

FINAL JEE-MAIN EXAMINATION – JULY, 2022
(Held On Tuesday 26th July, 2022)
TIME : 9 : 00 AM to 12 : 00 NOON
MATHEMATICS
TEST PAPER WITH SOLUTION
SECTION-A

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to :

- (A) 4 (B) 10
(C) 11 (D) 16

Official Ans. by NTA (B)
Allen Ans. (B)

Sol. $f(x) - f(x/3) = x/3$
 $f(x/3) - f(x/3^2) = x/3^2$

.... on adding

$$f(x) - \lim_{n \rightarrow \infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} + \dots + \infty\right)$$

$$f(x) - f(0) = \frac{x}{2}$$

$$f(8) = 7 ; f(0) = 3$$

$$f(x) = x/2 + 3$$

$$f(14) = 10$$

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2 , $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ?

- (A) $\arg z_2 = \pi - \tan^{-1} 3$
 (B) $\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$
 (C) $|z_2| = \sqrt{10}$
 (D) $|2z_1 - z_2| = 5$

Official Ans. by NTA (D)
Allen Ans. (D)

Sol. $AB = AO. z^{-i\pi/2} = -2 + i$

$$\text{So } OB = (-2 + i) + (1 + 2i)$$

$$z_2 = -1 + 3i$$

$$\therefore |2z_1 - z_2| = \sqrt{10}$$

3. If the system of linear equations.

$$8x + y + 4z = -2$$

$$x + y + z = 0$$

$$\lambda x - 3y = \mu$$

has infinitely many solutions, then the distance of

 the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z +$
 $2 = 0$ is :

- (A) $3\sqrt{5}$ (B) 4
(C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Official Ans. by NTA (D)
Allen Ans. (D)

Sol. $D = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0 \Rightarrow \lambda = 4$

$$\text{Also } D_1 = D_2 = D_3 = 0$$

$$\text{So } \mu = -2$$

$$\text{Point } \left(4, -2, -\frac{1}{2}\right)$$

$$\text{Distance from plane} = \frac{10}{3}$$

4. Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(\text{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be :

- (A) -1 (B) 2
(C) 1 (D) $-\sqrt{2}$

Official Ans. by NTA (B)
Allen Ans. (B)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc = -1$

$$|A + I| |\text{adj } A + I| = 4$$

$$\Rightarrow ad - bc + a + d + 1 = 2 \text{ or } -2$$

$$a + d = 2 \text{ or } -2$$

5. The odd natural number a , such that the area of the region bounded by $y = 1$, $y = 3$, $x = 0$, $x = y^a$ is

$$\frac{364}{3}, \text{ equal to :}$$

- (A) 3 (B) 5
(C) 7 (D) 9

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $A = \int_1^3 y^a \cdot dy = \frac{y^{a+1}}{a+1} \Big|_1^3 = \frac{364}{3}$

$$\Rightarrow a = 5$$

6. Consider two G.Ps. $2, 2^2, 2^3, \dots$ and $4, 4^2, 4^3, \dots$ of 60 and n terms respectively. If the geometric mean

of all the $60 + n$ terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^n k(n-k)$

is equal to :

- (A) 560 (B) 1540
(C) 1330 (D) 2600

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\left((2^1 2^2 \dots 2^{60}) (4^1 \cdot 4^2 \dots 4^n) \right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$

$$\left(2^{30 \times 61} 4^{\frac{n(n+1)}{2}} \right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$$

$$2^{1830+n^2+n} = 2^{\frac{(225)(60+n)}{8}}$$

$$= 8n^2 - 217n + 1140 = 0$$

$$n = 20, \frac{57}{8}$$

$$\sum_{k=1}^n nk - k^2 = \frac{n^2(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= 1330$$

7. If the function

$$f(x) = \begin{cases} \frac{\log_e(1-x+x^2) + \log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right) - \{0\} \\ k, & x=0 \end{cases}$$

is continuous at $x = 0$, then k is equal to :

- (A) 1 (B) -1
(C) e (D) 0

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\lim_{x \rightarrow 0} \frac{(\ln(1+x^2+x^4)) \cos x}{1 - \cos^2 x}$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{\ln(1+x^2+x^4)}{x^2+x^4} \right) x^2 (1+x^2) \cos x}{\left(\frac{\sin^2 x}{x^2} \right) x^2} = 1$$

$$\therefore k = 1$$

8. If $f(x) = \begin{cases} x+a, & x \leq 0 \\ |x-4|, & x > 0 \end{cases}$ and

$$g(x) = \begin{cases} x+1, & x < 0 \\ (x-4)^2 + b, & x \geq 0 \end{cases}$$

are continuous on \mathbb{R} , then $(g \circ f)(2) + (f \circ g)(-2)$ is equal to :

- (A) -10 (B) 10
(C) 8 (D) -8

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $f(x) = \begin{cases} x+a; & x \leq 0 \\ |x-4|; & x > 0 \end{cases}; g(x) = \begin{cases} x+1; & x < 0 \\ (x-4)^2 + b; & x \geq 0 \end{cases}$

For continuity $a = 4$ and $b = -15$

$$g(f(2)) + f(g(-2))$$

$$= g(2) + f(-1) = -8$$

9. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \leq 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$

Then the set of all values of b , for which $f(x)$ has maximum value at $x = 1$, is :

- (A) $(-6, -2)$
- (B) $(2, 6)$
- (C) $[-6, -2) \cup (2, 6]$
- (D) $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $f(1) = 3$

For $x < 1$, $f'(x) = 3x^2 - 2x + 10 > 0$

$\Rightarrow f(x)$ is increasing

For $x > 1$, $f'(x) < 0$

\Rightarrow function is decreasing.

$\lim_{x \rightarrow 1^+} f(x) = -2 + \log_2(b^2 - 4)$

For maximum value at $x = 1$

$3 \geq -2 + \log_2(b^2 - 4)$

$32 \geq b^2 - 4 > 0$

$b \in [-6, -2) \cup (2, 6]$

10. If $a = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2n}{n^2 + k^2}$ and $f(x) =$

$\sqrt{\frac{1 - \cos x}{1 + \cos x}}$, $x \in (0, 1)$, then :

(A) $2\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(B) $f\left(\frac{a}{2}\right)f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(D) $f\left(\frac{a}{2}\right) = \sqrt{2}f'\left(\frac{a}{2}\right)$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $a = \frac{1}{n} \sum_{k=1}^n \frac{2}{1 + \left(\frac{k}{n}\right)^2} = \int_0^1 \frac{2}{1 + x^2} dx = \frac{\pi}{2}$

$f(x) = \tan\left(\frac{x}{2}\right); x \in (0, 1)$

$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2} \sec^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{\sqrt{2} + 1}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$

11. If $\frac{dy}{dx} + 2y \tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) =$

0, then the maximum value of $y(x)$ is

(A) $\frac{1}{8}$ (B) $\frac{3}{4}$

(C) $\frac{1}{4}$ (D) $\frac{3}{8}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\frac{dy}{dx} + 2y \tan x = \sin x$

I.F = $e^{\int 2 \tan x dx} = e^{\ln(\sec x)^2} = \sec^2 x$

$y(\sec^2 x) = \int \sin x \sec^2 x dx + C$

$y \cdot \sec^2 x = \sec x + C$

Put $x = \frac{\pi}{3}$, $y = 0$

$y = \cos x - 2 \cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore y_{\max} = \frac{1}{8}$

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (-2, 1) is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the point C, D. Then the area of the quadrilateral ACBD is equal to

- (A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$
 (C) $\frac{3\sqrt{17}}{4}$ (D) 9

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $(x-1)^2 + (y-2)^2 + (x+2)^2 + (y-1)^2 = 14$

$$\Rightarrow x^2 + y^2 + x - 3y - 2 = 0$$

Put $x = 0$

$$\Rightarrow y^2 - 3y - 2 = 0$$

$$\Rightarrow y = \frac{3 \pm \sqrt{17}}{2}$$

Put $y = 0$

$$\Rightarrow x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$\therefore A(-2, 0), B(1, 0), C\left(0, \frac{3+\sqrt{17}}{2}\right), D\left(0, \frac{3-\sqrt{17}}{2}\right)$$

$$\text{Area} = \frac{1}{2} \cdot 3 \cdot \sqrt{17} = \frac{3\sqrt{17}}{2}$$

13. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does NOT pass through the point :

- (A) (25, 10) (B) (20, 12)
 (C) (30, 8) (D) (15, 13)

Official Ans. by NTA (D)

Allen Ans. (D)

- Sol.** Tangent at (α, β) has slope 1

$$\beta^2 = 24\alpha$$

$$\text{Equation of tangent } y\beta = 12(x + \alpha), \frac{12}{\beta} = 1$$

$$\Rightarrow \alpha = 6, \beta = 12$$

$$\therefore (\alpha + 4, \beta + 4) = (10, 16)$$

$$\text{Normal at } (10, 16) \text{ to } \frac{x^2}{36} - \frac{y^2}{144} = 1 \text{ is}$$

$$2x + 5y = 100$$

14. The length of the perpendicular from the point (1, -2, 5) on the line passing through (1, 2, 4) and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is :

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$ (D) 1

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. d.r's of the line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{i} - 4\hat{j} - 3\hat{k}$

\therefore equation of line is

$$\vec{r} = \hat{i} + 2\hat{j} + 4\hat{k} + \lambda(\hat{i} - 4\hat{j} - 3\hat{k})$$

Let A(1, 2, 4) and P be $(1 + \lambda, 2 - 4\lambda, 4 - 3\lambda)$

$$\therefore \overline{PA} \cdot (\hat{i} - 4\hat{j} - 3\hat{k}) = 0$$

$$\lambda = \frac{1}{2}$$

$$\Rightarrow P\left(\frac{1}{2}, 2, \frac{-5}{2}\right)$$

$$|AP| = \sqrt{\frac{21}{2}}$$

15. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to

- (A) $\frac{15}{2}$ (B) 8
(C) $\frac{13}{2}$ (D) 7

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\vec{a} \times \vec{b} = (1 - \alpha)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$

Projection of $\vec{a} \times \vec{b}$ on $-\hat{i} + 2\hat{j} - 2\hat{k}$

$$= \frac{(\vec{a} \times \vec{b}) \cdot (-\hat{i} + 2\hat{j} - 2\hat{k})}{3} = 30$$

$$\Rightarrow 2\alpha^2 - \alpha - 91 = 0$$

$$\Rightarrow \alpha = 7, -\frac{13}{2}$$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X = 1) = \frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to :

- (A) $\frac{5}{9}$ (B) $\frac{64}{81}$
(C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $np = \alpha$ (1)

$npq = \alpha/3$ (2)

From (1) & (2)

$$q = 1/3 \text{ \& } p = 2/3$$

$${}^nC_1 q^{n-1} p^1 = \frac{4}{243}$$

$$\frac{n}{3^n} = \frac{2}{243}$$

$$n = 6$$

$$P(4 \text{ or } 5) = {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^6C_5 \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right)^0$$

$$= \frac{16}{27}$$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2 , then $(p_1 + p_2)$ is equal to :

$$= \frac{1-p}{2}$$

are p_1 and p_2 , then $(p_1 + p_2)$ is equal to :

- (A) $\frac{2}{3}$ (B) $\frac{5}{3}$
(C) $\frac{5}{4}$ (D) 1

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $0 \leq P(E_i) \leq 1$ for $i = 1, 2, 3$

$$\Rightarrow -2/3 \leq p \leq 1$$

E_1 & E_2 & E_3 are mutually exclusive

$$P(E_1) + P(E_2) + P(E_3) \leq 1$$

$$\Rightarrow 2/3 \leq p \leq 1$$

$$p_1 = 1, p_2 = 2/3$$

$$p_1 + p_2 = 5/3$$

18. Let

$$S = \{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\}$$
. Then

$$n(S) + \sum_{\theta \in S} \left(\sec\left(\frac{\pi}{4} + 2\theta\right) \operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right) \right)$$
 is

equal to :

- (A) 0 (B) -2
(C) -4 (D) 12

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $8^{2\sin^2\theta} + 8^{2-2\sin^2\theta} = 16$

$$y + \frac{64}{y} = 16$$

$$\Rightarrow y = 8$$

$$\Rightarrow \sin^2\theta = 1/2$$

$$n(S) + \sum_{\theta \in S} \frac{1}{\cos(\pi/4 + 2\theta) \sin(\pi/4 + 2\theta)}$$

$$= 4 + (-2) \times 4 = -4$$

19. $\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ is equal to:

- (A) 1 (B) 2
 (C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\tan\left(2\left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$
 $= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right]$
 $= 2$

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

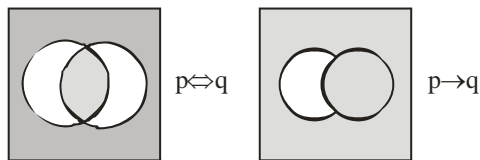
- (A) a tautology
 (B) a contradiction
 (C) equivalent to $(p \Rightarrow q) \wedge q$
 (D) equivalent to $(p \Rightarrow q) \wedge p$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



SECTION-B

1. If for some $p, q, r \in \mathbb{R}$, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p+r)x + q^2 + r^2 = 0$ is also a root of the equation

$x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to-

Official Ans. by NTA (272)

Allen Ans. (272)

Sol. $(px - q)^2 + (qx - r)^2 = 0$

$$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$$

$$\Rightarrow \frac{q^2 + r^2}{p^2} = 272$$

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is

Official Ans. by NTA (180)

Allen Ans. (180)

Sol. $3 \times \frac{5!}{2!2!} + \frac{5!}{3! \times 2!} + \frac{5!}{2!} + \frac{5!}{3!} = 180$

3. The series of positive multiples of 3 is divided into sets : $\{3\}$, $\{6, 9, 12\}$, $\{15, 18, 21, 24, 27\}$, ... Then the sum of the elements in the 11th set is equal to _____.

Official Ans. by NTA (6993)

Allen Ans. (6993)

Sol. $S_{11} = 3[101 + 102 + \dots + 121]$

$$= \frac{3}{2}(222) \times 21 = 6993$$

4. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$$\Rightarrow (x-1)^2(x+1)(x^5 + 3x - 1) = 0$$

$$\text{Let } f(x) = x^5 + 3x - 1$$

$$f'(x) > 0 \forall x \in \mathbb{R}$$

Hence 3 real distinct roots.

5. If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q$, $p, q \leq 15$, are -3 and -5 respectively, then the coefficient of x^3 is equal to _____.

Official Ans. by NTA (23)

Allen Ans. (23)

Sol. Since coefficient of x is -3

$$\Rightarrow {}^pC_1 - {}^qC_1 = -3$$

$$\Rightarrow p - q = -3 \quad \dots(1)$$

Comparing coefficients of x^2

$$-{}^pC_1 {}^qC_1 + {}^pC_2 + {}^qC_2 = -5$$

$$-pq + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5 \quad \dots(2)$$

Solving (1) and (2)

$$p = 8, q = 11$$

Coefficient of x^3 is

$$\begin{aligned} & -{}^qC_3 + {}^pC_3 + {}^pC_1 {}^qC_2 - {}^pC_2 {}^qC_1 \\ & = -{}^{11}C_3 + {}^8C_3 + {}^8C_1 {}^{11}C_2 - {}^8C_2 {}^{11}C_1 \\ & = 23 \end{aligned}$$

6. If

$$n(2n+1) \int_0^1 (1-x^n)^{2n} dx = 1177 \int_0^1 (1-x^n)^{2n+1} dx, \quad \text{then}$$

$n \in \mathbb{N}$ is equal to _____

Official Ans. by NTA (24)

Allen Ans. (24)

Sol. Let $I_1 = \int_0^1 (1-x^n)^{2n} dx$, $I_2 = \int_0^1 (1-x^n)^{2n+1} dx$

$$I_2 = \int_0^1 (1-x^n)^{2n+1} \cdot 1 dx$$

$$= (1-x^n)^{2n+1} \cdot x \Big|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n} (-nx^{n-1}) x dx$$

$$I_2 = -n(2n+1) \{I_2 - I_1\}$$

$$(2n^2 + n + 1)I_2 = n(2n+1)I_1$$

$$\frac{I_1}{I_2} = \frac{2n^2 + n + 1}{n(2n+1)} = \frac{1177}{n(2n+1)}$$

$$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$$

7. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(>3)$

be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the

point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to _____

Official Ans. by NTA (6)

Allen Ans. (6)

Sol. $x^4 = 3yx \cdot y' - 3y^2$

$$\Rightarrow 3xy \frac{dy}{dx} = 3y^2 + x^4$$

$$\text{Put } y^2 = t, y \frac{dy}{dx} = \frac{1}{2} \frac{dt}{dx}$$

$$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$$

$$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$$

$$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$$

Put (3, 3), $C = -2$

$$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$$

$$3y^2 = x^4 - 6x^2$$

$$x^4 - 6x^2 = 1080$$

$$\therefore x = 6$$

8. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is (2, a),

$-\frac{1}{2} < a < 2$, then p is equal to

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Coordinates of $A(1, -2)$, $B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and

orthocentre $H(2, a)$

Slope of AH = p

$$a + 2 = p \quad \dots(1)$$

Slope of BH = -1

$$31a - 2ab = 15a + 4p - 2 \quad \dots(2)$$

From (1) and (2)

$$a = 1 \text{ \& } p = 3$$

9. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a - 1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of

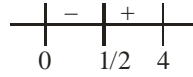
the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal

to-

Official Ans. by NTA (45)

Allen Ans. (45)

Sol. $f'(x) = 4x - \frac{1}{x}$



$$a = \frac{1}{2}$$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$$\frac{1}{y_1} = \frac{3 - y_1}{4 - x_1} \Rightarrow y_1^2 - 6y_1 + 8 = 0$$

$$y_1 = 2, 4$$

$\Rightarrow P(8, 4)$ as $P(2, 2)$ rejected

Equation of normal at P.

$$y - 4 = -4(x - 8)$$

$$\frac{x}{9} + \frac{y}{36} = 1$$

$$\alpha = 9, \beta = 36$$

$$\alpha + \beta = 45$$

10. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point $P(4, 2, 7)$. Then the square of the area of the triangle PQR is _____.

Official Ans. by NTA (153)

Allen Ans. (153)

- Sol.** Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$$

$$\lambda = 1, 3$$

$$Q(1, 1, 3); R(5, 7, 7); P(4, 2, 7)$$

$$\text{Area of triangle PQR} = \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$$

$$= \sqrt{153}$$