JEE-Mains-25-01-2023 (Memory Based) [Evening Shift]

Physics

Question: A wire with resistance 5Ω is redrawn to increase its length 5 times. What is the final resistance of the wire

Options:

(a) 25 Ω

(b) 16 Ω

(c) 125 Ω

(d) 32 Ω

Answer: (c)

Solution:

$R_i = 5\Omega$

$R \propto \frac{l}{A}$ & $l_1 A_1 = l_2 A_2$

$\therefore \frac{R_f}{R_i} = \frac{l_f}{A_f} \times \frac{A_i}{l_i} = \left(\frac{l_f}{l_i}\right)^2 = (5)^2$

$\therefore R_f = 25$

$R_i = 125\Omega$

Question: Find the velocity of the particle if position of the particle is given by $x = 2t^2$ at t = 2 sec.

Options:

(a) 8 m/s

(b) 4 m/s

(c) 16 m/s

(d) 32 m/s

Answer: (a)

Solution:

$x = 2t^2$

$\therefore v = \frac{dx}{dt} = 4t$

$\therefore$ At $t = 2$

$v = 4(2) = 8$ m/s

Question: A particle performing SHM with amplitude A starts from x = 0 and reaches x = A/2 in 2 sec. Find the time required for the particle to go from x = A/2 to x = A?

Options:

(a) 1.5 s

(b) 4 s

(c) 6 s
(d) 1 s
Answer: (b)
Solution:

$y = A\sin\omega t$

$\frac{A}{2} = A\sin\omega t$

$\frac{1}{2} = \sin\omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow \omega = \frac{\pi}{12}$

Now

$A = A\sin\omega t'$

$\Rightarrow \omega t' = \frac{\pi}{2} \Rightarrow t' = \frac{\pi}{2\omega}$

$\Rightarrow t' = \frac{\pi(12)}{2\pi} = 6 \text{ sec}$

Time taken = 6 – 2 = 4s

Question: An Object of mass m is placed at a height R_E from the surface of the earth. Find the increase in potential energy of the object if the height of the object is increased to $2R_E$. From the surface. (R_E : Radius of the earth)
Options:
(a) 1/3 mgR_e
(b) 1/6 mgR_e
(c) 1/2 mgR_e
(d) 1/4 mgR_e
Answer: (b)
Solution:

$U_i = \frac{-GMm}{2R_E}; U_f = \frac{-GMm}{3R_E}$

$|\Delta U| = \frac{-GMm}{6R_E}$

$\therefore |\Delta U| = \frac{1}{6}mgR_E \left(\because g = \frac{GM}{(R_E)^2}\right)$

Question: A moving coil galvanometer of coil N = 200 is connected to torsional spring of k = 100 SI units, and placed in B = 0.01 T. If at i = 4 mA deflection is 0.05 rad. Find area of coil.
Options:
(a) 675 m^2
(b) 665 m^2
(c) 655 m^2
(d) 685 m^2
Answer: (a)
Solution:

$$\tau = kQ = MB = NiAB$$

$$\theta = \frac{NiAB}{k}$$

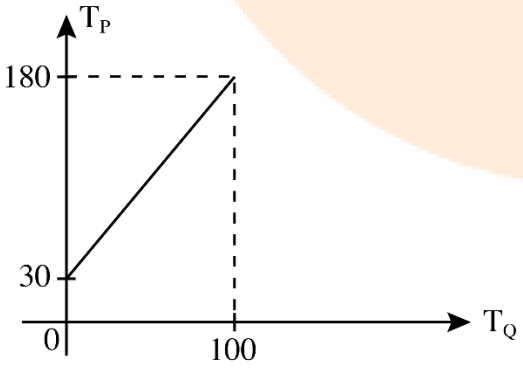
$$0.05 = \frac{200 \times 4 \times 10^{-3} A \times 0.01}{100}$$

$$A = \frac{0.05 \times 1000}{8 \times 0.01} = \frac{5 \times 1000}{8}$$

$$A = 675\,\text{m}^2$$

Question: Statement 1: Si when doped with 'B' is p-type and with 'As' is n type.
Statement 2: It is possible to measure current using ammeter if n & p type are joined.
Options:
(a) S1 and Reason are correct, S2 is correct explanation of assertion
(b) S1 and Reason are correct, S2 is NOT a correct explanation of assertion
(c) S1 is Correct, S2 is incorrect
(d) S1 is incorrect, S2 is correct
Answer: (c)
Solution:
1) B is trivalent whereas As is pentavalent.
2) It is not possible to measure current using ammeter if n and p type are joined.
∴ S1 is true, S2 is false

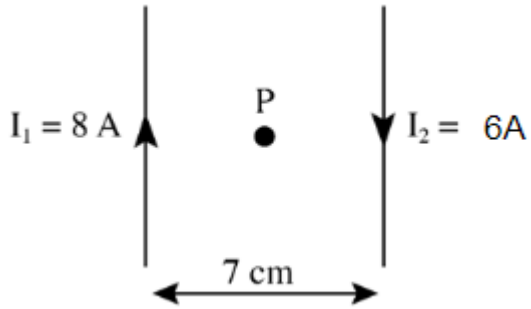
Question: Find the relation between T_P & T_Q.
Options:
(a) $T_P = T_Q$
(b) $3T_P + 10 = T_Q$
(c) $2T_P = 3T_Q + 60$
(d) None of the above
Answer: (c)
Solution:



$$\frac{T_p - 30}{T_Q - 0} = \frac{180 - 30}{100 - 0} = \frac{150}{100} = \frac{3}{2}$$

$$2T_p - 60 = 3T_Q$$

Question: Find magnetic field at P which is equidistant from both the wires.



Options:
(a) 13×10^{-4} T
(b) 8×10^{-5} T
(c) 13.14×10^{-5} T
(d) 14.10×10^{-5} T
Answer: (c)
Solution:

$$B_1 = \frac{\mu_0 I_1}{2\pi R} + \frac{\mu_0 I_2}{2\pi R}$$

$[\text{Here } R = 3.5 \text{ cm}]$

$$\therefore B = 8 \times 10^{-5}$$

Question: Statement 1: Stopping potential is independent of power of light.
Statement 2: Stopping potential depends on wavelength of light.
Options:
(a) S1 and Reason are correct, S2 is correct explanation of assertion
(b) S1 and Reason are correct, S2 is NOT a correct explanation of assertion
(c) S1 is Correct, S2 is incorrect
(d) S1 is incorrect, S2 is correct
Answer: (c)
Solution:
Power $\rightarrow$ Intensity $\rightarrow$ No. of photons
Stopping pot. depends on energy / wavelength or frequency.
Hence Statement 1 is right Statement 2 is wrong.

Question: A conductor l = 1m is moving prefeadicula to B = 2T with v = 8 m/s. Find emf induced.
Options:
(a) 4V
(b) 8V
(c) 16V
(d) 32V
Answer: (c)
Solution:

$$\varepsilon = VBl = 8 \times 2 \times 1 = 16 \text{ V}$$

Question: Match the matrix:

1. Adiabatic	a. No heat exchange
2. Isothermal	b. No change in internal energy

3. Isochoric	c. No change in pressure
4. Isobaric	d. Work is zero

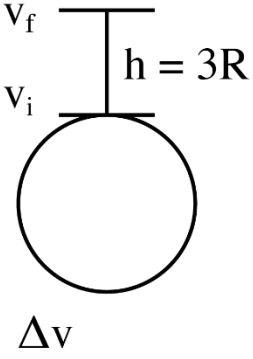
Options:
(a) 1-a, 2-b, 3-c, 4-d
(b) 1-d, 2-b, 3-a, 4-c
(c) 1-b, 2-a, 3-d, 4-c
(d) 1-a, 2-b, 3-d, 4-c
Answer: (d)
Solution:
1. By definition.
2. In isothermal process, no change in temp. Hence $\Delta U = 0$
3. $\Delta V = 0$; $\therefore \Delta W = 0$
4. By definition

Question: Diatomic gas with vibrational degree C_V = ?
Options:
(a) 7R/2
(b) 5R/2
(c) 3R/2
(d) 3R
Answer: (a)
Solution:

$$C_V = \frac{f}{2}R$$

'*f*' for diatomic with vibration = 7

Question: Find the change in potential energy if a particle of mass m is taken to a height 3 time the radius of earth above earth surface.
Options:
(a) 3 MgR
(b) MgR/4
(c) 5MgR/4
(d) 3MgR/4
Answer: (d)
Solution:



Δv

$$\Delta U = \frac{Mgh}{1+\frac{h}{R}} = \frac{Mg(3R)}{1+\frac{3R}{R}} = \frac{3MgR}{4}$$

Question: A charge of 10 μC is placed at origin. Where should a charge of 40 μC be placed on x-axis such that electric field is zero at $x = 2$.

Options:

(a) $x = -2$

(b) $x = 4$

(c) $x = 6$

(d) $x = 2$

Answer: (c)

Solution:

$\frac{k(10)}{(2)^2} = \frac{k(40)}{x^2}$ [x is the distance from 2]

$\Rightarrow \frac{10}{4} = \frac{40}{x^2}$

$\Rightarrow x^2 = 16 \Rightarrow x = 4$

$\therefore$ Distance from origin $= 4 + 2 = 6$

Question: If R = 80 Ohm, X_C = 130 ohm, X_L = 70 ohm, then find power factor.

Options:

(a) 2/3

(b) 4/5

(c) 1

(d) 3/8

Answer: (b)

Solution:

Power factor $= \cos\rho = \frac{R}{Z}$

$Z = \sqrt{(80)^2 + (130-70)^2} = 100\Omega$

$\therefore \cos\phi = \frac{80}{100} = \frac{4}{5}$

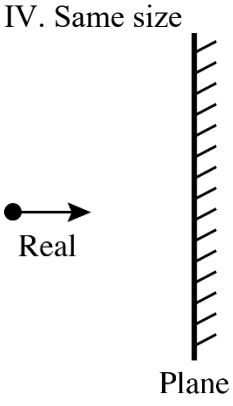
Question: Real object is placed in front of plane mirror, image will be

I. real

II. laterally inverted

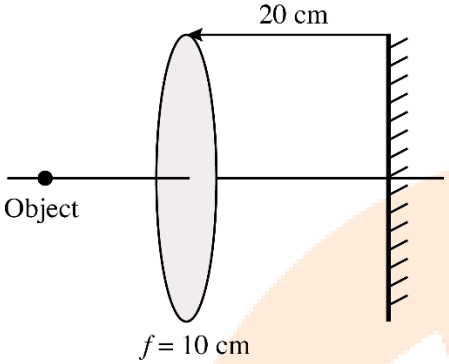
III. Erect

IV. Same size

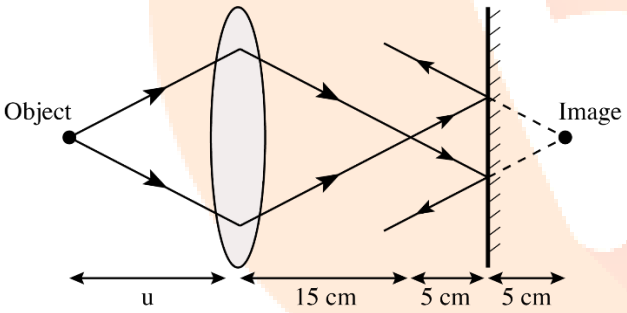


Options:
(a) I, III
(b) II, IV
(c) I, IV
(d) II, III, IV
Answer: (d)
Solution: Plane mirror creates laterally inverted virtual images and magnification is zero.

Question: Find the object distance such that final image is formed 5 cm behind plane mirror



Options:
(a) 30 cm
(b) 25 cm
(c) 45 cm
(d) 40 cm
Answer: (a)
Solution:



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{15}+\frac{1}{u}=\frac{1}{10}$$

$$\frac{1}{4}=\frac{-1}{30}\Rightarrow u=30\text{ cm}$$

Question: A big drop is broken into 1000 small drops. Find ratio of find surface energy to initial surface energy.
Options:
(a) 10
(b) 100
(c) 1/10
(d) 1000
Answer: (a)

Solution:

Surface energy $\propto$ A

$$\frac{4}{3}\pi R^3 = 1000\left(\frac{4}{3}\pi r^3\right)$$

$$\therefore r = \frac{R}{10}$$

[r = radius of small drops & R is radius of big drop]

Ratio of surface areas = $\frac{100r^2}{R^2} = \frac{10}{1}$

Question:

A) Planck constant	P) $[ML^2T^{-2}]$
B) Work Function	Q) $[ML^{-1}T^{-2}]$
C) Viscosity	R) $[ML^2T^{-1}]$
D) Young's Modulus	S) $[ML^{-1}T^{-1}]$

Options:

(a) A-R, B-P, C-S, D-Q
(b) A-R, B-S, C-P, D-Q
(c) A-S, B-R, C-P, D-Q
(d) A-Q, B-P, C-S, D-R

Answer: (a)

Solution:

A) $mvr = \frac{nh}{2\pi}$

$$\therefore h = [mvr] = \left[ML^2T^{-1}\right]$$

B) ϕ = Energy $= \left[ML^2T^{-2}\right]$

C) $\eta = \frac{F}{6\pi rV} \Rightarrow \eta = \left[ML^{-1}T^{-1}\right]$

D) $Y = \frac{\left(\frac{F}{A}\right)}{\Delta L / L} = \left[ML^{-1}T^{-2}\right]$

Question: Two projectiles are thrown at an angle of projection α and β with the horizontal. If α + β = 90° then ratio of range of two projectiles on horizontal plane is equal to

Options:

(a) 1 : 1
(b) 2 : 1
(c) 1 : 2
(d) 1 : 3

Answer: (a)

Solution:

Range is same for θ and (90° – θ) angles for projections.

JEE-Main-25-01-2023 (Memory Based)
[Evening Shift]

Chemistry

Question: Which of the following is the weakest reducing agent among the following?
Options:
(a) Li
(b) Na
(c) K
(d) Rb
Answer: (b)
Solution:

$E_o\left(\frac{Na^+}{Na}\right) = -2.71\ V$

$E_o\left(\frac{Li^+}{Li}\right) = -3.05\ V$

$E_o\left(\frac{K^+}{K}\right) = -2.93\ V$

$E_o\left(\frac{Rb^+}{Rb}\right) = -2.98\ V$

Question: Match Column I with Column II.

Column I (Name of Amine)	**Column II (PK_b)**
(A) Ethanamine	**(i)** 3
(B) N-ethyl ethanamine	**(ii)** 3.29
(C) N, N-diethyl ethanamine	**(iii)** 3.25
(D) Benzenamine	**(iv)** 9.38

Options:
(a) A – ii; B – i; C – iii; D – iv
(b) A – i; B – ii; C – iii; D – iv
(c) A – ii; B – iii; C – i; D – iv
(d) A – ii; B – iv; C – i; D – iii
Answer: (a)
Solution:
Ethanamine – 3.29

N-ethyl ethanamine – 3

N, N-Diethyl ethanamine – 3.25

Benzenamine – 9.38

Question: Assertion: Alkali metals and their salts impart characteristics colour in reduction flame.
Reason: Alkali metals can be identified using flame test.
Options:
(a) [A] and [R] both are correct and [R] is the correct explanation
(b) [A] and [R] both are correct and [R] is not the correct explanation
(c) [A] is correct and [R] is incorrect
(d) [A] is incorrect and [R] is correct
Answer: (d)
Solution: The alkali metals and their salts impart characteristic colour to an oxidizing flame. Alkali metals can be detected by the respective flame tests and can be determined by flame photometry or atomic absorption spectroscopy.

Question: Arrange the following elements in increasing order of metallic character Si, K, Mg and Be
Options:
(a) Si < Mg < Be < K
(b) Be < Mg < Si < K
(c) Si < Be < Mg < K
(d) K < Mg < Si < Be
Answer: (c)
Solution: Metallic character decreases from left to right in periodic table.

Question: Change of oxidation state of Cr in $K_2Cr_2O_7$ in acidic medium
Options:
(a) +3
(b) +4
(c) +2
(d) +6
Answer: (a)
Solution: $K_2Cr_2O_7 \xrightarrow{H^+} 3Cr^{+3} + H_2O$

Question: How many of the following orbitals is considered as axial orbital(s)

$p_x, p_y, p_z, d_{xy}, d_{yz}, d_{xz}, d_{x^2-y^2}, d_{z^2}$

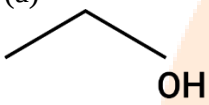
Options:
(a) 5
(b) 4
(c) 3
(d) 6
Answer: (a)
Solution: Orbital which lie on axis are $p_x, p_y, p_z, d_{x^2-y^2}, d_{z^2}$

Question: Match Column I with Column II.

Column I (Polymer)	**Column II (Uses)**

(A) Glyptal	**(i)** Conveyor belts, gaskets and hoses
(B) LDP	**(ii)** Paints and Lacquers
(C) Neoprene	**(iii)** Toys and flexible pipes
(D) Acrilan	**(iv)** Synthetic wool

Options:
(a) A – ii; B – i; C – iii; D – iv
(b) A – i; B – ii; C – iii; D – iv
(c) A – ii; B – iii; C – i; D – iv
(d) A – ii; B – iv; C – i; D – iii
Answer: (c)
Solution:
Glyptal – Paints and Lacquers
LDP – Toys and flexible pipes
Neoprene – Conveyor belts, gaskets and hoses
Acrilan – Synthetic wool

Question: Which of the following compound give positive CAN and Iodoform test?
Options:
(a)


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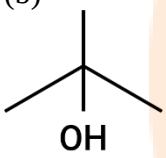

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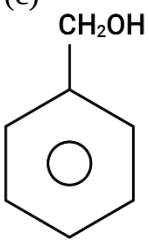

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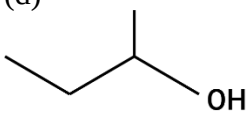
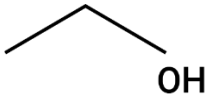
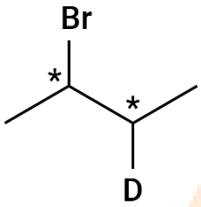


Answer: (a)
Solution:


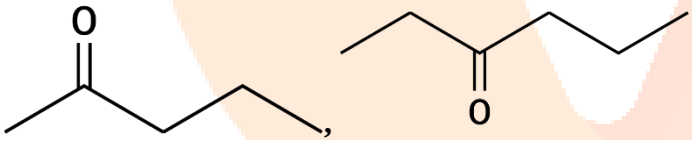
Question: Assertion [A]: Carbon form two oxides CO and CO_2 where CO is neutral, while CO_2 is acidic.
Reason [R]: CO_2 will combine with water to give carbonic acid and CO is soluble in water.
Options:
(a) [A] and [R] both are correct and [R] is the correct explanation

(b) [A] and [R] both are correct and [R] is not the correct explanation
(c) [A] is correct while [R] is incorrect
(d) [A] is incorrect while [R] is correct
Answer: (a)
Solution: $CO_2 + H_2O \rightarrow H_2CO_3$, and CO solubility is 27.6 mg/lt at 25°C

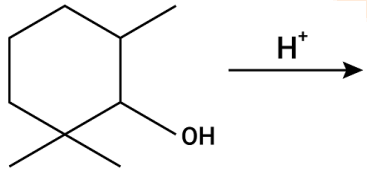
Question: Which of the following has two chiral centres?
Options:
(a) 2 - Bromo - 3 - duetrobutane
(b) 1 - Bromo - 2 - duetrobutane
(c) 1 - Bromo - 3 - duetrobutane
(d) 1 - Bromo - 4 - duetrobutane
Answer: (a)
Solution:



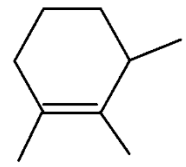
Question: Select the correct match.
Options:
(a) Hexan-2-one & Hexan-3-one — Position isomer
(b) Pentan-3-one & Pentan-2-one — Functional isomer
(c) 2-pentene & 1-pentene — Metamers
(d) Pentanoic acid & Hexanoic acid — Functional isomers
Answer: (a)
Solution:



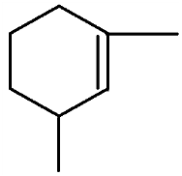
Question:



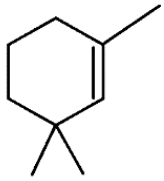
Options:
(a)



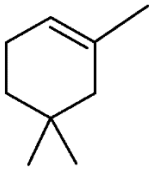
(b)



(c)



(d)



Answer: (a)
Solution: Dehydration will be followed by carbocation shift.

Question: Chloride salt of M is treated with the excess of $AgNO_3$. It forms curdy white precipitate'. When 'A' is treated with NH_4OH, it forms a soluble salt B. Then 'A' and 'B' respectively are:
Options:
(a) AgCl, $[Ag(NH_3)_2]^+$
(b) AgBr, $[Ag(OH)_2]^-$
(c) AgCl, $[Ag(OH)_4]^{2-}$
(d) AgBr, $[Ag(OH)_4]^{2-}$
Answer: (a)
Solution: $Cl^- + AgNO_3 \rightarrow \underset{\text{Curdy white}}{AgCl} \xrightarrow{NH_4OH} \left[Ag\left(NH_3\right)_2\right]^+$

JEE-Main-25-01-2023 (Memory Based)
[Evening Shift]

Mathematics

Question: If α, β are roots of $x^2+(60)^{\frac{1}{4}}x+a=0$ and $\alpha^4+\beta^4=-30$ then find product of possible values of a.

Answer: 45.00

Solution:

$\alpha^4+\beta^4=-30$

$\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2=-30$

$\left(\alpha+\beta\right)^2-2\alpha\beta$

$\Rightarrow\left(\sqrt{60}-2a\right)^2-2a^2=-30$

$\Rightarrow 60+2a^2-8a\sqrt{60}+30=0$

$\Rightarrow 2a^2-8a\sqrt{60}+90=0$

Product of roots = $\frac{90}{2}=45$

Question: Given that $f(x)=2x^n+\lambda$, $f(4)=133$ and $f(5)=255$. Find sum of positive divisors of $f(3)-f(2)$.

Answer: 60.00

Solution:

Given, $f(x)=2x^n+\lambda$

substitute $x=4$

$2\cdot 4^n+\lambda=133$

$2\cdot 5^n+\lambda=255$

$2\left(5^n-4^n\right)=122$

$n=3$

$2\times 4^3+\lambda=133$

$\lambda=133-128$

$\lambda=5$

$f(x)=2x^3+5$

$f(3)=2\times 27+5=59$

$f(2)=2\times 8+5=21$

$f(3)-f(2)=38$

Sum of positive integral divisors of 38 are 1, 2, 19, 38

$=1+2+19+38$

$=60$

Question: If $I=\int_1^2 \frac{dx}{x^3\left(x^2+2\right)^2}$ then the value of $16I$ is

Answer: $\frac{5}{3}-2\log 2$

Solution:

$I=\int_1^2 \frac{dx}{x^7\left(1+\frac{2}{x^2}\right)^2}$

Put $1+\frac{2}{x^2}=t$

$-\frac{4}{x^3}dx=dt$

$I=-\frac{1}{4}\int_3^{\frac{3}{2}} \frac{\left(\frac{t-1}{2}\right)^2}{t^2}dt$

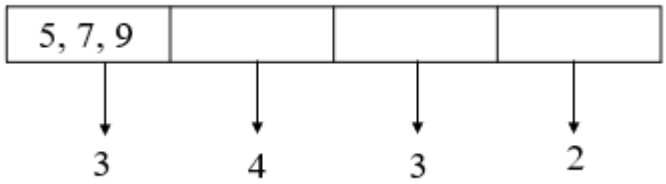
$=-\frac{1}{4\times 4}\int_3^{\frac{3}{2}} \frac{-2}{t}+\frac{1}{t^2}dt$

$16I=-\left[t-2\ln t-\frac{1}{t}\right]_3^{\frac{3}{2}}$

$16I=\frac{5}{3}-2\log 2$

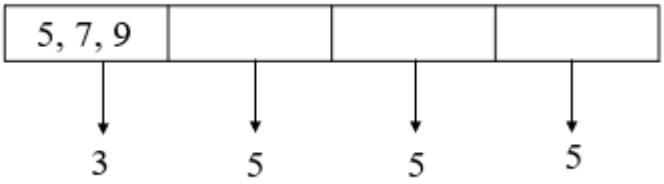
Question: How many numbers can be made between 5000 & 10000, using 1, 3, 5, 7, 9.
Answer: 375.00 or 72.00
Solution:
Four digit number

If repetition is not allowed



$3\times4\times3\times2=72$

If repetition allowed

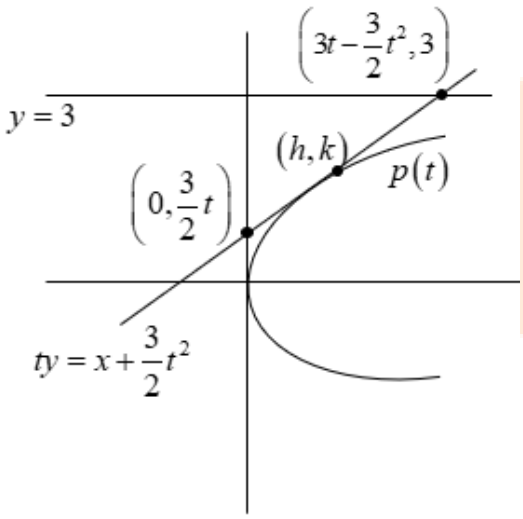


$3\times5\times5\times5=375$

Question: Find locus of circumcentre of triangle formed by tangent of $y^2=6x,\ x=0$ and $y=3$.

Answer: $4y^2-18y+3x+18=0$

Solution:



$$h=\frac{3t-\frac{3t^2}{2}}{2}$$

$$k=\frac{\frac{3}{2}t+3}{2}$$

$$t = \frac{2k-3}{\frac{3}{2}}$$

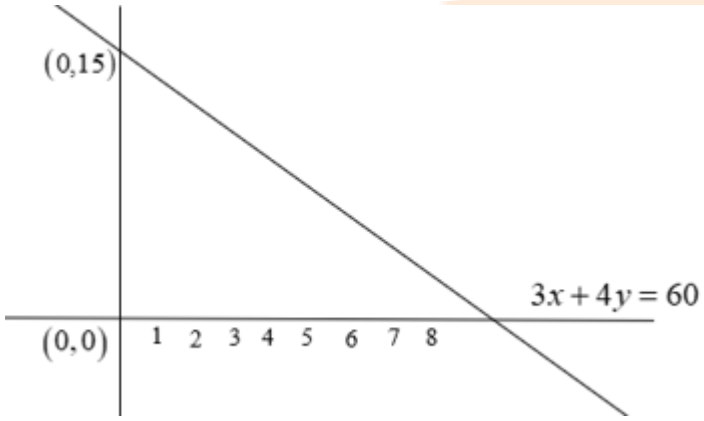
Substitute in h

$4y^2 - 18y + 3x + 18 = 0$

Question: The straight line $3x + 4y = 60$ makes a triangle with axes. How many points (a,b) lie inside it such that b is multiple of a?

Answer: 31.00

Solution:



$x = 1,\ y = \frac{57}{4} \Rightarrow 1,2,3,...14 \rightarrow 14$

$x = 2,\ y = 13 \Rightarrow y = 2,4,6,8,10,12 \rightarrow 6$

$x = 3,\ y = 12 \Rightarrow y = 3,6,9,12 \rightarrow 4$

$x = 4,\ y = 12 \Rightarrow y = 4,8 \rightarrow 2$

$x = 5,\ y = 16 \Rightarrow y = 5,10 \rightarrow 2$

$x = 6,\ y = 10 \Rightarrow y = 6 \rightarrow 1$

$x = 7,\ y = 9 \Rightarrow y = 7 \rightarrow 1$

$x = 8,\ y = 9 \Rightarrow y = 8 \rightarrow 1$

Total = 14 + 6 + 4 + 2 + 2 + 1 + 1 + 1 = 31

Question: Given: 8 oranges, 7 red Apples, 5 white apples. In how many ways 5 fruits can be selected, containing at least 2 oranges, at least 1 white apple and at least 1 red apple.

Answer: 6860.00

Solution:

Given 8 Oranges

7 Red Apples

5 White Apples

At least 5 Fruits can be selected

2 O	1	0	0
1 R	0	1	0
1 W	0	0	1

$${}^8C_3 \times {}^7C_1 \times {}^5C_1 + {}^8C_2 \times {}^7C_2 \times {}^5C_1 + {}^8C_2 \times {}^7C_1 \times {}^5C_2$$

$$= 6860$$

Question: If $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}.\vec{b} = 1$ & $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$, then find $\vec{a} - 6\vec{b}$.

Answer: $3\hat{i} + 3\hat{j} + 3\hat{k}$

Solution:

$$\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{a} \times \left(\hat{i} - \hat{j}\right)$$

$$(1)\,\vec{a} - 3\vec{b} = \left(-\hat{i} - \hat{j} + \hat{k}\right) \times \left(\hat{i} - \hat{j}\right)$$

$$-\hat{i} - \hat{j} + \hat{k} - 3\vec{b} = \left(-\hat{i} - \hat{j} + \hat{k}\right) \times \left(\hat{i} - \hat{j}\right)$$

$$-3\vec{b} = \left(-\hat{i} - \hat{j} + \hat{k}\right) \times \left(\hat{i} - \hat{j}\right) - \left(-\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{b} = \frac{\left(-\hat{i} - \hat{j} + \hat{k}\right) \times \left(\hat{i} - \hat{j}\right) - \left(-\hat{i} - \hat{j} + \hat{k}\right)}{3}$$

$$\vec{b} = \frac{\left(\hat{i} - \hat{j}\right)}{-3}$$

$$\left(\vec{a} \cdot \vec{b}\right)\vec{a} - \left(a^2\right)\vec{b}$$

$$\vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

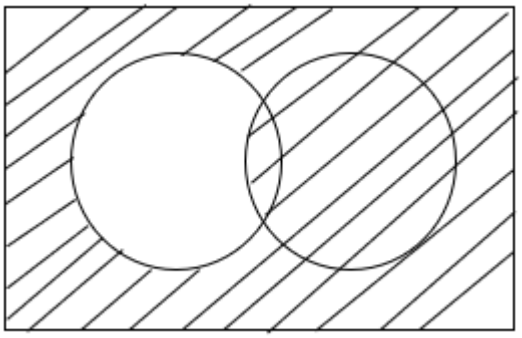
Question: $\Delta, \nabla \in \{\wedge, \vee\}$. If $(p \to q)\Delta(p \nabla q)$ is tautology.

Answer: ()

Solution:

$(p \to q)\Delta(p \nabla q)$ is tautology

$(\sim p \vee q)\Delta(p \nabla q)$ is tautology



∇ must be union

& Δ must be union

Question: $f(x)=2x^3+(2p-7)x^2+3(2p-9)x-6$. If maximum occurs at negative x and minima occurs at positive x then $p \in$ ____.

Answer: $p<\frac{9}{2}$

Solution:

$f(x)=2x^3+(2p-7)x^2+3(2p-9)x-6$

$f'(x)=6x^2+(4p-14)x+(6p-27)$

$\alpha\beta<0$

$\frac{6p-27}{6}<0$

$p-\frac{9}{2}<0$

$p<\frac{9}{2}$

Question: Value of $\sum_{k=0}^{6} {}^{51-k}C_3 = ?$

Answer: ${}^{52}C_4-{}^{45}C_4$

Solution:

${}^{51}C_3+{}^{51}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+{}^{46}C_3+{}^{45}C_3$

We know that

${}^{45}C_3+{}^{45}C_4={}^{46}C_4$

${}^{45}C_3={}^{46}C_4-{}^{45}C_4$

${}^{51}C_3+{}^{51}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+{}^{46}C_3+{}^{46}C_4-{}^{45}C_4$

$^{51}C_3 + {}^{51}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4 - {}^{45}C_4$

$^{51}C_3 + {}^{51}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4 - {}^{45}C_4$

$\vdots$

$^{52}C_4 - {}^{45}C_4$

Question: If $\left|\frac{z-2i}{z+i}\right| = 2$ is a circle, then centre of the circle is

Answer: (0, -2)

Solution:

$z = x + iy$

$|z-2i|^2 = 4|z+i|^2$

$x^2 + (y-2)^2 = 4\left(x^2 + (y+1)^2\right)$

$3x^2 + 3y^2 + 12y + 0 \cdot x + 0 \cdot x^2$

$x^2 + y^2 + 4y + 0 \cdot x + 0 \cdot x^2$

x coordinate is 0 and y coordinate is -2

(0, -2)

Question: If $\int_{\frac{1}{3}}^{3} |\ln x|\, dx = \frac{m}{n} \ln\left(\frac{n^2}{e}\right)$, then $m^2 + n^2 - 5$ is equal to

Answer: 20.00

Solution:

$\int_{\frac{1}{3}}^{3} |\ln x|\, dx = \frac{m}{n} \ln\left(\frac{n^2}{e}\right)$

$-\int_{\frac{1}{3}}^{1} \ln x \cdot dx + \int_{1}^{3} \ln x \cdot dx$

$= -\left(x \ln x - x\right)_{\frac{1}{3}}^{1} + \left(x \ln x - x\right)_{1}^{3}$

$= -\left((10-1) - \left(\frac{1}{3}\ln\left(\frac{1}{3}\right) + \frac{1}{3}\right)\right) + \left((3\ln 3 - 3) - (1 \cdot \ln - 1)\right)$

$= \left(\frac{2}{3} - \frac{1}{3}\ln 3\right) + (3\ln 3 - 2)$

$$= -\frac{4}{3} + \ln 3\left(3 - \frac{1}{3}\right) = \frac{-4}{3} + \ln 3\left(\frac{8}{3}\right) = \frac{4}{3}$$

$$= \frac{4}{3}\left(\ln \frac{9}{e}\right)$$

Comparing with $\frac{m}{n}\ln\left(\frac{n^2}{e}\right)$

$$m^2 + n^2 - 5 = 16 + 9 - 5 = 20$$

Question: If $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$; $B = \begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix}$ and $M = ABA^T$, then the inverse of $A^T M^{2023} A$ is

Answer: $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Solution:

Given, $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$; $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A^T M^{2023} A = B^{2023}$$

$$AB^{2023}A^T$$

$$B^2 = \begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2i \\ 0 & 1 \end{bmatrix}$$

$$B^{2023} = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $A^T M^{2023} A = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Question: a & b are positive, $\frac{1}{a}, 10, \frac{1}{b}$ are in AP & $a, b, \frac{1}{18}$ are in GP, then $12b + 16a = ?$

Answer: 3.00

Solution:

$20 = \frac{1}{a} + \frac{1}{b}$

$b^2 = a \times \frac{1}{18} \Rightarrow a = 18b^2$

$a + b = 20ab$

$18b^2 + b = 20 \times 18b^2 \times b$

$360b^2 - 18b - 1 = 0$

$b = \frac{-1}{30}$ or $b = \frac{1}{12}$

Rejected or $a = \frac{1}{8}$

$12b + 16a = 3$

Question: Find remainder when 2023^{2023} is divided by 35.

Answer: 7.00

Solution:

2023^{2023} is multiple of 7

35 is multiple of 7 i.e. 5 × 7

$n = (2023)^{2023} = 7t$

$(2023)^{2023} = (-2)^{2023}$ w.r.t 5

$= -2(2^2)^{1011}$

$= -2(5-1)^{1011}$

It will leave remainder 2 when divided by 5

$n = 7t \Rightarrow n - 7 = 7(t-1)$

$n = 5u + 2 \Rightarrow n - 7 = 5(u-1)$

$\therefore n - 7$ is multiple of 35