FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $S=\left\{x\in[-6,3]-\{-2,2\}:\frac{|x+3|-1}{|x|-2}\ge 0\right\}$

and $T=\{x\in Z: x^2-7|x|+9\le 0\}$. Then the number of elements in $S\cap T$ is

(A) 7 (B) 5

(C) 4 (D) 3

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $S\cap T=\{-5,-4,3\}$

2. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1,\frac{1}{\beta^2}+1$ be the roots of the equation $x^2+ax+b=0$. Then the roots of the equation $x^2-(a+b-2)x+(a+b+2)=0$ are :

(A) non-real complex numbers

(B) real and both negative

(C) real and both positive

(D) real and exactly one of them is positive

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $a=\frac{-1}{\alpha^2}-\frac{1}{\beta^2}-2$

$b=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+1+\frac{1}{\alpha^2\beta^2}$

$a+b=\frac{1}{(\alpha\beta)^2}-1=\frac{1}{6}-1=-\frac{5}{6}$

$x^2-\left(-\frac{5}{6}-2\right)x+\left(2-\frac{5}{6}\right)=0$

$6x^2+17x+7=0$

$x=-\frac{7}{3},\ x=-\frac{1}{2}$ are the roots

Both roots are real and negative.

3. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is **NOT** true?

(A) A^4-B^4 is a symmetric matrix

(B) AB – BA is a symmetric matrix

(C) B^5-A^5 is a skew-symmetric matrix

(D) AB + BA is a skew-symmetric matrix

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. Given that $A^T=A,\ B^T=-B$

(A) $C=A^4-B^4$

$C^T=(A^4-B^4)=(A^4)^T-(B^4)^T=A^4-B^4=C$

(B) $C=AB-BA$

$C^T=(AB-BA)^T=(AB)^T-(BA)^T$

$=B^TA^T-A^TB^T=-BA+AB=C$

(C) $C=B^5-A^5$

$C^T=(B^5-A^5)^T=(B^5)^T-(A^5)^T=-B^5-A^5$

(D) $C=AB+BA$

$C^T=(AB+BA)^T=(AB)^T+(BA)^T$

$=-BA-AB=-C$

∴ Option C is not true.

4. Let $f(x)=ax^2+bx+c$ be such that f(1) = 3, f(– 2) = λ and f(3) = 4. If f(0) + f(1) + f(–2) + f(3) = 14, then λ is equal to

(A) – 4 (B) $\frac{13}{2}$

(C) $\frac{23}{2}$ (D) 4

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. f(0) + 3 + λ + 4 = 14

∴ f(0) = 7 – λ = c

f(1) = a + b + c = 3 …(i)

f(3) = 9a + 3b + c = 4 …(ii)

f(–2) = 4a – 2b + c = λ …(iii)

(ii) – (iii)

$a+b=\frac{4-\lambda}{5}$ put in equation (i)

$\frac{4-\lambda}{5}+7-\lambda=3$

$6\lambda=24;\quad \lambda=4$

5. The function $f : R \to R$ defined by

$$f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$$ is

continuous for all x in

(A) $R - \{-1\}$ (B) $R - \{-1, 1\}$

(C) $R - \{1\}$ (D) $R - \{0\}$

Official Ans. by NTA (B)

Allen Ans. (B)

Note : n should be given as a natural number.

Sol. $$f(x) = \begin{cases} \frac{-\sin(x-1)}{x-1} & x < -1 \\ -(\sin 2 + 1) & x = -1 \\ \cos 2\pi x & -1 < x < 1 \\ 1 & x = 1 \\ \frac{-\sin(x-1)}{x-1} & x > 1 \end{cases}$$

f(x) is discontinuous at x = –1 and x = 1

6. The function $f(x) = xe^{x(1-x)}, x \in R$, is

(A) increasing in $\left(-\frac{1}{2}, 1\right)$

(B) decreasing in $\left(\frac{1}{2}, 2\right)$

(C) increasing in $\left(-1, -\frac{1}{2}\right)$

(D) decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $f(x) = x\ e^{x(1-x)}$

$f'(x) = -e^{x(1-x)}(2x + 1)(x - 1)$

f(x) is increasing in $\left(-\frac{1}{2}, 1\right)$

7. The sum of the absolute maximum and absolute minimum values of the function

$f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is

(A) 0 (B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) - \frac{\pi}{4}$ (D) $\frac{-\pi}{12}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $f(x) = \tan^{-1}(\sin x - \cos x)$

$$f'(x) = \frac{\cos x + \sin x}{(\sin x - \cos x)^2 + 1} = 0$$

$\therefore\ x = \frac{3\pi}{4}$

x	0	$\frac{3\pi}{4}$	π
f(x)	$-\frac{\pi}{4}$	$\tan^{-1}\sqrt{2}$	$\frac{\pi}{4}$

$\therefore$ $(f(x))_{max} = \tan^{-1}\sqrt{2}$, $(f(x))_{min} = -\frac{\pi}{4}$

$\text{sum} = \tan^{-1}\sqrt{2} - \frac{\pi}{4}$

$= \cos^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4}$

8. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and

$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$. Then

$$\frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$$ at $t = \frac{\pi}{4}$ is equal to

(A) $\frac{-2\sqrt{2}}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{-2}{3}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $x = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$$

$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$

$$\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\frac{dy}{dx} = \tan 3t$$

$$\frac{dy}{dx} = -1 \text{ at } t = \frac{\pi}{4}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{2}}\sec^3 3t \cdot \sqrt{\sin 2t} = -3 \text{ at } t = \frac{\pi}{4}$$

$$\therefore \frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{1+1}{-3} = -\frac{2}{3}$$

9. Let $I_n(x)=\int_0^x \frac{1}{(t^2+5)^n}dt,\ n = 1, 2, 3, \ldots$. Then

(A) $50I_6-9I_5=xI_5'$ (B) $50I_6-11I_5=xI_5'$

(C) $50I_6-9I_5=I_5'$ (D) $50I_6-11I_5=I_5'$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $I_n(x)=\int\limits_0^x \frac{dt}{(t^2+5)^n}$

Applying integral by parts

$$I_n(x)=\left[\frac{t}{(t^2+5)^n}\right]_0^x-\int\limits_0^x n(t^2+5)^{-n-1}\cdot 2t^2$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int\limits_0^x \frac{t^2}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\int\limits_0^x \frac{(t^2+5)-5}{(t^2+5)^{n+1}}dt$$

$$I_n(x)=\frac{x}{(x^2+5)^n}+2n\,I_n(x)-10n\,I_{n+1}(x)$$

$$10n\,I_{n+1}(x)+(1-2n)I_n(x)=\frac{x}{(x^2+5)^n}$$

Put n = 5

10. The area enclosed by the curves $y = \log_e(x + e^2)$, $x=\log_e\left(\frac{2}{y}\right)$ and $x = \log_e 2$, above the line $y = 1$ is

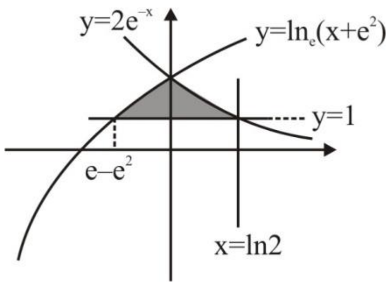
(A) $2+e-\log_e 2$ (B) $1+e-\log_e 2$

(C) $e-\log_e 2$ (D) $1+\log_e 2$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol.



Required area is

$$=\int\limits_{e-e^2}^{0}\ell n\left(x+e^2\right)-1dx+\int\limits_0^{\ell n2}2e^{-x}-1dx=1+e-\ell n2$$

11. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, $x > 1$ passing through the point $\left(2,\sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to

(A) $11+6\log_e 3$ (B) 19

(C) $12-2\log_e 3$ (D) $19-6\log_e 3$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$,

$$\frac{dy}{dx}+Py=Q$$

$$\text{I.F.}=e^{\int Pdx}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}=\int\left(\frac{x-1}{x+1}\right)^{1}dx$$

$$=x-2\log_e|x+1|+C$$

Curve passes through $\left(2,\frac{1}{\sqrt{3}}\right)$

$$\Rightarrow C=2\log_e 3-\frac{5}{3}$$

at x = 8,

$$\sqrt{7}y(8)=19-6\log_e 3$$

12. The differential equation of the family of circles passing through the points (0, 2) and (0, – 2) is

(A) $2xy\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$

(B) $2xy\frac{dy}{dx}+\left(x^2+y^2-4\right)=0$

(C) $2xy\frac{dy}{dx}+\left(y^2-x^2+4\right)=0$

(D) $2xy\frac{dy}{dx}-\left(x^2-y^2+4\right)=0$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Equation of circle passing through (0, –2) and (0, 2) is

$$x^2+(y^2-4)+\lambda x=0,\ (\lambda\in R)$$

Divided by x we get

$$\frac{x^2+(y^2-4)}{x}+\lambda=0$$

Differentiating with respect to x

$$\frac{x\left[2x+2y\cdot\frac{dy}{dx}\right]-\left[x^2+y^2-4\right]\cdot 1}{x^2}=0$$

$$\Rightarrow 2xy\cdot\frac{dy}{dx}+(x^2-y^2+4)=0$$

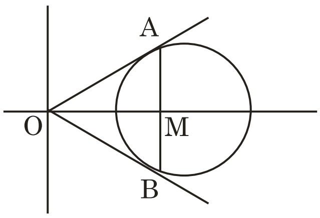
13. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin O (0, 0). Then the area of the triangle of OAB is

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$

(C) $\frac{3}{2\sqrt{3}}$ (D) $\frac{3}{4\sqrt{3}}$

Official Ans. by NTA (B)

Allen Ans. (B)



Sol. C : $(x-2)^2 + y^2 = 1$

Equation of chord AB : 2x = 3

OA = OB = $\sqrt{3}$

$$AM=\frac{\sqrt{3}}{2}$$

Area of triangle OAB $=\frac{1}{2}(2AM)(OM)$

$=\frac{3\sqrt{3}}{4}$ sq. units

14. Let the hyperbola H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ pass through the point $(2\sqrt{2},-2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

(A) $(2\sqrt{3},3\sqrt{2})$ (B) $(3\sqrt{3},-6\sqrt{2})$

(C) $(\sqrt{3},-\sqrt{6})$ (D) $(3\sqrt{6},6\sqrt{2})$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Foci : S (ae, 0), S' (–ae, 0)

Foot of directrix of parabola is (–ae, 0)

Focus of parabola is (ae, 0)

Now, semi latus rectum of parabola $=|SS'|=2ae$

Given, $4ae=e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2=2a^2$ ….. (1)

Given, $(2\sqrt{2},-2\sqrt{2})$ lies on H

$\Rightarrow \frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{8}$ ….. (2)

From (1) and (2)

$a^2 = 4,\ b^2 = 8$

$\because b^2=a^2(e^2-1)$

$\therefore e=\sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2=8\sqrt{3}x$

15. Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does **NOT** lies on P?

(A) (0, – 2, –2) (B) (– 5, 0, –1)

(C) (3, –1, 0) (D) (0, 4, 5)

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. Given, $L_1 : \frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$

and $L_2 : \frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$

are coplanar

$$\Rightarrow \begin{vmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 3$

Now, normal of plane P, which contains L_1 and L_2

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix}$$

$= -3\hat{i} - 13\hat{j} + 11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3x + 13y - 11z + 4 = 0$

(0, 4, 5) does not lie on plane P.

16. A plane P is parallel to two lines whose direction ratios are –2, 1, –3, and –1, 2, –2 and it contains the point (2, 2, –2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin and p = $\alpha+\beta+\gamma$, then the ordered pair (V, p) is equal to

(A) (48, –13) (B) (24, –13)

(C) (48, 11) (D) (24, –5)

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. Normal of plane P :

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ -1 & 2 & -2 \end{vmatrix} = 4\hat{i} - \hat{j} - 3\hat{k}$$

Equation of plane P which passes through (2, 2,–2) is $4x - y - 3z - 12 = 0$

Now, A (3, 0, 0), B (0, – 12, 0), C (0, 0, –4)

$\Rightarrow \alpha = 3, \beta = -12, \gamma = -4$

$\Rightarrow p = \alpha + \beta + \gamma = -13$

Now, volume of tetrahedron OABC

$$V = \left|\frac{1}{6}\overrightarrow{OA}\cdot\left(\overrightarrow{OB}\times\overrightarrow{OC}\right)\right| = 24$$

(V, p) = (24, –13)

17. Let S be the set of all $a \in R$ for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}, (b > 1)$ is acute. Then S is equal to

(A) $\left(-\infty, -\frac{4}{3}\right)$ (B) Φ

(C) $\left(-\frac{4}{3}, 0\right)$ (D) $\left(\frac{12}{7}, \infty\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. For angle to be acute

$\vec{u}\cdot\vec{v} > 0$

$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

$\forall\ b > 1$

let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$y = at^2 + 6at - 12\ \&\ y > 0,\ \forall\ t > 0$

$\Rightarrow a \in \phi$

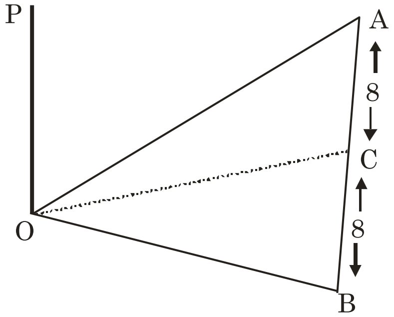
18. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(A) $\frac{32}{\sqrt{3}}(\sqrt{3}-1)$ (B) $\frac{32}{\sqrt{3}}(2-\sqrt{3})$

(C) $\frac{16}{\sqrt{3}}(\sqrt{3}-1)$ (D) $\frac{16}{\sqrt{3}}(2-\sqrt{3})$

Official Ans. by NTA (B)

Allen Ans. (B)



Sol.

$$\frac{OP}{OA} = \tan 15°$$

$$\Rightarrow OA = OP\cot 15°$$

$$\frac{OP}{OC} = \tan 45° \Rightarrow OP = OC$$

Now, $OP = \sqrt{OA^2 - 8^2}$

$$\Rightarrow OP^2 = (OP)^2 \cot^2 15° - 64$$

$$\Rightarrow OP^2 = \frac{32}{\sqrt{3}}(2-\sqrt{3})$$

19. Let A and B be two events such that $P(B|A) = \frac{2}{5}$, $P(A|B) = \frac{1}{7}$ and $P(A\cap B) = \frac{1}{9}$. Consider

$(S1)\, P(A'\cup B) = \frac{5}{6}$,

$(S2)\, P(A'\cap B') = \frac{1}{18}$. Then

(A) Both (S1) and (S2) are true

(B) Both (S1) and (S2) are false

(C) Only (S1) is true

(D) Only (S2) is true

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $P(A|B) = \frac{1}{7} \Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{1}{7}$

$$\Rightarrow P(B) = \frac{7}{9}$$

$$P(B|A) = \frac{2}{5} \Rightarrow \frac{P(A\cap B)}{P(A)} = \frac{2}{5}$$

$$\Rightarrow P(A) = \frac{5}{18}$$

Now, $P(A'\cup B) = 1 - P(A\cup B) + P(B)$

$$= 1 - P(A) + P(A\cap B) = \frac{5}{6}$$

$$P(A'\cap B') = 1 - P(A\cup B)$$

$$= 1 - P(A) - P(B) + P(A\cap B) = \frac{1}{18}$$

$\Rightarrow$ Both (S1) and (S2) are true.

20. Let

p : Ramesh listens to music.

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

(A) $((\sim q)\wedge(r\vee s)) \Rightarrow p$

(B) $(q\wedge(r\vee s)) \Rightarrow p$

(C) $p \Rightarrow (q\wedge(r\vee s))$

(D) $p \Rightarrow ((\sim q)\wedge(r\vee s))$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $p \equiv$ Ramesh listens to music

$\sim q \equiv$ He is in village.

$r\vee s \equiv$ Saturday or sunday

$$p \Rightarrow ((\sim q)\wedge(r\vee s))$$

SECTION-B

1. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}} + \beta x\right)^4$, $(1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50 - \frac{2d}{\beta^2}$ is equal to _____

Official Ans. by NTA (57)

Allen Ans. (57)

Sol. ${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$$

$$\beta = \frac{12}{5} \text{ or } \beta = -2 \quad \therefore \beta = \frac{12}{5}$$

$$d = -\frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$$

$$\therefore 50 - \frac{2d}{\beta^2} = 57$$

2. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3 g is equal to

Official Ans. by NTA (17)

Allen Ans. (17)

Sol. ${}^bC_3 \times {}^gC_2 = 168$

$b(b-1)(b-2)(g)(g-1) = 8 \times 7 \times 6 \times 3 \times 2$

$b + 3g = 17$

3. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ meet at the point $R\left(\sqrt{2}, 2\sqrt{2}-2\right)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to

Official Ans. by NTA (13)

Allen Ans. (13)

Sol. Ellipse is

$$\frac{x^2}{2} + \frac{y^2}{4} = 1; \; e = \frac{1}{\sqrt{2}}; S \equiv \left(0, -\sqrt{2}\right)$$

Chord of contact is

$$\frac{x}{\sqrt{2}} + \frac{\left(2\sqrt{2}-2\right)y}{4} = 1$$

$$\Rightarrow \frac{x}{\sqrt{2}} = 1 - \frac{\left(\sqrt{2}-1\right)y}{2}$$ solving with ellipse

$$\Rightarrow y = 0, \sqrt{2} \quad \therefore x = \sqrt{2}, 1$$

$$P \equiv \left(1, \sqrt{2}\right) \; Q \equiv \left(\sqrt{2}, 0\right)$$

$$\therefore (SP)^2 + (SQ)^2 = 13$$

4. If 1+ (2 + ${}^{49}C_1$ + ${}^{49}C_2$ + ….. + ${}^{49}C_{49}$) (${}^{50}C_2$ + ${}^{50}C_4$ + ….. + ${}^{50}C_{50}$) is equal to 2^n.m, where m is odd, then n + m is equal to _____

Official Ans. by NTA (99)

Allen Ans. (99)

Sol. $1 + \left(1 + 2^{49}\right)\left(2^{49} - 1\right) = 2^{98}$

$m = 1, \; n = 98$

$m + n = 99$

5. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then 17r is equal to

Official Ans. by NTA (9)

Allen Ans. (9)

Sol. $y^2 = -\frac{x}{2}$

$$y = mx - \frac{1}{8m}$$

this tangent pass through (2, 0)

$m = \pm\frac{1}{4}$ i.e., one tangent is $x - 4y - 2 = 0$

$17r = 9$

6. If $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots\ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then m.n is equal to _____

Official Ans. by NTA (12)

Allen Ans. (12)

Sol. $\frac{6}{3^{12}} + 10\left(\frac{1}{3^{11}} + \frac{2}{3^{10}} + \frac{2^2}{3^9} + \frac{2^3}{3^8} + \ldots\ldots + \frac{2^{10}}{3}\right)$

$$\frac{6}{3^{12}} + \frac{10}{3^{11}}\left(\frac{6^{11}-1}{6-1}\right)$$

$$= 2^{12} \cdot 1; \; m.n = 12$$

7. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$. Then the number of elements in the set

$$A=\left\{\theta\in S:\tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$$

is _____

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $\tan\theta+\sqrt{5}\tan2\theta\tan\theta=\sqrt{5}-\tan2\theta$

$\tan3\theta=\sqrt{5}$

$\theta=\frac{n\pi}{3}+\frac{\alpha}{3};\quad \tan\alpha=\sqrt{5}$

Five solution

8. Let $z = a + ib$, $b \neq 0$ be complex numbers satisfying $z^2=\bar{z}\cdot2^{1-|z|}$. Then the least value of $n \in N$, such that $z^n = (z+1)^n$, is equal to _____

Official Ans. by NTA (6)

Allen Ans. (6)

Sol. $|z^2|=|\bar{z}|\cdot2^{1-|z|}\Rightarrow|z|=1$

$z^2=\bar{z}\Rightarrow z^3=1\therefore z=\omega \text{ or } \omega^2$

$\omega^n=(1+\omega)^n=\left(-\omega^2\right)^n$

Least natural value of n is 6.

9. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then 100 σ^2 is equal to

Official Ans. by NTA (56)

Allen Ans. (56)

Sol.

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$$\sigma^2=\sum X^2P(X)-\left(\sum XP(X)\right)^2=\frac{56}{100}$$

$100\,\sigma^2=56$

10. The value of the integral $\int_0^{\frac{\pi}{2}}60\frac{\sin(6x)}{\sin x}dx$ is equal to

Official Ans. by NTA (104)

Allen Ans. (104)

Sol.

$$I=60\int_0^{\pi/2}\left(\frac{\sin6x-\sin4x}{\sin x}+\frac{\sin4x-\sin2x}{\sin x}+\frac{\sin2x}{\sin x}\right)dx$$

$$I=60\int_0^{\pi/2}(2\cos5x+2\cos3x+2\cos x)dx$$

$$I=60\left(\frac{2}{5}\sin5x+\frac{2}{3}\sin3x+2\sin x\right)\Bigg|_0^{\pi/2}=104$$