2022
MATHEMATICS

Full marks: 80 Time: 3 hours

General instructions:

i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
ii) The question paper consists of 18 questions. All questions are compulsory.
iii) Marks are indicated against each question.
iv) Internal and general choice has been provided in some questions.
v) Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check to ensure that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) Number of binary operation on the set $\{a,b\}$ are **1**
(i) 10 (ii) 16 (iii) 20 (iv) 8

(b) The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is **1**
(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{3}$ (iv) $\frac{\pi}{2}$

(c) If $y = e^{\cos x}$, then $\frac{dy}{dx}$ is equal to **1**
(i) $e^{\cos x}$ (ii) $e^{\sin x}$ (iii) $-\cos x e^{\sin x}$ (iv) $-\sin x e^{\cos x}$

(d) The rate of change of the area of a circle with respect to its radius at r = 6 cm is **1**
(i) 10π (ii) 12π (iii) 8π (iv) 11π

(e) $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ is equal to **1**
(i) $\frac{\pi}{3}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{\pi}{6}$ (iv) $\frac{\pi}{12}$

(f) If $\vec{a}$ is an unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$, then $|\vec{x}|$ is **1**
(i) 2 (ii) 3 (iii) 4 (iv) 5

(g) The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are **1**

(i) Perpendicular (ii) Parallel

(iii) Intersect y-axis (iv) Passes through $\left(0,0,\frac{5}{4}\right)$

(h) If P(A) = 0.8, P(B) = 0.5 and $P\left(\frac{B}{A}\right) = 0.4$, then $P(A \cap B)$ is **1**

(i) 0.32 (ii) 0.20 (iii) 0.40 (iv) None of these

Section – B

2. Prove: $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$ **2**

3. Find the value of k so that the given function
$f(x) = \begin{cases} kx + 1, \text{ if } x \le 5 \\ 3x - 5, \text{ if } x > 5 \end{cases}$ is continuous at $x = 5$ **2**

4. Find the points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the *x-axis*. **2**

5. Evaluate $\int \frac{(\log x)^3}{x}\, dx$ **2**

6. Form the differential equation representing the family of curves $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are arbitrary constants. **2**

7. Find a vector in the direction of the vector $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude 8 units. **2**

8. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other. **2**

9. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and P(B) = p. Find p if they are (i) mutually exclusive (ii) independent **2**

Section – C

10. **Case study based question:**
Students of class-XII, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L. **4**

Answer the following using the above information.

a). Let relation R be defined by R = $\{(L_1, L_2) : L_1 \text{ // } L_2 \text{ where } L_1, L_2 \in L\}$, then R is _____ relation.

(i) Equivalence (ii) Only reflexive
(iii) Not reflexive (iv) symmetric but not transitive

b). The function $f : R \to R$ defined by $f(x) = x - 4$ is

(i) bijective (ii) 1-1 but not onto
(iii) onto but not 1-1 (iv) neither 1-1 nor onto

c). Let $f : R \to R$ be defined by $f(x) = x - 4$, then the range of $f(x)$ is

(i) R (ii) Z
(iii) W (iv) Q

d). Let R = $\{(L_1, L_2) : L_1 \text{ // } L_2$ and $L_1 : y = x - 4\}$, then which of the following can be taken as L_2?

(i) $2x - 2y + 5 = 0$ (ii) $2x + y = 5$
(iii) $2x + 2y + 7 = 0$ (iv) $x + y = 7$

11. **a.** Express the following matrix as the sum of a symmetric and a skew symmetric matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Or **4**

b. Using properties of determinants, show that

$$\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix} = (a+b+c)^3$$

12. Answer **any three** (**3**) questions.

a. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the rate of 4 cm/min. When $x = 8$ cm and $y = 6$ cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle. **4**

b. Evaluate: $\int \frac{x}{(x-1)(x-2)(x-3)}\,dx$ **4**

c. Evaluate: $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$ **4**

d. Solve the differential equation: $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$ **4**

e. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ **4**

13. Answer **any two** (**2**) questions.

a. Find x such that the four points A(3, 2, 1) , B(4, x, 5), C(4, 2, -2) and D(6, 5, -1) are coplanar. **4**

b. Find the shortest distance between the lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$$

and $\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ **4**

c. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7$, $\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) = 9$ and through the point (2, 1, 3). **4**

14. **a.** An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers**.** The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Or **4**

b. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. find the probability that out of 5 such bulbs (i) none (ii) not more than one (iii) more than one
will fuse after 150 days of use.

Section – D

15. **a.** Solve the system of linear equations by using matrix method.

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

Or **6**

b. Using elementary transformation, find the inverse of the matrix

$$A = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

16. **a.** Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Or **6**

b. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

17. **a.** Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

Or **6**

b. Using the method of integration, find the area of region bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$.

18. **a.** A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is ₹5 each for type A and ₹6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

Or **6**

b. There are two types of fertilizer F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs ₹6/kg and F_2 costs ₹5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

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