

**FINAL JEE-MAIN EXAMINATION – JUNE, 2022**

 (Held On Tuesday 28<sup>th</sup> June, 2022)

TIME : 3 : 00 PM to 06 : 00 PM

**MATHEMATICS**
**TEST PAPER WITH SOLUTION**
**SECTION-A**

 1. Let  $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$  and

 $R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$ . Then on  $\mathbb{N}$ :

- (A) Both  $R_1$  and  $R_2$  are equivalence relations  
 (B) Neither  $R_1$  nor  $R_2$  is an equivalence relation  
 (C)  $R_1$  is an equivalence relation but  $R_2$  is not  
 (D)  $R_2$  is an equivalence relation but  $R_1$  is not

**Official Ans. by NTA (B)**
**Allen Ans. (B)**
**Sol.**  $R_1 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \leq 13\}$ 
 $R_2 = \{(a, b) \in \mathbb{N} \times \mathbb{N} : |a - b| \neq 13\}$ .

 For  $R_1$  :

- i) Reflexive relation  
 $(a, a) \in \mathbb{N} \times \mathbb{N} : |a - a| \leq 13$   
 ii) Symmetric relation  
 $(a, b) \in R_1, (b, a) \in R_1 : |b - a| \leq 13$   
 iii) Transitive relation  
 $(a, b) \in R_1, (b, c) \in R_1, (a, c) \in R_1$  :  
 $(1, 3) \in R_1, (3, 16) \in R_1$  but  $(1, 16) \notin R_1$

 For  $R_2$  :

- i) Reflexive relation  
 $(a, a) \in \mathbb{N} \times \mathbb{N} : |a - a| \neq 13$   
 ii) Symmetric relation  
 $(b, a) \in \mathbb{N} \times \mathbb{N} : |b - a| \neq 13$   
 iii) Transitive relation  
 $(a, b) \in R_2, (b, c) \in R_2, (a, c) \in R_2$

 $(1, 3) \in R_2, (3, 14) \in R_2$  but  $(1, 14) \notin R_2$ 

 2. Let  $f(x)$  be a quadratic polynomial such that  $f(-2) + f(3) = 0$ . If one of the roots of  $f(x) = 0$  is  $-1$ , then the sum of the roots of  $f(x) = 0$  is equal to :

- (A)  $\frac{11}{3}$  (B)  $\frac{7}{3}$   
 (C)  $\frac{13}{3}$  (D)  $\frac{14}{3}$

**Official Ans. by NTA (A)**
**Allen Ans. (A)**
**Sol.**  $f(-2) + f(3) = 0$ 

$$f(x) = (x + 1)(ax + b)$$

$$f(-2) + f(3) = -1(-2a + b) + 4(3a + b) = 0$$

$$2a - b + 12a + 4b = 0$$

$$14a + 3b = 0$$

$$\frac{-b}{a} = \frac{14}{3}$$

$$\text{Sum of roots} = \left(-1 + \frac{-b}{a}\right) = -1 + \frac{14}{3} = \frac{11}{3}$$

 3. The number of ways to distribute 30 identical candies among four children  $C_1, C_2, C_3$  and  $C_4$  so that  $C_2$  receives atleast 4 and atmost 7 candies,  $C_3$  receives atleast 2 and atmost 6 candies, is equal to

- (A) 205 (B) 615  
 (C) 510 (D) 430

**Official Ans. by NTA (D)**
**Allen Ans. (D)**
**Sol.**  $t_1 + t_2 + t_3 + t_4 = 30$ 

 Coefficient of  $x^{30}$  in  $(1 + x + x^2 + \dots + x^{30})^2$ 

$$(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$$

$$x^6 \left( \frac{1 - x^{31}}{1 - x} \right)^2 (1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)$$

$$x^6 (1 - x^3)^2 (1 - x^4)(1 - x^5)(1 - x)^4$$

$$x^6 (1 - x^4 - x^5 + x^9)(1 + x^{62} - 2x^{31}(1 - x)^{-4})$$

$$x^6 (1 - x^4 - x^5 + x^9)(1 - x)^{-4}$$

 Coefficient of  $x^n$  in  $(1 - x)^{-r}$  is  ${}^{n+r-1}C_{r-1}$ 

$$\Rightarrow {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$$

$$2925 - 1771 - 1540 + 816$$

$$= 430$$

**OR**

$$x_2 \in [4, 7], x_3 \in [2, 6]$$

$$\Rightarrow t_1 + t_2 + t_3 + t_4 = 24$$

total ways =

$${}^{24+4-1}C_{4-1} - {}^{20+4-1}C_{4-1} - {}^{19+4-1}C_{4-1} + {}^{15+4-1}C_{4-1}$$

$$= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3 = 430$$

4. The term independent of  $x$  in the expression of

$$(1 - x^2 + 3x^3) \left( \frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11}, x \neq 0 \text{ is}$$

- (A)  $\frac{7}{40}$  (B)  $\frac{33}{200}$   
 (C)  $\frac{39}{200}$  (D)  $\frac{11}{50}$

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

**Sol.**  $(1 - x^2 + 3x^3) \left( \frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11}$

General term of  $\left( \frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11}$  is

$${}^{11}C_r \left( \frac{5}{2}x^3 \right)^{11-r} \left( -\frac{1}{5x^2} \right)^r$$

General term is  ${}^{11}C_r \left( \frac{5}{2} \right)^{11-r} \left( -\frac{1}{5} \right)^r x^{33-5r}$

Now, term independent of  $x$

$$1 \times \text{coefficient of } x^0 \text{ in } \left( \frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11}$$

$$-1 \times \text{coefficient of } x^{-2} \text{ in } \left( \frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11} +$$

$$3 \times \text{coefficient of } x^{-3} \text{ in } \left( \frac{5}{2}x^3 - \frac{1}{5x^2} \right)^{11}$$

for coefficient of  $x^0$   $33 - 5r = 0$  not possible

for coefficient of  $x^{-2}$   $33 - 5r = -2$

$$35 = 5r \Rightarrow r = 7$$

for coefficient of  $x^{-3}$   $33 - 5r = -3$

$$36 = 5r \text{ not possible}$$

So term independent of  $x$  is

$$(-1) {}^{11}C_7 \left( \frac{5}{2} \right)^4 \left( -\frac{1}{5} \right)^7 = \frac{33}{200}$$

5. If  $n$  arithmetic means are inserted between  $a$  and  $100$  such that the ratio of the first mean to the last mean is  $1 : 7$  and  $a + n = 33$ , then the value of  $n$  is

- (A) 21 (B) 22  
 (C) 23 (D) 24

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.**  $d = \frac{100 - a}{n + 1}$

$$A_1 = a + d$$

$$A_n = 100 - d$$

$$\Rightarrow \frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{a + d}{100 - d} = \frac{1}{7}$$

$$\Rightarrow 7a + 8d = 100$$

$$\Rightarrow 7a + 8 \left( \frac{100 - a}{n + 1} \right) = 100 \quad \dots(1)$$

$$\therefore a + n = 33 \quad \dots(2)$$

Now, by Eq. (1) and (2)

$$7n^2 - 132n - 667 = 0$$

$$\boxed{n = 23} \text{ and } n = \frac{-29}{7} \text{ reject.}$$

6. Let  $f, g : \mathbf{R} \rightarrow \mathbf{R}$  be functions defined by

$$f(x) = \begin{cases} [x] & , x < 0 \\ |1 - x| & , x \geq 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} e^x - x & , x < 0 \\ (x - 1)^2 - 1 & , x \geq 0 \end{cases}$$

where  $[x]$  denote the greatest integer less than or equal to  $x$ . Then, the function  $f \circ g$  is discontinuous at exactly :

- (A) one point (B) two points  
 (C) three points (D) four points

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

**Sol.** Check continuity at  $x = 0$  and also check continuity at those  $x$  where  $g(x) = 0$

$$g(x) = 0 \text{ at } x = 0, 2$$

$$f \circ g(0^+) = -1$$

$$f \circ g(0) = 0$$

Hence, discontinuous at  $x = 0$

$$f \circ g(2^+) = 1$$

$$f \circ g(2^-) = -1$$

Hence, discontinuous at  $x = 2$

7. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be a differentiable function such that  $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$  and  $f'\left(\frac{\pi}{2}\right) = 1$  and

let  $g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t f(t)) dt$  for

$x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$ . Then  $\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} g(x)$  is equal to

- (A) 2 (B) 3  
(C) 4 (D) -3

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

**Sol.**  $g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t f(t)) dt$

$$g(x) = \int_x^{\pi/4} d(f(t) \cdot \sec t) = f(t) \sec t \Big|_x^{\pi/4}$$

$$g(x) = f\left(\frac{\pi}{4}\right) \sec \frac{\pi}{4} - f(x) \cdot \sec x$$

$$g(x) = 2 - f(x) \sec x = 2 - \left(\frac{f(x)}{\cos x}\right)$$

$$\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \left(\frac{f(x)}{\cos x}\right)$$

Using L'Hopital Rule

$$= 2 - \lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} \frac{f'(x)}{(-\sin x)}$$

$$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin \frac{\pi}{2}} = 2 + \frac{1}{1} = 3$$

8. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be continuous function satisfying  $f(x) + f(x+k) = n$ , for all  $x \in \mathbf{R}$  where  $k > 0$  and  $n$

is a positive integer. If  $I_1 = \int_0^{4nk} f(x) dx$  and

$$I_2 = \int_{-k}^{3k} f(x) dx, \text{ then}$$

- (A)  $I_1 + 2I_2 = 4nk$  (B)  $I_1 + 2I_2 = 2nk$   
(C)  $I_1 + nI_2 = 4n^2k$  (D)  $I_1 + nI_2 = 6n^2k$

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.**  $f(x) + f(x+k) = n$

$$\Rightarrow f(x) = f(x+2k)$$

$f(x)$  is periodic with period  $2k$

$$I_1 = \int_0^{4nk} f(x) dx = 2n \int_0^{2k} f(x) dx$$

$$I_2 = \int_{-k}^{3k} f(x) dx = 2 \int_0^{2k} f(x) dx$$

Now,

$$f(x) + f(x+k) = n$$

$$\Rightarrow \int_0^k f(x) dx + \int_0^k f(x+k) dx = nk$$

$$\Rightarrow \int_0^k f(x) dx + \int_k^{2k} f(x) dx = nk$$

$$\Rightarrow \int_0^{2k} f(x) dx = nk$$

$$\Rightarrow I_1 = 2n^2k, I_2 = 2nk$$

$$\Rightarrow I_1 + nI_2 = 4n^2k$$

9. The area of the bounded region enclosed by the

curve  $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$  and the x-axis is

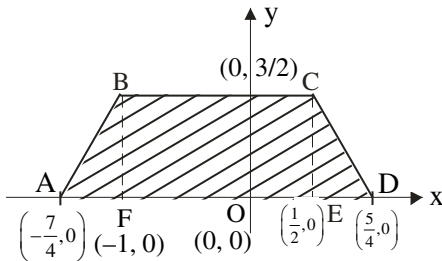
- (A)  $\frac{9}{4}$  (B)  $\frac{45}{16}$   
(C)  $\frac{27}{8}$  (D)  $\frac{63}{16}$

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

Sol. 
$$y = \begin{cases} 3 + (x+1) + \left(x - \frac{1}{2}\right), & x < -1 \\ 3 - (x+1) + \left(x - \frac{1}{2}\right), & -1 \leq x < \frac{1}{2} \\ 3 - (x+1) - \left(x - \frac{1}{2}\right), & \frac{1}{2} \leq x \end{cases}$$

$$y = \begin{cases} \frac{7}{2} + 2x, & x < -1 \\ \frac{3}{2}, & -1 \leq x < \frac{1}{2} \\ \frac{5}{2} - 2x, & \frac{1}{2} \leq x \end{cases}$$



Area bounded = ar ABF + ar BCEF + ar CDE

$$= \frac{1}{2} \left(\frac{3}{4}\right) \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) \left(\frac{3}{2}\right) + \frac{1}{2} \left(\frac{3}{4}\right) \left(\frac{3}{2}\right)$$

$$= \frac{27}{8} \text{ sq. units.}$$

10. Let  $x = x(y)$  be the solution of the differential equation  $2ye^{x/y^2} dx + (y^2 - 4xe^{x/y^2}) dy = 0$  such that  $x(1) = 0$ . Then,  $x(e)$  is equal to

- (A)  $e \log_e (2)$                       (B)  $-e \log_e (2)$   
(C)  $e^2 \log_e (2)$                       (D)  $-e^2 \log_e (2)$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol.  $2ye^{x/y^2} dx + (y^2 - 4xe^{x/y^2}) dy = 0$

$$2e^{x/y^2} [y dx - 2x dy] + y^2 dy = 0$$

$$2e^{x/y^2} \left[ \frac{y^2 dx - x \cdot (2y) dy}{y} \right] + y^2 dy = 0$$

Divide by  $y^3$

$$2e^{x/y^2} \left[ \frac{y^2 dx - x \cdot (2y) dy}{y^4} \right] + \frac{1}{y} dy = 0$$

$$2e^{x/y^2} d\left(\frac{x}{y^2}\right) + \frac{1}{y} dy = 0$$

Integrating

$$\int 2e^{x/y^2} d\left(\frac{x}{y^2}\right) + \int \frac{1}{y} dy = 0$$

$$2e^{x/y^2} + \ln y + c = 0$$

(0, 1) lies on it.

$$2e^0 + \ln 1 + c = 0 \Rightarrow c = -2$$

Required curve :  $2e^{x/y^2} + \ln y - 2 = 0$

For x (e)

$$2e^{x/e^2} + \ln e - 2 = 0 \Rightarrow x = -e^2 \log_e 2$$

11. Let the slope of the tangent to a curve  $y = f(x)$  at  $(x, y)$  be given by  $2 \tan x (\cos x - y)$ . If the curve passes through the point  $(\pi/4, 0)$ , then the value

of  $\int_0^{\pi/2} y dx$  is equal to

- (A)  $(2 - \sqrt{2}) + \frac{\pi}{\sqrt{2}}$                       (B)  $2 - \frac{\pi}{\sqrt{2}}$   
(C)  $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$                       (D)  $2 + \frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol.  $\frac{dy}{dx} = 2 \tan x \cos x - 2 \tan x \cdot y$

$$\frac{dy}{dx} + (2 \tan x) y = 2 \sin x$$

Integrating factor =  $e^{\int 2 \tan x dx} = \frac{1}{\cos^2 x}$

$$y \left( \frac{1}{\cos^2 x} \right) = \int \frac{2 \sin x}{\cos^2 x} dx$$

$$y \sec^2 x = \frac{2}{\cos x} + C$$

$$y = 2 \cos x + C \cos^2 x$$

Passes through  $\left(\frac{\pi}{4}, 0\right)$

$$0 = \sqrt{2} + \frac{C}{2} \Rightarrow C = -2\sqrt{2}$$

$f(x) = 2 \cos x - 2\sqrt{2} \cos^2 x$  : Required curve

$$\int_0^{\pi/2} y dx = 2 \int_0^{\pi/2} \cos x dx - 2\sqrt{2} \int_0^{\pi/2} \cos^2 x dx$$

$$= [2 \sin x]_0^{\pi/2} - 2\sqrt{2} \left[ \frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\pi/2}$$

$$= 2 - \frac{\pi}{\sqrt{2}}$$

12. Let a triangle be bounded by the lines  $L_1 : 2x + 5y = 10$ ;  $L_2 : -4x + 3y = 12$  and the line  $L_3$ , which passes through the point  $P(2, 3)$ , intersect  $L_2$  at A and  $L_1$  at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to

- (A)  $\frac{110}{13}$  (B)  $\frac{132}{13}$   
 (C)  $\frac{142}{13}$  (D)  $\frac{151}{13}$

**Official Ans. by NTA (B)**

**Allen Ans. (B)**

**Sol.** Points A lies on  $L_2$

$$A\left(\alpha, 4 + \frac{4}{3}\alpha\right)$$

Points B lies on  $L_1$

$$B\left(\beta, 2 - \frac{2}{5}\beta\right)$$

Points P divides AB internally in the ratio 1 : 3

$$\Rightarrow P(2, 3) = P\left(\frac{3\alpha + \beta}{4}, \frac{3\left(4 + \frac{4}{3}\alpha\right) + 1\left(2 - \frac{2}{5}\beta\right)}{4}\right)$$

$$\Rightarrow \alpha = \frac{3}{13}, \beta = \frac{95}{13}$$

$$\text{Point } A\left(\frac{3}{13}, \frac{56}{13}\right), B\left(\frac{95}{13}, -\frac{12}{13}\right)$$

Vertex C of triangle is the point of intersection of  $L_1$  &  $L_2$

$$\Rightarrow C\left(-\frac{15}{13}, \frac{32}{13}\right)$$

$$\text{area } \Delta ABC = \frac{1}{2} \begin{vmatrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & -\frac{12}{13} & 1 \\ -\frac{15}{13} & \frac{32}{13} & 1 \end{vmatrix}$$

$$= \frac{1}{2 \times 13^3} \begin{vmatrix} 3 & 56 & 13 \\ 95 & -12 & 13 \\ -15 & 32 & 13 \end{vmatrix}$$

$$\text{area } \Delta ABC = \frac{132}{13} \text{ sq. units.}$$

13. Let  $a > 0, b > 0$ . Let  $e$  and  $\ell$  respectively be the eccentricity and length of the latus rectum of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . Let  $e'$  and  $\ell'$  respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If  $e^2 = \frac{11}{14}\ell$  and  $(e')^2 = \frac{11}{8}\ell'$ ,

then the value of  $77a + 44b$  is equal to

- (A) 100 (B) 110  
 (C) 120 (D) 130

**Official Ans. by NTA (D)**

**Allen Ans. (D)**

**Sol.**  $e = \sqrt{1 + \frac{b^2}{a^2}}, \ell = \frac{2b^2}{a}$

Given  $e^2 = \frac{11}{14}\ell$

$$1 + \frac{b^2}{a^2} = \frac{11}{14} \cdot \frac{2b^2}{a}$$

$$\frac{a^2 + b^2}{a^2} = \frac{11}{7} \cdot \frac{b^2}{a} \dots\dots(1)$$

Also  $e' = \sqrt{1 + \frac{a^2}{b^2}}$ ,  $l' = \frac{2a^2}{b}$

Given  $(e')^2 = \frac{11}{8} l'$

$1 + \frac{a^2}{b^2} = \frac{11}{8} \cdot \frac{2a^2}{b}$

$\frac{a^2 + b^2}{b^2} = \frac{11}{4} \cdot \frac{a^2}{b}$  .....(2)

New (1) ÷ (2)

$\frac{b^2}{a^2} = \frac{4}{7} \cdot \frac{b^3}{a^3}$

∴  $7a = 4b$  ..... (3)

From (2)

$\frac{\frac{16b^2}{49} + b^2}{b^2} = \frac{11}{4} \cdot \frac{16b^2}{49b}$

$\frac{65}{49} = \frac{11}{4} \cdot \frac{16}{49} \cdot b$

∴  $b = \frac{4 \times 65}{11 \times 16}$  ..... (4)

We have to find value of

$77a + 44b$

$11(7a + 4b) = 11(4b + 4b) = 11 \times 8b$

∴ Value of  $11 \times 8b = 11 \times 8 \times \frac{4 \times 65}{16 \times 11} = 130$

14. Let  $\vec{a} = \alpha \hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{b} = -2\hat{i} + \alpha \hat{j} + \hat{k}$ , where  $\alpha \in \mathbf{R}$ . If the area of the parallelogram whose adjacent sides are represented by the vectors  $\vec{a}$  and  $\vec{b}$  is  $\sqrt{15(\alpha^2 + 4)}$ , then the value of

$2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$  is equal to

- (A) 10 (B) 7  
(C) 9 (D) 14

Official Ans. by NTA (D)

Allen Ans. (D)

Sol.  $\vec{a} = \alpha \hat{i} + 2\hat{j} - \hat{k}$ ,  $\vec{b} = -2\hat{i} + \alpha \hat{j} + \hat{k}$ ,  
area of parallelogram =  $|\hat{a} \times \hat{b}|$

$|\hat{a} \times \hat{b}| = \sqrt{(\alpha + 2)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2}$

Given  $|\hat{a} \times \hat{b}| = \sqrt{15(\alpha^2 + 4)}$

$2(\alpha^2 + 4) + (\alpha^2 + 4)^2 = 15(\alpha^2 + 4)$

$(\alpha^2 + 4)^2 = 13(\alpha^2 + 4)$

⇒  $\alpha^2 + 4 = 13$  ∴  $\alpha^2 = 9$

$2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$

$|\vec{a}|^2 = \alpha^2 + 4 + 1 = \alpha^2 + 5$

$|\vec{b}|^2 = 4 + \alpha^2 + 1 = \alpha^2 + 5$

$\vec{a} \cdot \vec{b} = -2\alpha + 2\alpha - 1 = -1$

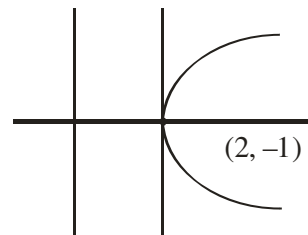
∴  $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$

$2(\alpha^2 + 5) - 1(\alpha^2 + 5) = \alpha^2 + 5 = 14$

15. If vertex of a parabola is (2, -1) and the equation of its directrix is  $4x - 3y = 21$ , then the length of its latus rectum is  
(A) 2 (B) 8  
(C) 12 (D) 16

Official Ans. by NTA (B)

Allen Ans. (B)



Sol.

$4x - 3y = 21$

$a = \frac{|8 + 3 - 21|}{5} = \frac{10}{5} = 2$

∴ latus rectum =  $4a = 8$

16. Let the plane  $ax + by + cz = d$  pass through (2, 3, -5) and is perpendicular to the planes  $2x + y - 5z = 10$  and  $3x + 5y - 7z = 12$ .

If a, b, c, d are integers  $d > 0$  and  $\gcd(|a|, |b|, |c|, d) = 1$ , then the value of  $a + 7b + c + 20d$  is equal to

- (A) 18 (B) 20  
(C) 24 (D) 22

**Official Ans. by NTA (D)**  
**Allen Ans. (D)**

**Sol.** DR'S normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

∴ eq<sup>n</sup> of plane

$$18x - y + 7z = d$$

It passes through (2, 3, -5)

$$36 - 3 - 35 = d \quad \therefore d = -2$$

∴ Eq<sup>n</sup> of plane

$$18x - y + 7z = -2$$

$$-18x + y - 7z = 2$$

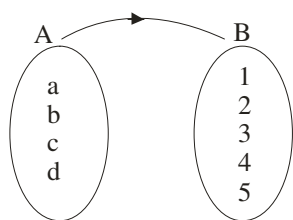
$$\therefore a = -18, b = 1, c = -7, d = 2$$

$$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$$

**17.** The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfies  $f(a) + 2f(b) - f(c) = f(d)$  is :

- (A)  $\frac{1}{24}$                       (B)  $\frac{1}{40}$   
(C)  $\frac{1}{30}$                       (D)  $\frac{1}{20}$

**Official Ans. by NTA (D)**  
**Allen Ans. (D)**



**Sol.**

$$n(s) = 5C_4 \times 4! = 120$$

f(a)	+	2f(b)	=	f(c)	+	f(d)
5		2×1		3		4
4		2×2		3		5
1		2×3		2		5

$$n(A) = 2 \times 3 = 6$$

$$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$$

**18.** The value of  $\lim_{n \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left( \frac{1}{r^2 + 3r + 3} \right) \right\}$

is equal to

- (A) 1                              (B) 2  
(C) 3                              (D) 6

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.**  $T_r = \tan^{-1} \left[ \frac{(r+2) - (r+1)}{1 + (r+2)(r+1)} \right]$

$$= \tan^{-1}(r+2) - \tan^{-1}(r+1)$$

$$T_1 = \tan^{-1} 3 - \tan^{-1} 2$$

$$T_2 = \tan^{-1} 4 - \tan^{-1} 3$$

$$T_n = \tan^{-1}(n+2) - \tan^{-1}(n+1)$$

$$S_n = \tan^{-1}(n+2) - \tan^{-1} 2 = \tan^{-1} \left( \frac{n+2-2}{1+2(n+2)} \right)$$

$$= \tan^{-1} \left( \frac{n}{2n+5} \right)$$

$$\lim_{n \rightarrow \infty} 6 \tan \left( \tan^{-1} \left( \frac{n}{2n+5} \right) \right)$$

$$= \lim_{n \rightarrow \infty} \frac{6n}{2n+5} = \frac{6}{2} = 3$$

**19.** Let  $\vec{a}$  be a vector which is perpendicular to the vector

$$3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}. \text{ If } \vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}, \text{ then}$$

the projection of the vector  $\vec{a}$  on the vector

$$2\hat{i} + 2\hat{j} + \hat{k} \text{ is}$$

- (A)  $\frac{1}{3}$                               (B) 1  
(C)  $\frac{5}{3}$                               (D)  $\frac{7}{3}$

**Official Ans. by NTA (C)**

**Allen Ans. (C)**

**Sol.**  $(\vec{a} \times (2\hat{i} + \hat{k})) \times \left( 3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$   
 $= (2\hat{i} - 13\hat{j} - 4\hat{k}) \times \left( 3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k} \right)$

$$-(6+2)\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$$

$$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

Projection of  $\vec{a}$  on vector  $2\hat{i} + 2\hat{j} + \hat{k}$  is

$$\vec{a} \cdot \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = \frac{5}{3}$$

20. If  $\cot \alpha = 1$  and  $\sec \beta = -\frac{5}{3}$ , where  $\pi < \alpha < \frac{3\pi}{2}$

and  $\frac{\pi}{2} < \beta < \pi$ , then the value of  $\tan(\alpha + \beta)$  and

the quadrant in which  $\alpha + \beta$  lies, respectively are

(A)  $-\frac{1}{7}$  and IV<sup>th</sup> quadrant

(B) 7 and I<sup>st</sup> quadrant

(C)  $-7$  and IV<sup>th</sup> quadrant

(D)  $\frac{1}{7}$  and I<sup>st</sup> quadrant

**Official Ans. by NTA (A)**

**Allen Ans. (A)**

**Sol.**  $\cot \alpha = 1, \sec \beta = \frac{-5}{3}, \cos \beta = \frac{-3}{5}, \tan \beta = \frac{-4}{3}$

$$\tan(\alpha + \beta) = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3} \times 1} = \frac{-1}{7}$$

### SECTION-B

1. Let the image of the point P(1, 2, 3) in the line

$$L: \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3} \text{ be } Q. \text{ let } R(\alpha, \beta, \gamma) \text{ be}$$

a point that divides internally the line segment PQ

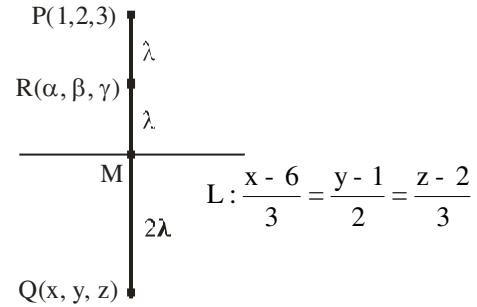
in the ratio 1 : 3. Then the value of  $22(\alpha + \beta + \gamma)$

is equal to

**Official Ans. by NTA (125)**

**Allen Ans. (125)**

**Sol.**



Let M be the mid-point of PQ

$$\therefore M = (3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$$

$$\text{Now, } \vec{PM} = (3\lambda + 5)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\therefore \vec{PM} \perp (3\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\therefore 3(3\lambda + 5) + 2(2\lambda - 1) + 3(3\lambda - 1) = 0$$

$$\lambda = \frac{-5}{11}$$

$$\therefore M \left( \frac{51}{11}, \frac{1}{11}, \frac{7}{11} \right)$$

Since R is mid-point of PM

$$22(\alpha + \beta + \gamma) = 125$$

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

**Official Ans. by NTA (0)**

**Allen Ans. (0)**

$$\text{Sol. } 20 = \frac{\sum_{i=1}^7 |x_i - 62|^2}{7}$$

$$\Rightarrow |x_1 - 62|^2 + |x_2 - 62|^2 + \dots + |x_7 - 62|^2 = 140$$

$$\text{If } x_1 = 49$$

$$|49 - 62|^2 = 169$$

then,

$|x_2 - 62|^2 + \dots + |x_7 - 62|^2 = \text{Negative Number}$   
which is not possible, therefore, no student can fail.

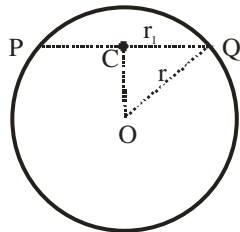


3. If one of the diameters of the circle  $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$  is a chord of the circle  $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$ , then the value of  $r^2$  is equal to

**Official Ans. by NTA (10)**

**Allen Ans. (10)**

Sol.



PQ is diameter of circle

$$S: x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$$

$$C(\sqrt{2}, 3\sqrt{2}), O(2\sqrt{2}, 2\sqrt{2})$$

$$r_1 = \sqrt{6}$$

$$S_1: (x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$$

Now in  $\Delta OCQ$

$$|OC|^2 + |CQ|^2 = |OQ|^2$$

$$4 + 6 = r^2$$

$$r^2 = 10$$

4. If  $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$ , then the

value of  $(a - b)$  is equal to

**Official Ans. by NTA (11)**

**Allen Ans. (11)**

Sol. 
$$\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$$

For finite limit

$$a + b - 5 = 0 \quad \dots(1)$$

Apply L'H rule

$$\lim_{x \rightarrow 1} \frac{\cos(3x^2 - 4x + 1)(6x - 4) - 2x}{(6x^2 - 14x + a)} = -2$$

For finite limit

$$6 - 14 + a = 0$$

$$\boxed{a = 8}$$

From (1)  $\boxed{b = -3}$

Now  $(a - b) = 11$

5. Let for  $n = 1, 2, \dots, 50$ ,  $S_n$  be the sum of the infinite geometric progression whose first term is

$n^2$  and whose common ratio is  $\frac{1}{(n+1)^2}$ . Then the

value of  $\frac{1}{26} + \sum_{n=1}^{50} \left( S_n + \frac{2}{n+1} - n - 1 \right)$  is equal to

**Official Ans. by NTA (41651)**

**Allen Ans. (41651)**

$$S_n = \frac{n^2}{1 - \frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{(n+2)}$$

$$S_n = \frac{n(n^2 + 2n + 1)}{(n+2)}$$

$$S_n = \frac{n[n(n+2)+1]}{(n+2)}$$

$$S_n = n \left[ n + \frac{1}{n+2} \right]$$

$$S_n = n^2 + \frac{n+2-2}{(n+2)}$$

$$S_n = n^2 + 1 - \frac{2}{(n+2)}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50} \left[ (n^2 - n) - 2 \left( \frac{1}{n+2} - \frac{1}{n+1} \right) \right]$$

$$= \frac{1}{26} + \left[ \frac{50 \times 51 \times 101}{6} - \frac{50 \times 51}{2} - 2 \left( \frac{1}{52} - \frac{1}{2} \right) \right]$$

$$= 41651$$

6. If the system of linear equations

$$2x - 3y = \gamma + 5,$$

$$\alpha x + 5y = \beta + 1, \text{ where } \alpha, \beta, \gamma \in \mathbf{R} \text{ has infinitely}$$

many solutions, then the value of  $|9\alpha + 3\beta + 5\gamma|$  is equal to

**Official Ans. by NTA (58)**

**Allen Ans. (58)**

Sol.  $2x - 3y = \gamma + 5$

$$\alpha x + 5y = \beta + 1$$

Infinite many solution

$$\frac{\alpha}{2} = \frac{5}{-3} = \frac{\beta+1}{\gamma+5}$$

$$\alpha = \frac{-10}{3}, \quad 5\gamma + 25 = -3\beta - 3$$

$$9\alpha = -30, \quad 3\beta + 5\gamma = -28$$

$$\text{Now, } 9\alpha + 3\beta + 5\gamma = -58$$

$$|9\alpha + 3\beta + 5\gamma| = 58$$

7. Let  $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$  where  $i = \sqrt{-1}$ .

Then, the number of elements in the set

$$\{n \in \{1, 2, \dots, 100\} : A^n = A\} \text{ is}$$

**Official Ans. by NTA (25)**

**Allen Ans. (25)**

**Sol.**  $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{4n+1} = A$$

$$n = 1, 5, 9, \dots, 97$$

$\Rightarrow$  total elements in the set is 25.

8. Sum of squares of modulus of all the complex numbers  $z$  satisfying  $\bar{z} = iz^2 + z^2 - z$  is equal to

**Official Ans. by NTA (2)**

**Allen Ans. (2)**

**Sol.**  $z + \bar{z} = iz^2 + z^2$

Consider  $z = x + iy$

$$2x = (i+1)(x^2 - y^2 + 2xyi)$$

$$\Rightarrow 2x = x^2 - y^2 - 2xy \text{ and } x^2 - y^2 + 2xy = 0$$

$$\Rightarrow 2x = -4xy$$

$$\Rightarrow x = 0 \text{ or } y = \frac{-1}{2}$$

Case 1 :  $x = 0 \Rightarrow y = 0$  here  $z = 0$

Case 2 :  $y = \frac{-1}{2}$

$$\Rightarrow 4x^2 - 4x - 1 = 0$$

$$(2x-1)^2 = 2$$

$$2x-1 = \pm\sqrt{2}$$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

Here  $z = \frac{1+\sqrt{2}}{2} - \frac{i}{2}$  or  $z = \frac{1-\sqrt{2}}{2} - \frac{i}{2}$

Sum of squares of modulus of  $z$

$$= 0 + \frac{(1+\sqrt{2})^2 + 1}{4} + \frac{(1-\sqrt{2})^2 + 1}{4} = \frac{8}{4} = 2$$

9. Let  $S = \{1, 2, 3, 4\}$ . Then the number of elements in the set  $\{f : S \times S \rightarrow S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) \in S \times S\}$  is

**Official Ans. by NTA (37)**

**Allen Ans. (37)**

**Sol.** (1, 1), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4) – all have one choice for image.

(2, 1), (1, 2), (2, 2) – all have three choices for image

(3, 2), (2, 3), (3, 1), (1, 3), (3, 3) – all have two choices for image.

So the total functions =  $3 \times 3 \times 2 \times 2 \times 2 = 72$

Case 1 : None of the pre-images have 3 as image

Total functions =  $2 \times 2 \times 1 \times 1 \times 1 = 4$

Case 2 : None of the pre-images have 2 as image

Total functions =  $2 \times 2 \times 2 \times 2 \times 2 = 32$

Case 3 : None of the pre-images have either 3 or 2 as image

Total functions =  $1 \times 1 \times 1 \times 1 \times 1 = 1$

$\therefore$  Total onto functions =  $72 - 4 - 32 + 1 = 37$

10. The maximum number of compound propositions, out of  $p \vee r \vee s$ ,  $p \vee r \vee \sim s$ ,  $p \vee \sim q \vee s$ ,  $\sim p \vee \sim r \vee s$ ,  $\sim p \vee \sim r \vee \sim s$ ,  $\sim p \vee q \vee \sim s$ ,  $q \vee r \vee \sim s$ ,  $q \vee \sim r \vee \sim s$ ,  $\sim p \vee \sim q \vee \sim s$  that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

**Official Ans. by NTA (9)**

**Allen Ans. (9)**

**Sol.** If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.