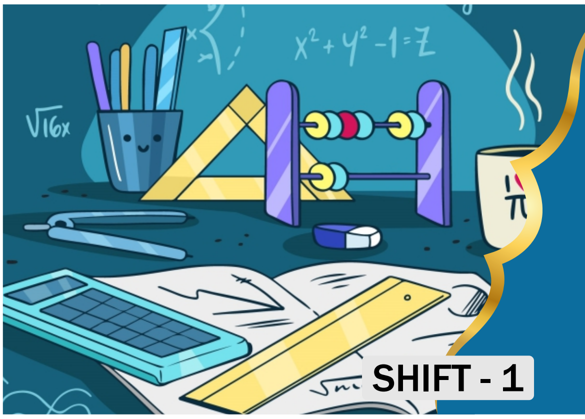
JEE MAIN 2023

JAN ATTEMPT

PAPER-1 (B.Tech / B.E.)



QUESTIONS & SOLUTIONS

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24 JANUARY, 2023

9:00 AM to 12:00 Noon

Duration : 3 Hours | Maximum Marks : 300

SUBJECT - MATHEMATICS

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MATHEMATICS

1. If $I = \int_0^{\pi/2} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}}\,dx$, then the value of I is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

Ans. (1)

Sol. $I = \int_0^{\pi/2} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}}\,dx$

$I = \int_0^{\pi/2} \frac{(\cos x)^{2023}}{(\cos x)^{2023} + (\sin x)^{2023}}$ .... (a + b – X property)

$2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{\pi}{4}$

2. If $I = \int_0^3 \left|x^2 - 3x + 2\right| dx$, then find the value of 12I

Ans. (22)

Sol. $I = \int_0^1 \left(x^2 - 3x + 2\right)dx - \int_1^2 \left(x^2 - 3x + 2\right)dx$

$+\int_2^3 \left(x^2 - 3x + 2\right)dx$

$I = \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_0^1 - \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_1^2$

$+\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_2^3$

$= \left(\frac{1}{3} - \frac{3}{2} + 2\right) - \left(\frac{8}{3} - 6 + 4 - \left(\frac{1}{3} - \frac{3}{2} + 2\right)\right) + 9 - \frac{27}{2} + 6 - \left(\frac{8}{3} - 6 + 4\right)$

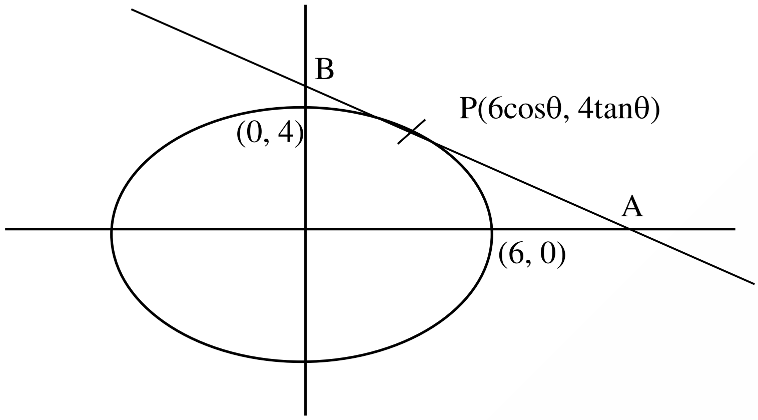
$= \frac{5}{6} - \left(\frac{2}{3} - \frac{5}{6}\right) + \frac{3}{2} - \frac{2}{3}$

$I = \frac{1}{3} + \frac{3}{2} = \frac{11}{6} \Rightarrow 12I = 22$

3. A tangent at P on the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ is drawn. If this tangent cuts x -axis & y-axis at the points A and B respectively then find minimum possible value of AB.

Ans. **(10)**

Sol.



Let P = (6 cosθ, 4 sinθ)

Equation of tangent will be

$$\frac{x\cos\theta}{6}+\frac{y\sin\theta}{4}=1$$

$$\therefore\ AB = \sqrt{\frac{36}{\cos^2\theta}+\frac{16}{\sin^2\theta}} = \sqrt{36\left(1+\tan^2\theta\right)+16\left(1+\cot^2\theta\right)}$$

Since $\frac{36\tan^2\theta+16\cot^2\theta}{2} \ge \sqrt{36\tan^2\theta\cdot 16\cot^2\theta}$

$36\tan^2\theta + 16\cot^2\theta \ge 2 \times 6 \times 4$

$AB_{min} = \sqrt{52+36\tan^2\theta+16\cot^2\theta} = 10$

4. If $\sum_{r=0}^{2023} r^2 . {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$, then the value of α is-

(1) 1011 (2) 1012 (3) 2022 (4) 2024

Ans. **(2)**

Sol. $\sum_{r=0}^{n} r^2 . {}^{n}C_r$

$\sum_{r=0}^{n} (r(r-1)+r)\,{}^{n}C_r$

$\sum_{r=0}^{n} \left\{n(n-1)\,{}^{n-2}C_{r-2} + n\,{}^{n-1}C_{r-1}\right\}$

$= n(n-1)2^{n-2} + n \times 2^{n-1}$

$= 2023\ [2022\ 2^{2021} + 2^{2022}]$

$= 2023 \times 2^{2022} \times 1012$

5. There are 12 subjects in a class, out of which 5 are language subjects. A student has to choose 5 subjects in which atmost 2 are language subjects. Find no. of ways to do so.

(1) 546 (2) 540 (3) 456 (4) 567

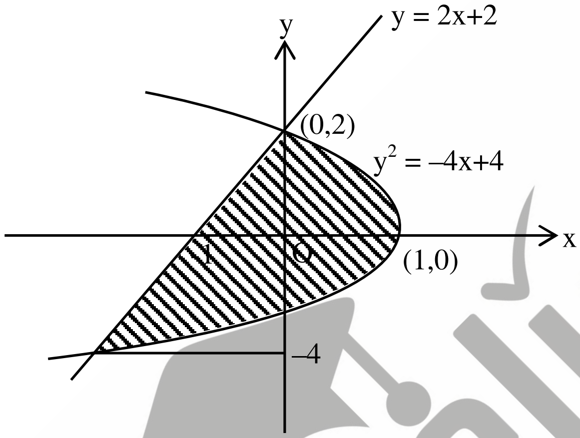
Ans. **(1)**

6. Find the area bounded by the curves $y^2 = -4x + 4$ and $y = 2x + 2$.

(1) 27 (2) 9 (3) $\frac{27}{4}$ (4) $\frac{9}{2}$

Ans. (2)

Sol. $A = \int_{-4}^{2}\left(\frac{4-y^2}{4}-\frac{2y-4}{4}\right)dy = \int_{-4}^{2}\frac{1}{4}\left(4-y^2-2y+4\right)dy = \frac{1}{4}\int_{-4}^{2}\left(-y^2-2y+8\right)dy$



$= \frac{1}{4}\left[-\frac{y^3}{3}-y^2+8y\right]_{-4}^{2} = \frac{1}{4}\left[\left(-\frac{8}{3}-4+16\right)-\left(\frac{64}{3}-16-32\right)\right] = \frac{1}{4}\times\left(-\frac{72}{3}+60\right) = 9$ sq. units

7. If $x^2 - 4x + 3 = x[x] - [x]$, where [.] represents the greatest integer function then :

(1) No. of solutions in $(-\infty, 1)$ are 1 (2) No. of solutions in $(-\infty,\infty)$ are 1

(3) No. of solutions in $(1,\infty)$ are 2 (4) No. of solutions in $(3,\infty)$ are infinite

Ans. (2)

Sol. $(x-1)(x-3) = (x-1)\,.\,[x]$

$x - 3 = [x]$ or $x = 1$

Case-I : $x \in I$ Case-II : $x \notin I$

$x - 3 = x$

No solution No solution

$\therefore$ only 1 solution in R.

8. If $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ c & a & b \\ 1 & 1 & 1 \end{bmatrix}$ is a singular matrix and α is a root of the equation $(a-b)x^2 + (b-c)x + (c-a) = 0$,

then the value of $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$ is

(1) 3 (2) 6 (3) 9 (4) 12

Ans. **(1)**

Sol. $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ c & a & b \\ 1 & 1 & 1 \end{bmatrix} = 0$

$\Rightarrow (a-b) + (b-c)\alpha + (c-a)\alpha^2 = 0$

Also, $(a-b)\alpha^2 + (b-c)\alpha + (c-a) = 0$

$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)} = \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

9. Two lines are given by $\frac{x-2}{3} = \frac{y-1}{3} = \frac{z-0}{2}$ and $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-1}{3}$ then shortest distance between lines is-

(1) $\frac{6}{\sqrt{43}}$ (2) $\frac{11}{\sqrt{43}}$ (3) $\frac{3}{\sqrt{43}}$ (4) $\frac{5}{\sqrt{43}}$

Ans. **(2)**

Sol. s.d. = $\left|\frac{(a_1 - a_2).(\overline{p} \times \overline{q})}{(\overline{p} \times \overline{q})}\right|$

$= \left|\frac{(\hat{i} - \hat{j} - \hat{k}).(5\hat{i} - 3\hat{j} - 3\hat{k})}{|5\hat{i} - 3\hat{j} - 3\hat{k}|}\right|$

$= \frac{11}{\sqrt{43}}$

10. Let $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & ; \quad x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$, then

(1) f is continuous and f' is discontinuous at x = 0

(2) f and f' both are continuous at x = 0

(3) f and f' both are discontinuous at x = 0

(4) f is discontinuous and f' is continuous at x = 0

Ans. **(1)**

Sol. $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\frac{1}{x} = 0 = f(0)$

$\therefore$ f(x) continuous at x = 0

$f'(x) = 2x\sin\frac{1}{x} + x^2\left(\cos\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)$

$f'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

$\lim_{x\to 0} f'(x)$ does not exists

$\therefore$ f'(x) is discontinuous at x = 0

11. $\lim_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + ... + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) n^2 (2) n (3) $\frac{n(n+1)}{2}$ (4) $n^2 + n$

Ans. (1)

Sol. $n\left[\left(\frac{1}{n}\right)^{\frac{1}{\sin^2 t}} + \left(\frac{2}{n}\right)^{\frac{1}{\sin^2 t}} + ... + \left(\frac{n-1}{n}\right)^{\frac{1}{\sin^2 t}} + 1\right]^{\sin^2 t} = n$

12. Find the minimum distance of the point (7, –4, –3) from the plane formed by the points (2, 2, –1), (3, 4, 2) and (7, 0, 6).

(1) $\frac{\sqrt{19}}{4}$ (2) $\sqrt{19}$ (3) $\frac{\sqrt{19}}{3}$ (4) $\sqrt{\frac{19}{2}}$

Ans. (4)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & -4 & 4 \end{vmatrix} = <5, 2, -3>$

Plane : 5x + 2y – 3z = 17

Distance = $\left|\frac{35-8+9-17}{\sqrt{25+4+9}}\right| = \frac{19}{\sqrt{38}} = \sqrt{\frac{19}{2}}$

13. If 'N' is decided by rolling a normal die and $\frac{'k'}{6}$ is the probability that the system of equations

x + y + z = 0

Nx + y + z = 2

3x + (N – 3)y + z = 6

has a unique solution, then find sum of all possible values of 'k' and 'n'

Ans. **(20)**

Sol. D ≠ 0 for unique solution,

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ N & 1 & 1 \\ 3 & N-3 & 1 \end{vmatrix} \neq 0$$

$\Rightarrow (N-1)(N-4) \neq 0$

$\Rightarrow N \neq 1, 4$

$\therefore$ N can be 2, 3, 5, 6

Also, required probability = $\frac{4}{6} \Rightarrow k = 4$

Hence, sum = (2 + 3 + 5 + 6) + 4

= 20

14. Numbers are formed using digits 1, 2, 3, 4, 1, 2, 3, 4 & 1 then the number of 9 digits numbers such that even digits occupy even places are-

Ans. **(60)**

Sol. 2, 2, 4, 4 occupy 2^{nd}, 4^{th}, 6^{th} and 8^{th} places

no. of numbers = $\frac{4!}{2!.2!}.\frac{5!}{3!.2!} = 60$

15. A circle with centre ≡ (2, 0) with largest possible radius is inscribed in ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

If circle passes through the point (1, α), then find value of $5\alpha^2$.

Ans. 59

Sol. P ≡ (6 cos θ, 4 sin θ)

N ; $\frac{36x}{6\cos\theta} - \frac{16y}{4\sin\theta} = 20$

Passes (2, 0)

$\frac{6}{\cos\theta} = 10 \Rightarrow \cos\theta = \frac{3}{5}$

$\Rightarrow P \equiv \left(\frac{18}{5}, \frac{16}{5}\right)$

$R = \sqrt{\frac{64}{25} + \frac{256}{25}} = \sqrt{\frac{320}{25}}$

S : $(x-2)^2 + y^2 = \frac{320}{25}$

Passes (1, α)

$\alpha^2 = \frac{64}{5} - 1 = \frac{59}{5}$

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