

FINAL JEE-MAIN EXAMINATION – JULY, 2022

 (Held On Friday 29th July, 2022)

TIME : 3 : 00 PM to 06 : 00 PM

MATHEMATICS
TEST PAPER WITH SOLUTION
SECTION-A

1. If $z \neq 0$ be a complex number such that $|z - \frac{1}{z}| = 2$,

 then the maximum value of $|z|$ is:

- (A) $\sqrt{2}$ (B) 1
 (C) $\sqrt{2} - 1$ (D) $\sqrt{2} + 1$

Official Ans. by NTA (D)
Allen Ans. (D)

Sol. $|z - 1/z| = 2$

$$\left| |z| - \frac{1}{|z|} \right| \leq \left| z - \frac{1}{z} \right| \leq |z| + \frac{1}{|z|} \quad \text{Let } |z| = r$$

$$\left| r - \frac{1}{r} \right| \leq 2 \leq r + \frac{1}{r}$$

$$\left| r - \frac{1}{r} \right| \leq 2 \text{ \& } r + \frac{1}{r} \geq 2 \text{ always true}$$

$$r - \frac{1}{r} \geq -2 \text{ \& } r - \frac{1}{r} \leq 2$$

$$r^2 - 1 \leq 2r$$

$$r^2 - 2r \leq 1$$

$$(r - 1)^2 \leq 2$$

$$r - 1 \leq \sqrt{2}$$

$$\therefore |z|_{\max} = 1 + \sqrt{2}$$

2. Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

(A) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Official Ans. by NTA (C)
Allen Ans. (C)

Sol. $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

(1) $R_1 \rightarrow R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \rightarrow R_2 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

3. If the system of equations

$$x + y + z = 6$$

$$2x + 5y + \alpha z = \beta$$

$$x + 2y + 3z = 14$$

has infinitely many solutions, then $\alpha + \beta$ is equal to :

- (A) 8 (B) 36
 (C) 44 (D) 48

Official Ans. by NTA (C)
Allen Ans. (C)

Sol. $x + y + z = 6$ ____ (1)

$$2x + 5y + \alpha z = \beta$$
 ____ (2)

$$x + 2y + 3z = 14$$
 ____ (3)

$$x + y = 6 - z$$

$$x + 2y = 14 - 3z$$

On solving

$$x = z - 2 \Rightarrow y = 8 - 2z \text{ in (2)}$$

$$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$$

$$(\alpha - 8)z = \beta - 36 \text{ For having infinite solutions}$$

$$\alpha - 8 = 0 \quad \& \quad \beta - 36 = 0$$

$$\alpha = 8, \beta = 36 \quad (\alpha + \beta = 44)$$

4. Let the function

$$f(x) = \begin{cases} \frac{\log_e(1+5x) - \log_e(1+\alpha x)}{x} & ; \text{if } x \neq 0 \\ 10 & ; \text{if } x = 0 \end{cases}$$

be continuous at $x = 0$.

The α is equal to :

- (A) 10 (B) -10
(C) 5 (D) -5

Official Ans. by NTA (D)

Allen Ans. (D)

Sol.
$$f(x) = \begin{cases} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x} & ; x \neq 0 \\ 10 & ; x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x} = 10$$

Using expansion

$$\lim_{x \rightarrow 0} \frac{(5x + \dots) - (\alpha x + \dots)}{x} = 10$$

$$5 - \alpha = 10 \Rightarrow \alpha = -5$$

5. If $[t]$ denotes the greatest integer $\leq t$, then the value of $\int_0^1 [2x - |3x^2 - 5x + 2| + 1] dx$ is:

- (A) $\frac{\sqrt{37} + \sqrt{13} - 4}{6}$ (B) $\frac{\sqrt{37} - \sqrt{13} - 4}{6}$
(C) $\frac{-\sqrt{37} - \sqrt{13} + 4}{6}$ (D) $\frac{-\sqrt{37} + \sqrt{13} + 4}{6}$

Official Ans. by NTA (A)

Allen Ans. (A)

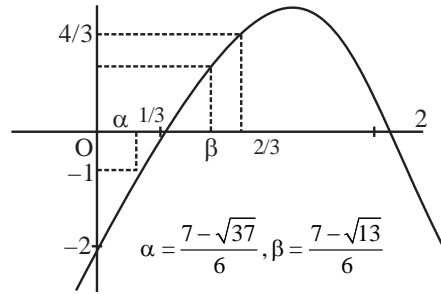
Sol.
$$I = \int_0^1 [2x - |3x^2 - 5x + 2| + 1] dx$$

$$I = \int_0^1 [2x - (3x - 2)(x - 1)] dx + \int_1^2 1 dx$$

$$I = \int_0^{2/3} [(2x - (3x^2 - 5x + 2))] dx + \int_{2/3}^1 (2x + (3x^2 - 5x + 2)) dx + 1$$

$$I = \int_0^{2/3} [-3x^2 + 7x - 2] dx + \int_{2/3}^1 (3x^2 - 3x + 2) dx + 1$$

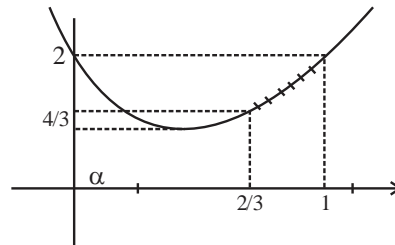
$$y = -3x^2 + 7x - 2$$



$$\int_0^\alpha (-2) dx + \int_\alpha^{1/3} (-1) dx + \int_{1/3}^\beta 0 dx + \int_\beta^2 1 dx$$

$$= -2\alpha - \left(\frac{1}{3} - \alpha\right) + \frac{2}{3} - \beta = -\alpha - \beta + \frac{1}{3}$$

$$y = 3x^2 - 3x + 2$$



When $x \in \left(\frac{2}{3}, 1\right)$

$$3x^2 - 3x + 2 \in \left(\frac{4}{3}, 2\right)$$

$$[3x^2 - 3x + 2] = 1$$

$$\therefore \int_{2/3}^1 [3x^2 - 3x + 2] dx = 1 \left(1 - \frac{2}{3}\right) = \frac{1}{3}$$

Hence $I = \left(\frac{1}{3} - (\alpha + \beta)\right) + \left(\frac{1}{3}\right) + 1$

$$= \frac{5}{3} - \left(\frac{7 - \sqrt{37}}{6} + \frac{7 - \sqrt{13}}{6}\right)$$

$$= \frac{-2}{3} + \frac{\sqrt{37} + \sqrt{13}}{6}$$

$$= \frac{\sqrt{37} + \sqrt{13} - 4}{6}$$

6. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0$.
Then $a_{25} a_{23} - 2 a_{25} a_{22} - 2 a_{23} a_{24} + 4 a_{22} a_{24}$ is equal to:
- (A) 483 (B) 528 (C) 575 (D) 624

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $a_0 = 0, a_1 = 0$

$$a_{n+2} = 3 a_{n+1} - 2 a_n : n \geq 0$$

$$a_{n+2} - a_{n+1} = 2 (a_{n+1} - a_n) + 1$$

$$n = 0 \quad a_2 - a_1 = 2 (a_1 - a_0) + 1$$

$$n = 1 \quad a_3 - a_2 = 2 (a_2 - a_1) + 1$$

$$n = 2 \quad a_4 - a_3 = 2 (a_3 - a_2) + 1$$

$$n = n \quad a_{n+2} - a_{n+1} = 2 (a_{n+1} - a_n) + 1$$

$$(a_{n+2} - a_1) - 2 (a_{n+1} - a_0) - (n + 1) = 0$$

$$a_{n+2} = 2a_{n+1} + (n + 1)$$

$$n \rightarrow n - 2$$

$$a_n - 2a_{n-1} = n - 1$$

$$\text{Now } a_{25} a_{23} - 2a_{25} a_{22} - 2a_{23} a_{24} + 4a_{22} a_{24}$$

$$= (a_{25} - 2a_{24}) (a_{23} - 2a_{22}) = (24) (22) = 528$$

7. $\sum_{r=1}^{20} (r^2 + 1)(r!)$ is equal to:

(A) $22! - 21!$ (B) $22! - 2(21!)$

(C) $21! - 2(20!)$ (D) $21! - 20!$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\sum_{x=1}^{20} (r^2 + 1)r!$

$$\sum_{x=1}^{20} ((r+1)^2 - 2r)r!$$

$$\sum_{x=1}^{20} ((r+1)(r+1)! - r.r!) - \sum_{r=1}^{20} r.r!$$

$$\sum_{x=1}^{20} ((r+1)(r+1)! - r.r!) - \sum_{r=1}^{20} ((r+1)! - r!)$$

$$= (21 \cdot 21!) - (21! - 1)$$

$$= 20 \cdot 21! = 22! - 2 \cdot 21!$$

8. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then

(A) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(B) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

(C) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(D) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $I(x) = \int \sec^2 x \cdot \sin^{-2022} x dx - 2022 \int \sin^{-2022} x dx$

$$= \tan x \cdot (\sin x)^{-2022} + \int (2022) \tan x \cdot (\sin x)^{-2023} \cos x dx$$

$$- 2022 \int (\sin x)^{-2022} dx$$

$$I(x) = (\tan x) (\sin x)^{-2022} + C$$

$$\text{At } X = \pi/4, 2^{1011} = \left(\frac{1}{\sqrt{2}}\right)^{-2022} + C \therefore C = 0$$

$$\text{Hence } I(x) = \frac{\tan x}{(\sin x)^{2022}}$$

$$I(\pi/6) = \frac{1}{\sqrt{3} \left(\frac{1}{2}\right)^{2022}} = \frac{2^{2022}}{\sqrt{3}}$$

$$I(\pi/3) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}} = \frac{2^{2022}}{(\sqrt{3})^{2021}} = \frac{1}{3^{1010}} I\left(\frac{\pi}{6}\right)$$

$$3^{1010} I(\pi/3) = I(\pi/6)$$

9. If the solution curve of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passes through the point (2,1) and

(k + 1, 2), k > 0, then

(A) $2 \tan^{-1}\left(\frac{1}{k}\right) = \log_e (k^2 + 1)$

(B) $\tan^{-1}\left(\frac{1}{k}\right) = \log_e (k^2 + 1)$

(C) $2 \tan^{-1}\left(\frac{1}{k+1}\right) = \log_e (k^2 + 2k + 2)$

(D) $2 \tan^{-1}\left(\frac{1}{k}\right) = \log_e \left(\frac{k^2 + 1}{k^2}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y} = \frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

$$x-1 = X, y-1 = Y$$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$Y = VX \quad \frac{dY}{dX} = V + X \frac{dV}{dX}$$

$$V + X \frac{dV}{dX} = \frac{1+V}{1-V} \quad X \frac{dV}{dX} = \frac{V^2+1}{1-V}$$

$$\int \frac{1-V}{1+V^2} dV = \int \frac{dX}{X}$$

$$\int \frac{dV}{1+V^2} - \frac{1}{2} \int \frac{2VdV}{1+V^2} = \int \frac{dX}{X}$$

$$\tan^{-1} V - \frac{1}{2} \ln(1+V^2) = \ln X + c$$

$$\tan^{-1} \left(\frac{Y}{X} \right) - \frac{1}{2} \ln \left(1 + \frac{Y^2}{X^2} \right) = \ln(X) + c$$

$$\tan^{-1} \left(\frac{y-1}{x-1} \right) - \frac{1}{2} \ln \left(1 + \frac{(y-1)^2}{(x-1)^2} \right) = \ln(x-1) + c$$

Passes through (2,1)

$$0 - \frac{1}{2} \ln 1 = \ln 1 + c \therefore c = 0$$

Passes through (k+1, 2)

$$\therefore \tan^{-1} \left(\frac{1}{k} \right) - \frac{1}{2} \ln \left(1 + \frac{1}{k^2} \right) = \ln k$$

$$2 \tan^{-1} \left(\frac{1}{k} \right) = \ln \left(\frac{1+k^2}{k^2} \right) + 2 \ln k$$

$$2 \tan^{-1} \left(\frac{1}{k} \right) = \ln(1+k^2)$$

- 10.** Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6} \right)$

$$y = \frac{(x+3)}{x+1}, x > -1, \text{ which passes through the point}$$

(0,1). Then $y(1)$ is equal to:

- (A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6} \right) y = \frac{x+3}{x+1}$

$$\int p(x) dx \quad \text{I.F.} = e^{\int p(x) dx}$$

$$\int p(x) dx = \int \frac{(2x^2+11x+13) dx}{(x+1)(x+2)(x+3)}$$

Using partial fraction

$$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$$A = \frac{4}{2} = 2$$

$$B = 1$$

$$C = -1$$

$$\therefore \int p(x) dx = A \ln(x+1) + B \ln(x+2) + C \ln(x+3)$$

$$= \ln \left(\frac{(x+1)^2(x+2)}{x+3} \right)$$

$$\text{I.F.} = e^{\int p(x) dx} = \frac{(x+1)^2(x+2)}{(x+3)}$$

$$\text{Solution } y(\text{IF}) = \int Q(\text{IF}) dx$$

$$y \left(\frac{(x+1)^2(x+2)}{x+3} \right) = \int \left(\frac{x+3}{x+1} \right) \frac{(x+1)^2(x+2)}{(x+3)} dx$$

$$y \left(\frac{(x+1)^2(x+2)}{x+3} \right) = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$$

$$\text{Passes through } (0, 1) \quad C = \frac{2}{3}$$

Now put $x = 1$

$$\Rightarrow y(1) = \frac{3}{2}$$

- 11.** Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2 + 11a + 3(m_1^2 + m_2^2) = 220$. If one vertex of the square is $(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$, then $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$ is equal to:

- (A) 119 (B) 128

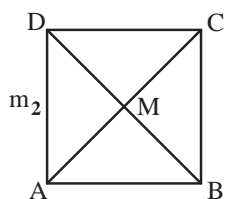
- (C) 145 (D) 155

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $m_1 m_2 = -1$

$$a^2 + 11a + 3 \left(m_1^2 + \frac{1}{m_1^2} \right) = 220$$



Eq. of AC

$$AC = (\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$$

$$BD = (\sin\alpha - \cos\alpha)x + (\sin\alpha - \cos\alpha)y = 0$$

$$(10(\cos\alpha - \sin\alpha), 10(\sin\alpha - \cos\alpha))$$

$$\text{Slope of AC} = \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} \right) = \tan\theta = M$$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m \pm \tan \frac{\pi}{4}}{1 \pm m \tan \frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} + 1}{1 - \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} \right)}, \frac{\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha} - 1}{1 + \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}}$$

$$m_1, m_2 = \tan\alpha, \cot\alpha$$

mid point of AC & BD

$$= M(5(\cos\alpha - \sin\alpha), 5(\cos\alpha + \sin\alpha))$$

$$B(10(\cos\alpha - \sin\alpha), 10(\cos\alpha + \sin\alpha))$$

$$a = AB = \sqrt{2} BM = \sqrt{2}(5\sqrt{2}) = 10$$

$$a = 10$$

$$\therefore a^2 + 11a + 3 \left(m_1^2 + \frac{1}{m_1^2} \right) = 220$$

$$100 + 110 + 3(\tan^2\alpha + \cot^2\alpha) = 220$$

$$\text{Hence } \tan^2\alpha = 3, \tan\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3} \text{ or } \frac{\pi}{6}$$

$$\text{Now } 72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$$

$$= 72 \left(\frac{9}{16} + \frac{1}{16} \right) + 100 - 30 + 13$$

$$= 72 \left(\frac{5}{8} \right) + 83 = 45 + 83 = 128$$

12. The number of elements in the set

$$S = \left\{ x \in \mathbb{R} : 2 \cos \left(\frac{x^2 + x}{6} \right) = 4^x + 4^{-x} \right\} \text{ is:}$$

- (A) 1 (B) 3
(C) 0 (D) infinite

Official Ans. by NTA (A)

Allen Ans. (A)

Sol. $2 \cos \left(\frac{x^2 + x}{6} \right) = 4^x + 4^{-x}$

$$\text{L.H.S} \leq 2. \text{ \& R.H.S.} \geq 2$$

$$\text{Hence L.H.S} = 2 \text{ \& R.H.S} = 2$$

$$2 \cos \left(\frac{x^2 + x}{6} \right) = 2 \quad 4^x + 4^{-x} = 2$$

Check $x = 0$ Possible hence only one solution.

13. Let A $(\alpha, -2)$, B $(\alpha, 6)$ and C $\left(\frac{\alpha}{4}, -2 \right)$ be vertices

of a ΔABC . If $\left(5, \frac{\alpha}{4} \right)$ is the circumcentre of

ΔABC , then which of the following is NOT correct about ΔABC :

- (A) area is 24 (B) perimeter is 25
(C) circumradius is 5 (D) inradius is 2

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. A $(\alpha, -2) : B(\alpha, 6) : C\left(\frac{\alpha}{4}, -2\right)$

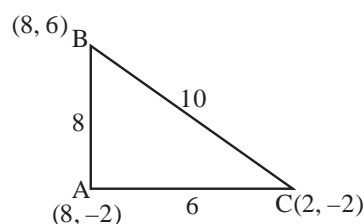
since AC is perpendicular to AB.

So, ΔABC is right angled at A.

$$\text{Circumcentre} = \text{mid point of BC} = \left(\frac{5\alpha}{8}, 2 \right)$$

$$\therefore \frac{5\alpha}{8} = 5 \text{ \& } \frac{\alpha}{4} = 2$$

$$\alpha = 8$$



$$\text{Area} = \frac{1}{2}(6)(8) = 24$$

$$\text{Perimeter} = 24$$

$$\text{Circumradius} = 5$$

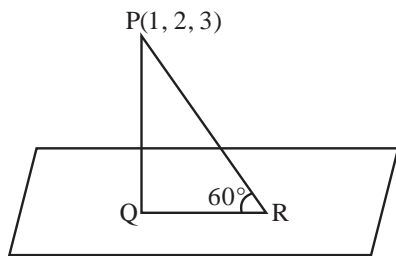
$$\text{Inradius} = \frac{\Delta}{s} = \frac{24}{12} = 2$$

14. Let Q be the foot of perpendicular drawn from the point P (1, 2, 3) to the plane $x + 2y + z = 14$. If R is a point on the plane such that $\angle PRQ = 60^\circ$, then the area of ΔPQR is equal to:

- (A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{3}$
(C) $2\sqrt{3}$ (D) 3

Official Ans. by NTA (B)

Allen Ans. (B)



Sol. $x + 2y + z = 14$

Length of perpendicular

$$PQ = \frac{|1 + 4 + 3 - 14|}{\sqrt{6}} = \sqrt{6}$$

$$QR = (PQ) \cot 60^\circ = \sqrt{2}$$

$$\therefore \text{Area of } \Delta PQR = \frac{1}{2}(PQ)(QR) = \sqrt{3}$$

15. If (2, 3, 9), (5, 2, 1), (1, λ , 8) and (λ , 2, 3) are coplanar, then the product of all possible values of λ is:

- (A) $\frac{21}{2}$ (B) $\frac{59}{8}$
(C) $\frac{57}{8}$ (D) $\frac{95}{8}$

Official Ans. by NTA (D)

Allen Ans. (D)

Sol. $A(2, 3, 9); B(5, 2, 1); C(1, \lambda, 8); D(\lambda, 2, 3)$

$$[\overline{AB} \ \overline{AC} \ \overline{AD}] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda - 3 & -1 \\ \lambda - 2 & -1 & -6 \end{vmatrix} = 0$$

$$\Rightarrow [-6(\lambda - 3) - 1] - 8(1 - (\lambda - 3)(\lambda - 2)) + (6 + (\lambda - 2)) = 0$$

$$3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$$

$$8\lambda^2 - 57\lambda + 95 = 0$$

$$\lambda_1 \lambda_2 = \frac{95}{8}$$

16. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:

- (A) $\frac{4}{9}$ (B) $\frac{5}{18}$ (C) $\frac{1}{6}$ (D) $\frac{3}{10}$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol.

3R	2R
4B	5B
3W	2W

A : Drawn ball from bag II is black

B : Red ball transferred

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{9} \times \frac{5}{10}}{\frac{3}{9} \times \frac{5}{10} + \frac{4}{9} \times \frac{6}{10} + \frac{3}{9} \times \frac{5}{10}}$$

$$= \frac{15}{15 + 24 + 15} = \frac{15}{54} = \frac{5}{18}$$

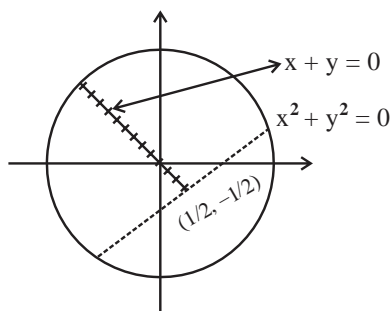
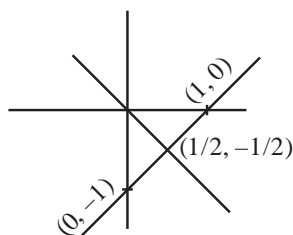
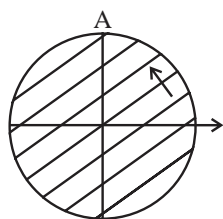
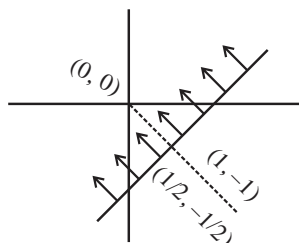
17. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x , for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

- (A) $\left[-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (B) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$
 (C) $\left[-\sqrt{2}, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $|z - 1 + i| \geq |z|$; $|z| < 2$; $|z + i| = |z - 1|$



Hence

$$w = 2x + iy \in S$$

$$2x \leq \frac{1}{2} \therefore x \leq \frac{1}{4}$$

Now

$$(2x)^2 + (2x)^2 < 4$$

$$x^2 < \frac{1}{2} \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\therefore x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{4}\right]$$

18. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = 168$ then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to:

- (A) 10 (B) 14
 (C) 16 (D) 18

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $|\vec{a}| |\vec{b}| |\vec{c}| = 14$

$$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \frac{2\pi}{3}$$

$$\text{So, } \vec{a} \cdot \vec{b} = -\frac{1}{2}ab, \vec{b} \cdot \vec{c} = -\frac{1}{2}bc, \vec{a} \cdot \vec{c} = -\frac{1}{2}ac$$

(let)

$$\begin{aligned} (\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) &= (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{b}) \\ &= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c \end{aligned}$$

Similarly

$$(\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) = \frac{3}{4}abc^2$$

$$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \frac{3}{4}a^2bc$$

$$168 = \frac{3}{4}abc(a + b + c)$$

$$\text{So, } (a + b + c) = 16$$

19. The domain of the function

$$f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right) \text{ is :}$$

- (A) $[1, \infty)$ (B) $(-1, 2]$
 (C) $[-1, \infty)$ (D) $(-\infty, 2]$

Official Ans. by NTA (C)

Allen Ans. (C)

Sol. $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ Domain

$$\frac{x^2 - 3x + 2}{x^2 + 2x + 7} \geq -1 \text{ and } \frac{x^2 - 3x + 2}{x^2 + 2x + 7} \leq 1$$

$$2x^2 - x + 9 \geq 0 \text{ and } 5x \geq -5 \Rightarrow x \geq -1$$

$$x \in \mathbb{R}$$

$$\text{Hence Domain } x \in [-1, \infty)$$

20. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to:

- (A) $(p \wedge (\sim r)) \Rightarrow q$ (B) $(\sim q) \Rightarrow ((\sim r) \vee p)$
 (C) $p \Rightarrow (q \vee r)$ (D) $(p \wedge (\sim q)) \Rightarrow r$

Official Ans. by NTA (B)

Allen Ans. (B)

Sol. $(p \rightarrow q) \vee (p \rightarrow r)$

$$(\sim p \vee q) \vee (\sim p \vee r)$$

$$= \sim p \vee (q \vee r)$$

$$= p \rightarrow (q \vee r) \equiv (3) \text{ is true.}$$

Now (1) $(p \wedge \sim r) \rightarrow q$

$$\sim(p \wedge \sim r) \vee q = (\sim p \vee r) \vee q$$

$$= \sim p \vee (r \vee q) = p \rightarrow (q \vee r)$$

$$(4) (p \wedge \sim q) \rightarrow r = p \rightarrow (q \vee r)$$

SECTION-B

1. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. They the number of trials in the binomial distribution is:

Official Ans. by NTA (96)

Allen Ans. (96)

Sol. Let, mean = $m = np$

$$\& \text{ variance} = v = npq, p + q = 1$$

$$\text{Sum} = m + v = \frac{165}{2}$$

$$\text{Product} = mv = 1350$$

On solving,

$$m = np = 60 \& v = npq = \frac{45}{2} \therefore q = \frac{3}{8} \therefore P = \frac{5}{8}$$

Hence $n = 96$

2. Let α, β ($\alpha > \beta$) be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$, then

$$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$
 is equal to _____.

Official Ans. by NTA (16)

Allen Ans. (16)

Sol. $P_n = \alpha^n - \beta^n \quad x^2 - x - 4 = 0$

$$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}} \text{ ---(1)}$$

$$\text{As } P_n - P_{n-1} = (\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})$$

$$= \alpha^{n-2}(\alpha^2 - \alpha) - \beta^{n-2}(\beta^2 - \beta)$$

$$= 4(\alpha^{n-2} - \beta^{n-2})$$

$$P_n - P_{n-1} = 4 P_{n-2}$$

Hence Expression (1)

$$\frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$$

$$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}} = 16$$

3. Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if

$X^T A^k X = 33$, then k is equal to:

Official Ans. by NTA (10)

Allen Ans. (Dropped or 10)

Sol. $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$$X^T A^k X = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\text{As } A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{for } K \rightarrow \text{Even } A^K = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X^T A^K X = 33 \text{ (This is not correct)}$$

$$[1 \ 1 \ 1] \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= [1 \ 1 \ 3K+1] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3K+3]$$

$$\therefore 3K + 3 = 33 \therefore K = 10$$

But it should be dropped as 33 is not matrix

If K is odd

$$X^T A^K X = 33$$

$$X^T A A^{K-1} X = 33$$

$$[1 \ 1 \ 1] \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3k-3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$[-1 \ 3 \ 8] \begin{bmatrix} 3k-2 \\ 1 \\ 1 \end{bmatrix} = [33]$$

$$[-3k + 13] = [33]$$

$$k = 20/3 \text{ (not possible)}$$

4. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is _____,

Official Ans. by NTA (6)

Allen Ans. (6)

Sol. 4 digit numbers

For divisibility by 55, no. should be

div. by 5 and 11 both

			5
a	b	c	d

Also, for divisibility by 11

$$a + c = b + 5$$

$$\text{for } b = 1 \quad a = 2, \quad c = 4$$

$$a = 4, \quad c = 2$$

$$\text{for } b = 2 \quad a = 3, \quad c = 4$$

$$a = 4, \quad c = 3$$

$$\text{for } b = 3 \quad a = 6, \quad c = 2$$

$$a = 2, \quad c = 6$$

\therefore 6 possible four digit no.s are div. by 55

(II) 5 digit number is not possible

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(Not possible)

5. If $\sum_{k=1}^{10} K^2 \binom{10}{C_k} = 22000L$, then L is equal to ____.

Official Ans. by NTA (221)

Allen Ans. (221)

Sol. $\sum_{k=1}^{10} K^2 \binom{10}{C_k}^2$

$$\sum_{k=1}^{10} (K \cdot \binom{10}{C_k})^2 = \sum_{k=1}^{10} (10 \cdot \binom{9}{C_{k-1}})^2$$

$$= 100 \sum_{k=1}^{10} \binom{9}{C_{k-1}} \cdot \binom{9}{C_{10-k}}$$

$$= 100 \binom{18}{C_9} = 100 \left(\frac{18!}{9!9!} \right)$$

$$\Rightarrow 4862000 = 22000L$$

$$\text{Hence } L = 221$$

6. If $[t]$ denotes the greatest integer $\leq t$, then number of points, at which the function $f(x) = 4|2x + 3| + 9 \left[x + \frac{1}{2} \right] - 12[x + 20]$ is not differentiable in the open interval $(-20, 20)$, is ____.

Official Ans. by NTA (79)

Allen Ans. (79)

Sol. $f(x) = 4|2x + 3| + 9 \left[x + \frac{1}{2} \right] - 12[x + 20]$

$$x \in (-20, 20)$$

$$f(x) \text{ is not Diff. at } x = I \in \{-19, -18, \dots, 0, \dots, 19\} = 39$$

$$\text{at } x = I + \frac{1}{2}, \quad f(x) \text{ Non diff. at 39 points}$$

$$\text{Check at } x = \frac{-3}{2} \text{ Discount at } x = \frac{-3}{2} \therefore \text{N. R(1)}$$

$$\text{No. of point of non-differentiability}$$

$$= 39 + 39 + 1 = 79$$

7. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to _____.

Official Ans. by NTA (195)

Allen Ans. (195)

Sol. $y = 5x^2 + 2x - 25$ $P(2, -1)$

$$y' = 10x + 2$$

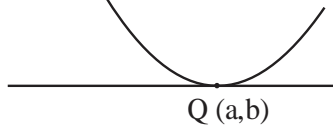
$$y'_P = 22$$

\therefore tangent to curve at P

$$y + 1 = 22(x - 2)$$

$$y = 22x - 45$$

$$y = x^3 - x^2 + x$$



$$\left. \frac{dy}{dx} \right|_{C_2} = 3x^2 - 2x + 1$$

$$\left. \frac{dy}{dx} \right|_Q = 3a^2 - 2a + 1$$

$$\text{Hence } 3a^2 - 2a + 1 = 22$$

$$\therefore 3a^2 - 2a - 21 = 0$$

$$3a^2 - 9a + 7a - 21 = 0$$

$$(3a + 7)(a - 3) = 0 \begin{cases} a = 3 \\ a = -7/3 \end{cases}$$

from curve $b = a^3 - a^2 + a$

$$a = 3$$

$$b = 21 \quad |2a + 9b| = 195$$

at $a = -7/3$ tangent will be parallel

Hence it is rejected

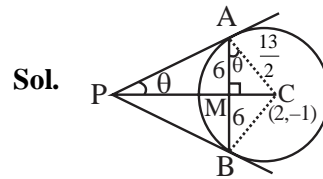
8. Let AB be a chord of length 12 of the circle

$$(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$$

If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ____.

Official Ans. by NTA (72)

Allen Ans. (72)



$$\cos \theta = \frac{6}{\frac{13}{2}} = \frac{12}{13}$$

$$\sin \theta = \frac{5}{13}$$

$$PM = AM \cot \theta$$

$$PM = 6 \left(\frac{12}{5} \right) \therefore 5(PM) = 72$$

9. Let \vec{a} and \vec{b} be two vectors such that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a} \cdot \vec{b} = 3$ and $|\vec{a} \times \vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to ____.

Official Ans. by NTA (14)

Allen Ans. (14)

Sol. $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$; $\vec{a} \cdot \vec{b} = 3$

$$\text{As } |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2$$

$$|\vec{b}|^2 = 2\vec{a} \cdot \vec{b} = 6$$

$$|\vec{a} \times \vec{b}|^2 = 75$$

$$|\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = 75$$

$$6|\vec{a}|^2 - 9 = 75 \Rightarrow |\vec{a}|^2 = 14$$

10. Let

$$S = \{(x, y) \in \mathbb{N} \times \mathbb{N} : 9(x-3)^2 + 16(y-4)^2 \leq 144\}$$

and $T = \{(x, y) \in \mathbb{R} \times \mathbb{R} : (x-7)^2 + (y-4)^2 \leq 36\}$.

The $n(S \cap T)$ is equal to_____.

Official Ans. by NTA (27)

Allen Ans. (27)

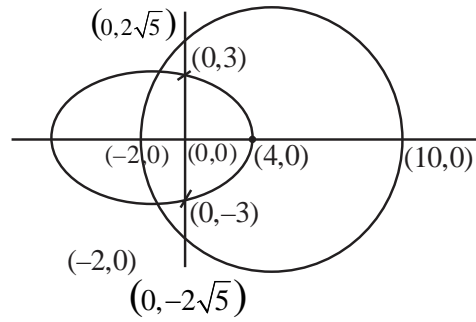
Sol. $S : \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \leq 1 ; x, y \in \{1, 2, 3, \dots\}$

$T : (x-7)^2 + (y-4)^2 \leq 36 ; x, y \in \mathbb{R}$

Let $x-3 = x ; y-4 = y$

$S : \frac{x^2}{16} + \frac{y^2}{9} \leq 1 ; x \in \{-2, -1, 0, 1, \dots\}$

$T : (x-4)^2 + y^2 \leq 36 ; y \in \{-3, -2, -1, 0, \dots\}$



$S \cap T = (-2, 0), (-1, 0), \dots, (4, 0) \rightarrow (7)$

$(-1, 1), (0, 1), \dots, (3, 1) \rightarrow (5)$

$(-1, -1), (0, -1), \dots, (3, -1) \rightarrow (5)$

$(-1, 2), (0, 2), (1, 2), (2, 2) \rightarrow (4)$

$(-1, -2), (0, -2), (1, -2), (2, -2) \rightarrow (4)$

$(0, 3) (0, -3) \rightarrow (2)$