FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Consider the efficiency of Carnot's engine is given by $\eta = \frac{\alpha\beta}{\sin\theta}\log_e\frac{\beta x}{kT}$, where α and β are constants. If T is temperature, k is Boltzman constant, θ is angular displacement and x has the dimensions of length. Then, choose the **incorrect** option.

(A) Dimensions of β is same as that of force.

(B) Dimensions of α^{-1} x is same as that of energy.

(C) Dimensions of η^{-1} sinθ is same as that of $\alpha\beta$

(D) Dimensions of α is same as that of β

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $[\alpha\beta] = [\eta] = [\sin\theta] = \text{Dimensionless}$

$\left[\eta^{-1}\sin\theta\right] = [\alpha\beta] = \text{D.L.}$

2. At time t = 0 a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the x and y directions are defined as 3t and $5t^3$ respectively. At t = 1s acceleration of the particle will be

(A) $-30y$ (B) $30y$

(C) $3x + 15y$ (D) $3x + 15y + 7\hat{z}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\vec{r} = 3t\hat{i} + 5t^3\hat{j} + 7k$

$\frac{d^2\vec{r}}{dt^2} = 30t\,\hat{j}$

At $t = 1 \Rightarrow \boxed{\frac{d^2\vec{r}}{dt^2} = 30\hat{j}}$

3. A pressure-pump has a horizontal tube of cross-sectional area 10 cm^2 for the outflow of water at a speed of 20 m/s. The force exerted on the vertical wall just in front of the tube which stops water horizontally flowing out of the tube, is: [given : density of water = 1000 kg/m^3]

(A) 300 N (B) 500 N

(C) 250 N (D) 400 N

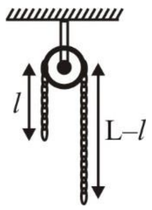
Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $F = \rho a \upsilon^2 = 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$F = 400$

4. A uniform metal chain of mass m and length 'L' passes over a massless and frictionless pulley. It is released from rest with a part of its length 'l' is hanging on one side and rest of its length '$L - l$' is hanging on the other side of the pulley. At a certain point of time, when $l = \frac{L}{x}$, the acceleration of the chain is $\frac{g}{2}$. The value of x is ....

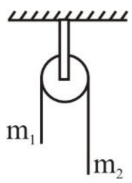


(A) 6 (B) 2 (C) 1.5 (D) 4

Official Ans. by NTA (D)

Allen Ans. (D)

Sol.

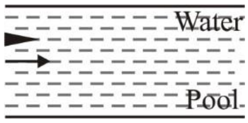


$a = \frac{(m_2 - m_1)}{(m_2 + m_1)}g$

$\frac{g}{2} = \frac{(\lambda(L-\ell) - \lambda\ell)g}{\lambda L} \Rightarrow L = \frac{L}{4} = \frac{L}{x}$

$x = 4$

5. A bullet of mass 200 g having initial kinetic energy 90 J is shot inside a long swimming pool as shown in the figure. If it's kinetic energy reduces to 40 J within 1s, the minimum length of the pool, the bullet has a to travel so that it completely comes to rest is



(A) 45 m (B) 90 m
(C) 125 m (D) 25 m

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. Using $mv = \sqrt{2mk}$

$$u = \frac{1}{0.2}\sqrt{2\times0.2\times90} = 30 \text{ m/s}$$

$$v = \frac{1}{0.2}\sqrt{2\times0.2\times40} = 20 \text{ m/s}$$

$$a = \frac{20-30}{1} = -10 \text{ m/s}^2$$

$$s = \frac{-u^2}{2a} = 45 \text{ m}$$

6. Assume there are two identical simple pendulum Clocks-1 is placed on the earth and Clock-2 is placed on a space station located at a height h above the earth surface. Clock-1 and Clock-2 operate at time periods 4s and 6s respectively. Then the value of h is –

(consider radius of earth R_E = 6400 km and g on earth 10 m/s^2)

(A) 1200 km
(B) 1600 km
(C) 3200 km
(D) 4800 km

Official Ans. by NTA (C)

Allen Ans. (C)

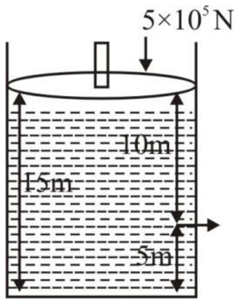
Sol. $t \propto \frac{1}{\sqrt{g}}$ and $g \propto \frac{1}{(R+h)^2}$

$$\frac{t_1}{t_2} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{R^2}{(R+h)^2}}$$

$$\frac{t_1}{t_2} = \frac{4}{6} = \frac{R}{(R+h)} \Rightarrow h = 3200\,\text{km}$$

7. Consider a cylindrical tank of radius 1m is filled with water. The top surface of water is at 15 m from the bottom of the cylinder. There is a hole on the wall of cylinder at a height of 5m from the bottom. A force of 5×10^5 N is applied an the top surface of water using a piston. The speed of efflux from the hole will be :

(given atmospheric pressure $P_A = 1.01 \times 10^5$ Pa, density of water ρ_w = 1000 kg/m^3 and gravitational acceleration g = 10 m/s^2)



(A) 11.6 m/s (B) 10.8 m/s
(C) 17.8 m/s (D) 14.4 m/s

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. Apply Bernoulli's theorem between Piston and hole $P_A + \rho gh = P_0 + \frac{1}{2}\rho v_e^2$

Assuming there is no atmospheric pressure on piston

$$\frac{5\times10^5}{\pi} + 10^3\times10\times10 = 1.01\times10^5 + \frac{1}{2}\times10^3\times v_e^2$$

v_e = 17.8 m/s

8. A vessel contains 14 g of nitrogen gas at a temperature of 27°C. The amount of heat to be transferred to the gap to double the r.m.s. speed of its molecules will be : (Take R = 8.32 J mol^{-1}k^{-1})

(A) 2229 J (B) 5616 J
(C) 9360 J (D) 13,104 J

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $v_{rms} \propto \sqrt{T}$

$v_{rms} \propto \sqrt{300K},\ v_{rms_f} = 2v_{rms_i}$

$v_{rms_f} \propto \sqrt{1200K}$

$T_f = 1200K,\ T_i = 300K,\ n = \frac{14}{28} = \frac{1}{2}$

$Q = nC_v\Delta T = \frac{1}{2} \times \frac{5R}{2} \times 900$

$Q = 9360$ J

9. A slab of dielectric constant K has the same cross-sectional area as the plates of a parallel plate capacitor and thickness $\frac{3}{4}d$, where d is the separation of the plates. The capacitance of the capacitor when the slab is inserted between the plates will be :

(Given C_o = capacitance of capacitor with air as medium between plates.)

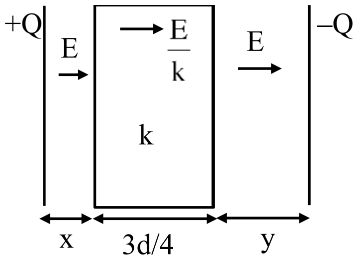
(A) $\frac{4KC_0}{3+K}$ (B) $\frac{3KC_0}{3+K}$

(C) $\frac{3+K}{4KC_0}$ (D) $\frac{K}{4+K}$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol.



$x + y + \frac{3d}{4} = d$

$x + y = \frac{d}{4}$

$\frac{A\in_0}{d} = C_0$

$\Delta V = Ex + \frac{E}{k} \times \frac{3d}{4} + Ey$

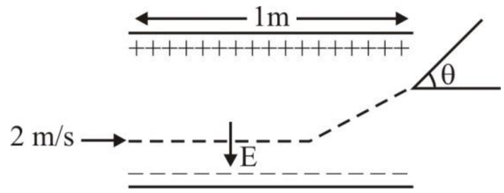
$= \frac{3Ed}{4k} + E(x+y)$

$\Delta V = E\left[\frac{3d}{4k} + \frac{d}{4}\right]$

$\Delta V = \frac{\sigma}{\in_0}\left[\frac{3d+dk}{4k}\right] = \frac{Qd}{A\in_0}\left[\frac{3+k}{4k}\right]$

$\frac{Q}{\Delta V} = C = \frac{A\in_0}{d}\left[\frac{4k}{3+k}\right] = \frac{4kC_0}{k+3}$

10. A uniform electric field E = (8m/e) V/m is created between two parallel plates of length 1m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be ....



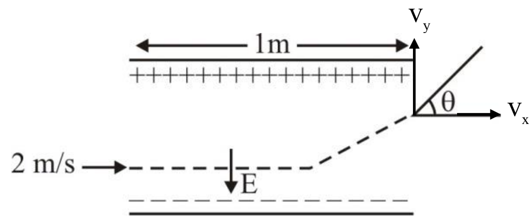
(A) $\tan^{-1}(4)$ (B) $\tan^{-1}(2)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$ (D) $\tan^{-1}(3)$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol.



$a_y = \frac{F_y}{m} = \frac{e(E)}{m} = \frac{e\left(\frac{8m}{e}\right)}{m} = 8m/s^2$

$s_x = u_x t$

$1 = 2 \times t$

$t = \frac{1}{2}$ sec

$v_y = u_y + a_y t$

$v_y = 0 + 8 \times \frac{1}{2}$

$v_y = 4$ m/s

$\tan\theta = \frac{v_y}{v_x} = \frac{4}{2} = 2 \Rightarrow \theta = \tan^{-1}(2)$

11. Given below are two statements :

Statement I : A uniform wire of resistance 80Ω is cut into four equal parts. These parts are now connected in parallel. The equivalent resistance of the combination will be $5\ \Omega$.

Statement II : Two resistance 2R and 3R are connected in parallel in a electric circuit. The value of thermal energy developed in 3R and 2R will be in the ratio 3 : 2.

In the light of the above statements, choose the most appropriate answer from the options given below

(A) Both statement I and statement II are correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

(D) Statement I is incorrect but statement II is correct.

Official Ans. by NTA (C)

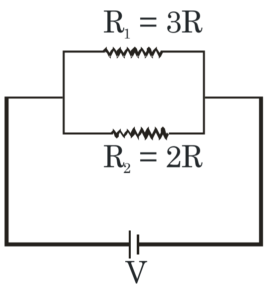
Allen Ans. (C)

Sol. **Statement 1 -** $R = 80\Omega$

$R_1 = R_2 = R_3 = R_4 = 20\ \Omega$

In parallel $R_{eq} = \frac{20}{4} = 5\Omega$

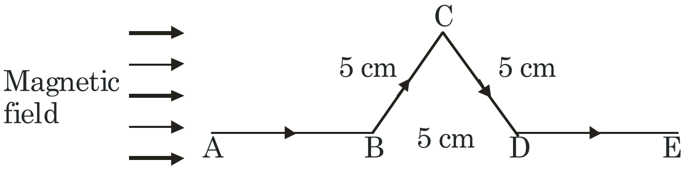
Statement 2 –



$P_{th} = \frac{v^2}{R}$

$\frac{P_1}{P_2} = \left(\frac{R_2}{R_1}\right) = \frac{2}{3}$ (where P is power)

12. A triangular shaped wire carrying 10A current is placed in a uniform magnetic field of 0.5T, as shown in figure. The magnetic force on segment CD is (Given BC = CD = BD = 5 cm).



(A) 0.126 N (B) 0.312 N

(C) 0.216 N (D) 0.245 N

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $F_M(CD) = BI\ell_{eff}$

$= 0.5\times(10) \times (5 \sin 60 \times 10^{-2})$

$= 0.216$ N

13. The magnetic field at the center of current carrying circular loop is B_1. The magnetic field at a distance of $\sqrt{3}$ times radius of the given circular loop from the center on its axis is B_2. The value of B_1/B_2 will be

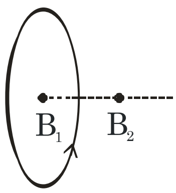
(A) 9 : 4 (B) $12 : \sqrt{5}$

(C) 8 : 1 (D) $5 : \sqrt{3}$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol.



$B_1 = \frac{\mu_0 I}{2R}$

$B_2 = \frac{\mu_0 I R^2}{2\left(R^2 + 3R^2\right)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right) = \frac{B_1}{8}$

$\frac{B_1}{B_2} = \frac{8}{1}$

14. A transformer operating at primary voltage 8 kV and secondary voltage 160 V serves a load of 80 kW. Assuming the transformer to be ideal with purely resistive load and working on unity power factor, the loads in the primary and secondary circuit would be

(A) 800 Ω and 1.06 Ω (B) 10 Ω and 500 Ω

(C) 800 Ω and 0.32 Ω (D) 1.06 Ω and 500 Ω

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $$\frac{(8\times10^3)^2}{R_P}=80\times10^3$$

$R_P = 800\Omega$

$$\frac{(160)^2}{R_S}=80\times10^3$$

$R_S = 0.32\Omega$

15. Sun light falls normally on a surface of area 36 cm^2 and exerts an average force of 7.2×10^{-9} N within a time period of 20 minutes. Considering a case of complete absorption, the energy flux of incident light is

(A) 25.92×10^2 W/cm^2 (B) 8.64×10^{-6} W/cm^2

(C) 6.0 W/cm^2 (D) 0.06 W/cm^2

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $$\frac{I}{C}\times\text{area}=\text{force}$$

$$\frac{I}{C}\times36\times10^{-4}=7.2\times10^{-9}$$

$$I=\frac{7.2\times10^{-9}\times3\times10^8}{36\times10^{-9}\times10}$$

$$=\frac{6\times10^{-1}}{10^{-3}}$$

$$I=6\times10^2\frac{w}{m^2}$$

$$=0.06\frac{w}{cm^2}$$

16. The power of a lens (biconvex) is 1.25 m^{-1} in particular medium. Refractive index of the lens is 1.5 and radii of curvature are 20 cm and 40 cm respectively. The refractive index of surrounding medium :

(A) 1.0 (B) $\frac{9}{7}$

(C) $\frac{3}{2}$ (D) $\frac{4}{3}$

Official Ans. by NTA (B)

Allen Ans. (D)

Sol. $P=\frac{\mu_2}{f}=(\mu_1-\mu_2)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (For this formula refer to NCERT Part-2, Chapter-9, Page no. 328, solved example 8)

(μ_1 is refractive index of lens and μ_2 is of surrounding medium)

$$1.25=(1.5-\mu_2)\left(\frac{1}{0.2}+\frac{1}{0.4}\right)$$

$$\frac{1.25\times0.08}{0.6}=(1.5-\mu_2)$$

$$\Rightarrow\mu_2=\frac{4}{3}$$

17. Two streams of photons, possessing energies to five and ten times the work function of metal are incident on the metal surface successively. The ratio of the maximum velocities of the photoelectron emitted, in the two cases respectively, will be

(A) 1 : 2 (B) 1 : 3

(C) 2 : 3 (D) 3 : 2

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $\frac{1}{2}mv_1^2 = 4\phi$

$\frac{1}{2}mv_2^2 = 9\phi$

$\frac{v_1}{v_2} = \frac{2}{3}$

18. A radioactive sample decays $\frac{7}{4}$ times its original quantity in 15 minutes. The half-life of the sample is

(A) 5 min (B) 7.5 min

(C) 15 min (D) 30 min

Official Ans. by NTA (A)

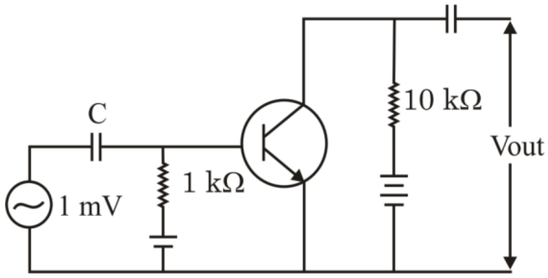
Allen Ans. (A)

Sol. Remaining $= \frac{1}{8}$

$3t_{1/2} = 15\,\text{min}$

$t_{1/2} = 5\,\text{min}$

19. An n.p.n transistor with current gain β = 100 in common emitter configuration is shown in figure. The output voltage of the amplifier will be



(A) 0.1 V (B) 1.0 V

(C) 10 V (D) 100 V

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\frac{V_{out}}{V_{in}} = \beta\frac{R_{out}}{R_{in}}$

$V_{out} = \frac{100 \times 10 \times 10^3}{10^3} \times 10^{-3}$

= 1 V

20. A FM Broad cast transmitter, using modulating signal of frequency 20 kHz has a deviation ratio of 10. The Bandwidth required for transmission is :

(A) 220 kHz

(B) 180 kHz

(C) 360 kHz

(D) 440 kHz

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. Given

FM broadcast

Modulating frequency = 20 k Hz = f

$\text{Deviation ratio} = \frac{\text{frequency deviation}}{\text{modulating frequency}} = \frac{\Delta f}{f}$

⇒ frequency deviation – $\Delta f = f \times 10$

= 20 kHz × 10 = 200 kHz

⇒ Bandwidth = $2(f + \Delta f)$

= 2 (20 + 200) kHz

= 440 kHz

SECTION-B

1. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)$m. The value of k is ….. (use $g = 9.8\ m/s^2$)

Official Ans. by NTA (392)

Allen Ans. (392)

Sol. $t_a = \frac{u}{g} = \frac{19.6}{9.8} = 2s$

$$t_d = 6 - 2s = \sqrt{\frac{2h_{max}}{g}}$$

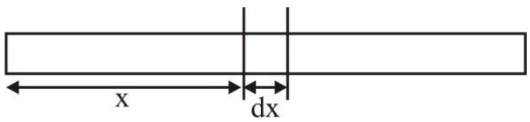
$$\Rightarrow h_{max} = \frac{16 \times 9.8}{2} = \frac{392}{5}$$

2. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1 - \frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$m. The value of α is ………. (where x is the distance form end A)

Official Ans. by NTA (8)

Allen Ans. (8)

Sol.



$$dm = \lambda \cdot dx = \lambda_0\left(1 - \frac{x^2}{\ell^2}\right)$$

$$X_{cm} = \frac{\int x dm}{\int dm_\ell}$$

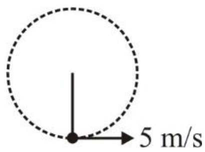
$$= \frac{\lambda_0 \int_0^\ell \left(x - \frac{x^3}{\ell^2}\right) dx}{\int_0^\ell \lambda_0 \left(1 - \frac{x^2}{\ell^2}\right) dx} = \frac{\frac{\ell^2}{2} - \frac{\ell^4}{4\ell^2}}{\ell - \frac{\ell^3}{3\ell^2}} = \frac{3\ell}{8}$$

3. A string of area of cross-section 4 mm^2 and length 0.5 is connected with a rigid body of mass 2 kg. The body is rotated in a vertical circular path of radius 0.5 m. The body acquires a speed of 5 m/s at the bottom of the circular path. Strain produced in the string when the body is at the bottom of the circle is …… × 10^{-5}. (Use Young's modulus $10^{11}\ N/m^2$ and $g = 10\ m/s^2$)

Official Ans. by NTA (30)

Allen Ans. (30)

Sol.



Strain = F/AY

$$= \frac{mg + \frac{mv^2}{R}}{AY}$$

$$= \frac{20 + \frac{2(5)^2}{0.5}}{3 \times 10^{-6} \times 10^{11}} = 30 \times 10^{-5}$$

4. At a certain temperature, the degrees of freedom per molecule for gas is 8. The gas performs 150 J of work when it expands under constant pressure. The amount of heat absorbed by the gas will be …….. J.

Official Ans. by NTA (750)

Allen Ans. (750)

Sol. $W = nR\Delta T = 150$ J

$$\mathbf{Q} = \left(\frac{f}{2} + 1\right) nR\Delta T = \left(\frac{8}{2} + 1\right) 150 = 750\,J$$

5. The potential energy of a particle of mass 4 kg in motion along the x-axis is given by U = 4(1 – cos 4x) J. The time period of the particle for small oscillation $(\sin\theta \simeq \theta)$ is $\left(\frac{\pi}{K}\right)$s. The value of K is ……..

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. U = 4(1 – cos 4x)

$$F = -\frac{dU}{dx} = -4(+\sin 4x)4 = -16\sin(4x)$$

For small θ

$\sin\theta \approx \theta$

F = –64x

a = –64x/m = –16x

$\omega^2 = 16$

$$T = \frac{2\pi}{\omega} = \frac{\pi}{2}$$

6. An electrical bulb rated 220 V, 100 W, is connected in series with another bulb rated 220 V, 60 W. If the voltage across combination is 220 V, the power consumed by the 100 W bulb will be about ………. W.

Official Ans. by NTA (14)

Allen Ans. (14)

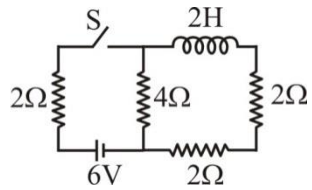
Sol. $R_1 = \frac{V^2}{P} = \frac{220^2}{100} = 484$

$$R_2 = \frac{V^2}{P} = \frac{220^2}{60} = 484\left(\frac{10}{6}\right)$$

$$I = \frac{220}{484 + 484 \times \frac{10}{6}}$$

$P_1 = I^2R_1 = 14.06\,W$

7. For the given circuit the current through battery of 6 V just after closing the switch 'S' will be ………. A.



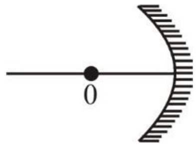
Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Just after closing the switch S, inductor behaves like an open circuit.

$$I = \frac{6}{2+4} = 1A$$

8. An object 'o' is placed at a distance of 100 cm in front of a concave mirror of radius of curvature 200 cm as shown in the figure. The object starts moving towards the mirror at a speed 2 cm/s. The position of the image from the mirror after 10s will be at ……. cm.



Official Ans. by NTA (400)

Allen Ans. (400)

Sol. After 10 sec.

u = –80 cm

f = –100 cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

v = 400 cm

9. In an experiment with a convex lens. The plot of the image distance (v') against the object distance (μ') measured from the focus gives a curve v' μ' = 225. If all the distances are measured in cm. The magnitude of the focal length of the lens is ……….. cm.

Official Ans. by NTA (15)

Allen Ans. (15)

Sol. $vu = f^2$ (by Newton's formula)

$f^2 = 225$

$f = 15$ cm

10. In an experiment to find acceleration due to gravity (g) using simple pendulum, time period of 0.5 s is measured from time of 100 oscillation with a watch of 1s resolution. If measured value of length is 10 cm known to 1mm accuracy. The accuracy in the determination of g is found to be x %. The value of x is

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$g = \frac{1}{4\pi^2}\frac{T^2}{\ell}$$

$$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta \ell}{\ell}$$

$$\frac{\Delta g}{g} = 2\cdot\frac{1}{100\times 0.5} + \frac{1mm}{10cm}$$

$$\frac{\Delta g}{g} = \frac{5}{100}$$