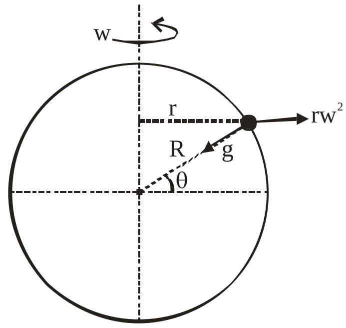
(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (D)

Allen Ans. (D)

Sol.



Effective acceleration due to gravity is the resultant of g & rw^2 whose direction & magnitude depends upon θ. Hence assertion is false.

When θ = 0° (at equator), effective acceleration is radially inward.

6. If ρ is the density and η is coefficient of viscosity of fluid which flows with a speed v in the pipe of diameter d, the correct formula for Reynolds number R_e is :

(A) $R_e = \frac{\eta d}{\rho v}$ (B) $R_e = \frac{\rho v}{\eta d}$

(C) $R_e = \frac{\rho v d}{\eta}$ (D) $R_e = \frac{\eta}{\rho v d}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. Reynold's number is given by $\frac{\rho v d}{\eta}$

7. A flask contains argon and oxygen in the ratio of 3:2 in mass and the mixture is kept at 27°C. The ratio of their average kinetic energy per molecule respectively will be :

(A) 3 : 2 (B) 9 : 4

(C) 2 : 3 (D) 1 : 1

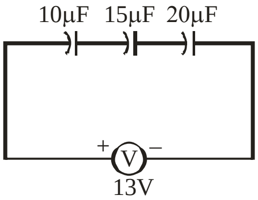
Official Ans. by NTA (D)

Allen Ans. (Bonus)

Sol. Average K.E./molecule $= \frac{f}{2}kT$

So, $\frac{K_{Ar}}{K_{O_2}} = \frac{\frac{3}{2}kT}{\frac{5}{2}kT} = \frac{3}{5}$

8. The charge on capacitor of capacitance $15\mu F$ in the figure given below is :

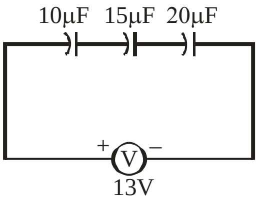


(A) $60\mu c$ (B) $130\mu c$ (C) $260\ \mu c$ (D) $585\ \mu c$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol.



$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20} = \frac{12+8+6}{120} = \frac{26}{120}$$

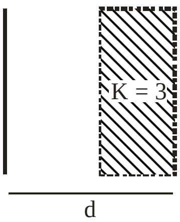
$$C_{eq} = \frac{60}{13}\mu F$$

$$Q = \frac{13 \times 60}{13} = 60\mu C$$

Charge on each capacitor is same

$\because$ they are in series.

9. A parallel plate capacitor with plate area A and plate separation d=2 m has a capacitance of 4 μF. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant K=3 (as shown in figure) will be :

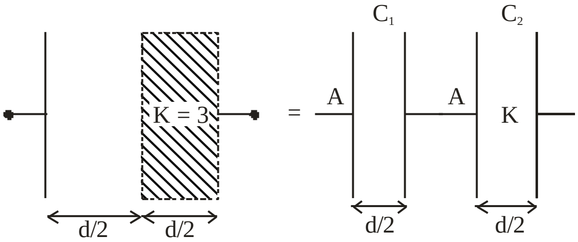


(A) $2\mu F$ (B) $32\mu F$ (C) $6\mu F$ (D) $8\mu F$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $C_{original} = \frac{A\varepsilon_0}{d}$



$$C_1 = \frac{A\varepsilon_0}{d/2} = \frac{2A\varepsilon_0}{d} = C$$

$$C_2 = \frac{KA\varepsilon_0}{d/2} = \frac{2KA\varepsilon_0}{d} = \frac{6A\varepsilon_0}{d} = 3C$$

C_1 & C_2 are in series

$$C_{new} = \frac{C_1C_2}{C_1 + C_2} = \frac{C \times 3C}{C + 3C} = \frac{3C}{4}$$

$$= \frac{3}{4} \times \frac{2A\varepsilon_0}{d} = \frac{3}{2} \times \frac{A\varepsilon_0}{d}$$

$$C_{new} = \frac{3}{2}C_{original}$$

$$= \frac{3}{2} \times 4 = 6\mu F$$

10. Sixty four conducting drops each of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface density of bigger drop to the smaller drop will be :

(A) 1 : 4 (B) 4 : 1 (C) 1 : 8 (D) 8 : 1

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. Let R = radius of combined drop

r = radius of smaller drop

Volume will remain same

$$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

R = 4r

Q = 64q ;

q : charge of smaller drop

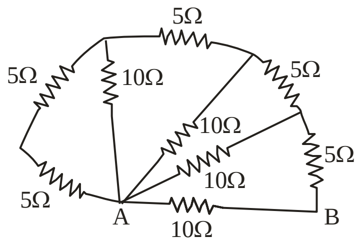
Q : Charge of combined drop

$$\frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{\frac{Q}{4\pi R^2}}{\frac{q}{4\pi r^2}} = \frac{Q}{q} \cdot \frac{r^2}{R^2}$$

$$= 64\frac{r^2}{16r^2} = 4$$

$$\frac{\sigma_{bigger}}{\sigma_{smaller}} = \frac{4}{1}$$

11. The equivalent resistance between points A and B in the given network is :



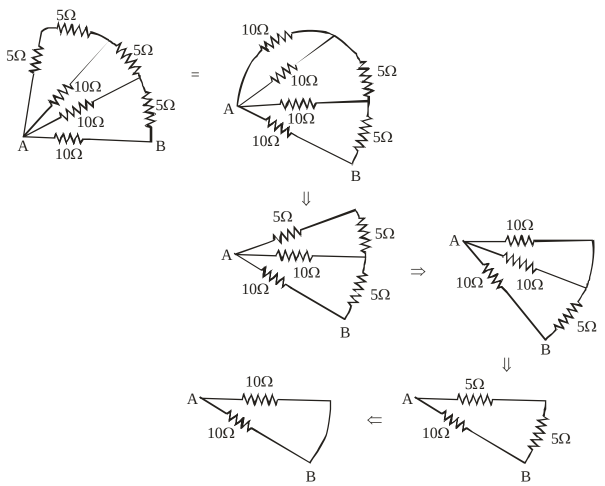
(A) 65Ω (B) 20Ω

(C) 5Ω (D) 2Ω

Official Ans. by NTA (C)

Allen Ans. (C)

Sol.



$R_{AB} = 5\Omega$

12. A bar magnet having a magnetic moment of 2.0 × 10^5 JT^{-1}, is placed along the direction of uniform magnetic field of magnitude B= 14 × 10^{-5} T. The work done in rotating the magnet slowly through 60° from the direction of field is :

(A) 14 J (B) 8.4 J (C) 4 J (D) 1.4 J

Official Ans. by NTA (A)

Allen Ans. (A)

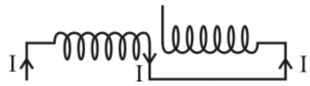
Sol. Work done = MB ($\cos\theta_1 - \cos\theta_2$)

$\theta_1 = 0°, \theta_2 = 60°$

$= 2 \times 10^5 \times 14 \times 10^{-5} (1 - 1/2)$

= 14 J

13. Two coils of self inductance L_1 and L_2 are connected in series combination having mutual inductance of the coils as M. The equivalent self inductance of the combination will be :



(A) $\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{M}$ (B) $L_1 + L_2 + M$

(C) $L_1 + L_2 + 2M$ (D) $L_1 + L_2 - 2M$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. Current on both the inductor is in opposite direction.

Hence :

$L_{eq} = L_1 + L_2 - 2M$

14. A metallic conductor of length 1m rotates in a vertical plane parallel to east-west direction about one of its end with angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is 0.2 × 10^{-4} T, then emf induced between the two ends of the conductor is :

(A) 5μV (B) 50μV (C) 5mV (D) 50mV

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. emf induced between the two ends = $\frac{B_H \omega l^2}{2}$

$$\frac{0.2 \times 10^{-4} \times 5 \times 1}{2} = 0.5 \times 10^{-4} = 50 \times 10^{-6} V = 50\mu V$$

15. Which is the correct ascending order of wavelengths?

(A) $\lambda_{visible} < \lambda_{X-ray} < \lambda_{gamma\text{-}ray} < \lambda_{microwave}$

(B) $\lambda_{gamma\text{-}ray} < \lambda_{X-ray} < \lambda_{visible} < \lambda_{microwave}$

(C) $\lambda_{X\text{-}ray} < \lambda_{gamma\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

(D) $\lambda_{microwave} < \lambda_{visible} < \lambda_{gamma\text{-}ray} < \lambda_{X\text{-}ray}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. From electromagnetic wave spectrum.

λ increases ⟶

γ-ray	x-rays	ultra violet	visible	infrared	microwave	Radio wave

$\lambda_{\text{gamma-ray}} < \lambda_{X\text{-ray}} < \lambda_{\text{visible}} < \lambda_{\text{microwave}}$

16. For a specific wavelength 670 nm of light coming from a galaxy moving with velocity v, the observed wavelength is 670.7 nm.

The value of v is :

(A) $3 \times 10^8\ ms^{-1}$ (B) $3 \times 10^{10}\ ms^{-1}$

(C) $3.13 \times 10^5\ ms^{-1}$ (D) $4.48 \times 10^5\ ms^{-1}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\lambda_{\text{emitted}} = 670$ nm

$\lambda_{\text{obs}} = 670.7$ nm

v = ?

$c = 3 \times 10^8$ m/s

If v << c

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{v}{c}$$

$$\frac{670.7 - 670}{670} = \frac{v}{c}$$

$V = 3.13 \times 10^5$ m/s

17. A metal surface is illuminated by a radiation of wavelength 4500 Å. The ejected photo-electron enters a constant magnetic field of 2 mT making an angle of 90° with the magnetic field. If it starts revolving in a circular path of radius 2 mm, the work function of the metal is approximately :

(A) 1.36 eV (B) 1.69 eV (C) 2.78 eV (D) 2.23 eV

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\lambda = 4500$ Å

B = 2mT, R = 2mm

$$R = \frac{\sqrt{2Km}}{qB}$$

$$\frac{(qBR)^2}{2m} = K$$

$$\frac{\left(1.6\times10^{-19}\times2\times10^{-3}\times2\times10^{-3}\right)^2}{2\times9.1\times10^{-31}} = K$$

$$\frac{(6.4)^2}{2\times9.1}\times\frac{10^{-50}}{10^{-31}} = K$$

$K = 2.25 \times 10^{-19}$ J

$$= \frac{2.25\times10^{-19}}{1.6\times10^{-19}}\text{eV} = 1.40\text{eV}$$

$$E = \frac{12400}{4500} = 2.76\text{eV}$$

$\phi = E - K = (2.76 - 1.40)$ eV $= 1.36$ eV

18. A radioactive nucleus can decay by two different processes. Half-life for the first process is 3.0 hours while it is 4.5 hours for the second process. The effective half- life of the nucleus will be :

(A) 3.75 hours (B) 0.56 hours

(C) 0.26 hours (D) 1.80 hours

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{\ln 2}{(t_{1/2})_{eq}} = \frac{\ln 2}{(t_{1/2})_1} + \frac{\ln 2}{(t_{1/2})_2}$$

$$(t_{1/2})_{eq} = \frac{(t_{1/2})_1 \times (t_{1/2})_2}{(t_{1/2})_1 + (t_{1/2})_2}$$

$$= \frac{3\times4.5}{3+4.5} = \frac{3\times4.5}{7.5} = \frac{3\times3}{5} = 1.8\text{ hr}$$

19. The positive feedback is required by an amplifier to act an oscillator. The feedback here means :

(A) External input is necessary to sustain ac signal in output.

(B) A portion of the output power is returned back to the input.

(C) Feedback can be achieved by LR network.

(D) The base-collector junction must be forward biased.

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. When the amplifier connects with positive feedback, it acts as the oscillator the feedback here is positive feedback which means some amount of voltage is given to the input.

20. A sinusoidal wave $y(t) = 40\sin(10 \times 10^6 \pi t)$ is amplitude modulated by another sinusoidal wave $x(t) = 20\sin(1000\pi t)$. The amplitude of minimum frequency component of modulated signal is :

(A) 0.5 (B) 0.25 (C) 20 (D) 10

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $y(t) = 40 \sin(10 \times 10^6 \pi t)$

$x(t) = 20\sin(1000\pi t)$

$\Rightarrow \omega_c = 10^7 \pi$

$\omega_m = 10^3 \pi$

$A_C = 40$

$A_m = 20$

Equation of modulated wave = $(A_C + A_m \sin \omega_m t) \sin \omega_c t$

$$= A_c\left(1+\frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$$

$$= A_c(1 + \mu \sin \omega_m t) \sin \omega_c t, \qquad \mu = \frac{A_m}{A_c}$$

$$= A_c \sin\omega_c t + \frac{\mu A_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

Amplitude of minimum frequency =

$$\frac{\mu A_c}{2} = \frac{A_m}{A_c} \times \frac{A_c}{2} = \frac{A_m}{2} = 10$$

SECTION-B

1. A ball is projected vertically upward with an initial velocity of 50 ms^{-1} at t = 0s. At t = 2s. another ball is projected vertically upward with same velocity. At t = ________s, second ball will meet the first ball ($g = 10\ ms^{-2}$).

Official Ans. by NTA (6)

Allen Ans. (6)

Sol. Let they meet at t = t

So first ball gets t sec.

& 2nd gets (t – 2) sec. & they will meet at same height

$$h_1 = 50t - \frac{1}{2}gt^2$$

$$h_2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$h_1 = h_2$

$$50t - \frac{1}{2}gt^2 = 50(t-2) - \frac{1}{2}g(t-2)^2$$

$$100 = \frac{1}{2}g\left[t^2 - (t-2)^2\right]$$

$$100 = \frac{10}{2}[4t-4]$$

$5 = t - 1$

t = 6 sec.

2. A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is ______________Ns.

Official Ans. by NTA (12)

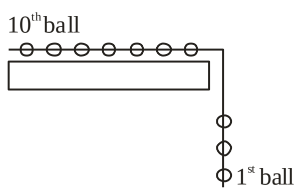
Allen Ans. (12)

Sol. Impulse = change in momentum

$= m[v - (-v)] = 2\ mv$

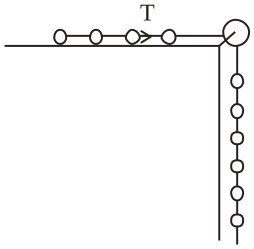
$= 2 \times 0.4 \times 15 = 12$ Ns

3. A system to 10 balls each of mass 2 kg are connected via massless and unstretchable string. The system is allowed to slip over the edge of a smooth table as shown in figure. Tension on the string between the 7th and 8th ball is____________N when 6th ball just leaves the table.



Official Ans. by NTA (36)

Allen Ans. (36)

Sol. 

$a = \frac{6mg}{10m} = \frac{6g}{10} = \frac{3g}{5}$

taking 8,9,10 together 

$T = 3\ ma$

$= 3m \times \frac{3g}{5}$

$= 36\ N$

4. A geyser heats water flowing at a rate of 2.0 kg per minute from 30°C to 70°C. If geyser operates on a gas burner, the rate of combustion of fuel will be ____________g min^{-1}

[Heat of combustion = $8 \times 10^3\ Jg^{-1}$

Specific heat of water = $4.2\ Jg^{-1}\ °C^{-1}$]

Official Ans. by NTA (42)

Allen Ans. (42)

Sol. m = 2000 gm/min

Heat required by water/min = $mS\Delta T$

$= (2000) \times 4.2 \times 40$ J/min

$= 336000$ J/min

The rate of combustion = $\left(\frac{dm}{dt} L\right) = 336000 J/min$

$\frac{dm}{dt} = \frac{336000}{8 \times 10^3} g/min$

$= 42$ gm/min

5. A heat engine operates with the cold reservoir at temperature 324 K.

The minimum temperature of the hot reservoir, if the heat engine takes 300 J heat from the hot reservoir and delivers 180 J heat to the cold reservoir per cycle, is ___________K.

Official Ans. by NTA (540)

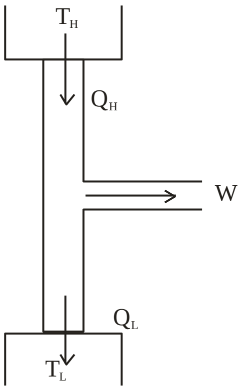
Allen Ans. (540)

Sol. $T_c = 324$ k

$T_H = ?$

$Q_H = 300$ J

$Q_L = 180$ J



$1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$

$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$

$T_H = \frac{Q_H}{Q_L} \times T_L = \frac{300}{180} \times 324 = 540 K$

6. A set of 20 tuning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is_____Hz.

Official Ans. by NTA (152)

Allen Ans. (152)

Sol. $f_1 = f$

$f_2 = f + 4$

$f_3 = f + 2 \times 4$

$f_4 = f + 3 \times 4$

$f_{20} = f + 19 \times 4$

$f + (19 \times 4) = 2 \times f$

$f = 76$ Hz.

Frequency of last tuning forks = 2f

= 152 Hz

7. Two 10 cm long, straight wires, each carrying a current of 5A are kept parallel to each other. If each wire experienced a force of 10^{-5} N, then separation between the wires is _________cm.

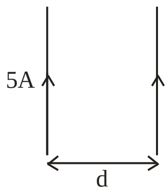
Official Ans. by NTA (5)

Allen Ans. (5)

Sol. It should be mentioned, 10 cm wire is part of long wire.

Force experienced by unit length of wire

$$= \frac{\mu_0 I_1 I_2}{2\pi d}, \; I_1 = I_2 = 5A$$



Force experienced by wires of length 10 cm

$$= \frac{\mu_0 I_1 I_2}{2\pi d} \times 10 \times 10^{-2}$$

$$10^{-5} = \frac{2\times10^{-7}\times5\times5}{d}\times10\times10^{-2}$$

$d = 50 \times 10^{-3}$ m

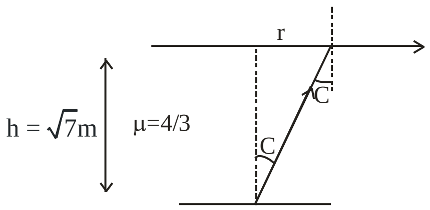
$d = 50 \times 10^{-1}$ cm = 5 cm.

8. A small bulb is placed at the bottom of a tank containing water to a depth of $\sqrt{7}$ m. The refractive index of water is $\frac{4}{3}$. The area of the surface of water through which light from the bulb can emerge out is $x\pi$ m^2. The value of x is_______.

Official Ans. by NTA (9)

Allen Ans. (9)

Sol. C : Criticle angle



$$\tan C = \frac{r}{h}$$

r = h tan C

$$\sin C = \frac{1}{\mu} = \frac{3}{4}$$

$$\tan C = \frac{3}{\sqrt{7}}$$

$$r = \sqrt{7} \times \frac{3}{\sqrt{7}} = 3$$

Area of surface = $\pi r^2 = 9\pi m^2$

9. A travelling microscope is used to determine the refractive index of a glass slab. If 40 divisions are there in 1 cm on main scale and 50 Vernier scale divisions are equal to 49 main scale divisions, then least count of the travelling microscope is ______$\times 10^{-6}$ m.

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. 50 VSD = 49 MSD

$$1VSD = \frac{49}{50} MSD$$

Least count = 1 MSD – 1 VSD

$$= \left(1 - \frac{49}{50}\right) MSD = \frac{1}{50} MSD$$

$$1MSD = \frac{1}{40} cm$$

$$\text{Least count} = \frac{1}{50 \times 40} \text{ cm}$$

$$= \frac{1}{2000} \text{ cm} = \frac{1}{2} \times 10^{-5} \text{m}$$

$= 0.5 \times 10^{-5}$ m

$= 5 \times 10^{-6}$ m

10. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 6630 Å is 0.42 V. If the threshold frequency is $x \times 10^{13}$/s, where x is ______ (nearest integer).

(Given, speed light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} Js)

Official Ans. by NTA (35)

Allen Ans. (35)

Sol. Stopping potential $V_0 = 0.42$ V

$\lambda = 6630$ Å

$E = \phi + eV_0$

E : energy of incident photon

V_0 : Stopping potential

$\phi = E - eV_0$

$$E = \frac{12400}{6630} eV = 1.87 \text{ eV}$$

$\phi = (1.87 - 0.42) = 1.45$ eV

$\phi = h\nu_0$; ν_0 : threshold frequency

$1.45 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34} \times \nu_0$

$\nu_0 = 0.35 \times 10^{15}$

$= 35 \times 10^{13}$ sec^{-1}

$= 35$