JEE-Main-29-01-2023 (Memory Based)
[Morning Shift]

Physics

Question: A projectile was projected at an angle θ = 30°, then find the ratio of kinetic energy at a starting point to the kinetic energy at highest point.

Options:

(a) $\frac{\sqrt{3}}{2}$

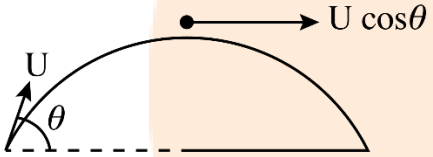
(b) $\frac{3}{2}$

(c) $\frac{4}{3}$

(d) $\frac{2}{\sqrt{3}}$

Answer: (c)

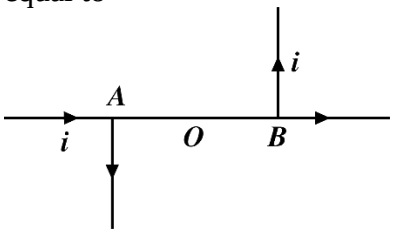
Solution:



$$\frac{K=\frac{1}{2}mu^2}{K'=\frac{1}{2}m\left(4\frac{\sqrt{3}}{2}\right)^2}$$

$$\frac{K}{K'}=\frac{4}{3}$$

Question: Point 0 and two long wires are kept in same plane such that point 0 lies at middle of the line. Then magnetic field at point 0 due to the current i flowing in both the wires is equal to



Options:

(a) $\frac{\mu_0 i}{2\pi l}$

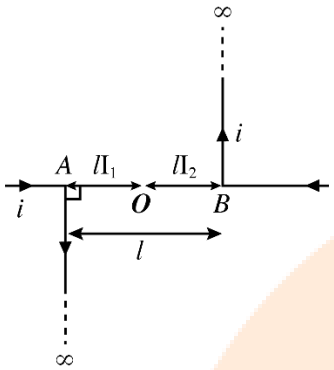
(b) $\frac{\mu_0 i}{\pi l}$

(c) $\frac{2\pi\mu_0 i}{l}$

(d) $\frac{\mu_0 i}{2l}$

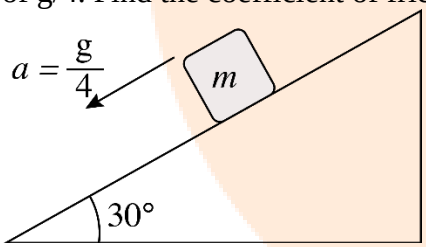
Answer: (b)

Solution:



$$B = \frac{2 \times \mu_0 i}{2\mu\pi\left(\frac{l}{2}\right)} = \frac{\mu_0 i}{\pi l}$$

Question: A block is sliding down an inclined plane of inclination 30° with an acceleration of g/4. Find the coefficient of friction between the block and incline



Options:

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{2\sqrt{3}}$

(c) $1/3$

(d) $1/2$

Answer: (b)

Solution:

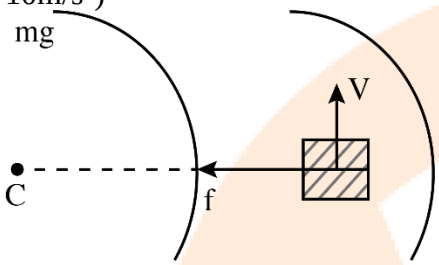
$$a = g\sin\theta - Mg\cos\theta$$

$$\frac{g}{4} = \frac{g}{2} - \mu g\frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2}\mu g = \frac{g}{4}$$

$$\mu = \frac{1}{2\sqrt{3}}$$

Question: A car is moving on a circular track of radius 50 cm with coefficient of friction being 0.34. On this horizontal track the maximum safe speed for turning is equal to (g = 10m/s^2)



Options:
(a) 1.03
(b) 1.7
(c) 1.3
(d) 1.8
Answer: (c)
Solution:

$$f = \frac{mV^2}{R} \le \mu mg$$

$$V \le \sqrt{\mu Rg}$$

$$V_{max} = \sqrt{\mu Rg} = \sqrt{0.34\times 0.5\times 10} = \sqrt{1.7} = 1.3$$

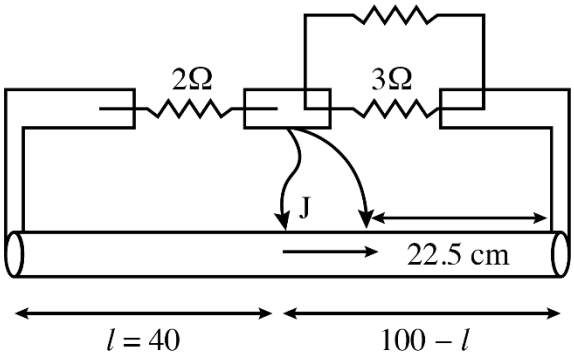
Question: Two resistors R and 3R are in parallel, find the ratio of power dissipated in them.
Options:
(a) 1 : 2
(b) 2 : 1
(c) 3 : 1
(d) 1 : 3
Answer: (c)
Solution:

$$P = \frac{V^2}{R}$$

$$P \propto \frac{1}{R}$$

$$\frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{3R}{R}$$

Question: In a meter bridge experiment, null point is found at a particular point for resistance 2 Ω and 3 Ω. Then resistance of X Ω is shunted on 3 Ω resistor, and null point shifts by 22.5 cm. Then find the value of X.



Options:
(a) 1
(b) 2
(c) 3
(d) 4
Answer: (b)
Solution:

$$\frac{2}{3} = \frac{l}{100 - l} \Rightarrow 200 - 2l = 3l$$

$$l = 40cm$$

Finally, after attaching X in parallel with 3Ω

New Resistance = $\frac{3x}{x+3}$

So $\frac{2}{\left(\frac{3x}{x+3}\right)} = \frac{40 + 22.5}{60 - 22.5} = \frac{62.5}{37.5} = 2\Omega$

Question: Find the ratio of maximum wavelength of Lyman series of Hydrogen atom to minimum wavelength of Balmer series of Helium atom.
Options:
(a) 4/3
(b) 1
(c) 3/2
(d) 3/4
Answer: (a)
Solution:

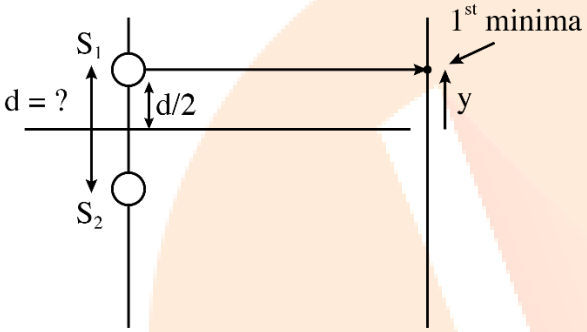
$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2 - \frac{1}{n_i^2}}\right] Z^2$$

$$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \times 1^2$$

$$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] \times 2^2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

Question: In YDSE 1st minima is formed directly opposite to upper slit, then find d (λ = 800 nm, D = 5 cm)



Options:

(a) 0.4 mm

(b) 0.3 mm

(c) 0.2 mm

(d) 0.1 mm

Answer: (c)

Solution:

$$\frac{d^2}{2D} = \frac{\lambda}{2}$$

$$d = \sqrt{\lambda D}$$

$$\Delta x = \frac{dy}{D} = \frac{d\left(d/2\right)}{D}$$

Question: Two objects of equal mass m are moving in a circular path of radius "r" because of their mutual gravitational attraction force. Find the velocity of each particle.

Options:

(a) $\sqrt{\frac{Gm}{r}}$

(b) $\sqrt{\frac{Gm}{2r}}$

(c) $\sqrt{\frac{Gm}{4r}}$

(d) $\sqrt{\frac{2Gm}{r}}$

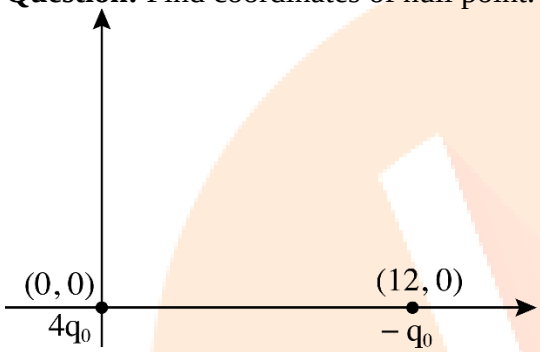
Answer: (c)

Solution:

$$F = \frac{Gm^2}{4r^2} = \frac{Mv^2}{r}$$

$$V = \sqrt{\frac{Gm}{4r}}$$

Question: Find coordinates of null point.



Options:

(a) (12, 0)

(b) (0, 12)

(c) (0, 24)

(d) (24, 0)

Answer: (d)

Solution:

Suppose the null point is at 'x'

Then

$$\frac{K(4q_0)}{(12+x)^2} = \frac{Kq_0}{x^2}$$

$$\Rightarrow x = 24$$

Question: A soap bubble of Radius R & surface tension S is blown to double the radius. Find charge in surface energy.

Options:

(a) $24\pi SR^2$

(b) $12\pi SR^2$

(c) $6\pi SR^2$

(d) $3\pi SR^2$

Answer: (a)

Solution:

$$U_i = \left(S \times 4\pi R^2\right) \times 2$$

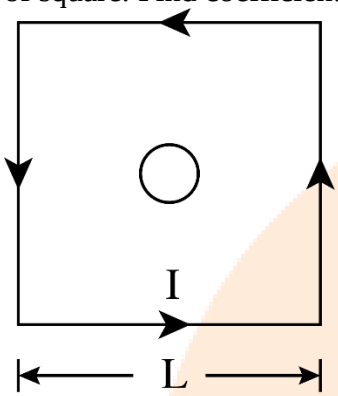
$$U_i = 8\pi SR^2$$

$$U_{final} = S \times 4\pi \left(2R\right)^2 \times 2$$

$$U_f = 32S\pi R^2$$

$$\Delta U = 24\pi SR^2$$

Question: There is square loop of length 'L'. A very small ring of radius r is placed at centre of square. Find coefficient of mutual inductance.

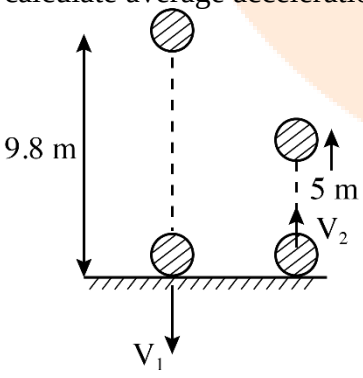


Solution:

$$B = \frac{\mu_o i_1}{4\pi \frac{L}{2}} \left(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \right) \times \pi r^2$$

$$\phi_{2,1} = \left[\frac{2\sqrt{2}\mu_0 r^2}{L} \right] i_1$$

Question: A Tennis ball is dropped from 9.8 m it hits the ground and rebounds to 5m calculate average acceleration at point of contact t = 0.2 sec



Options:

(a) 50 m/s^2

(b) 120 m/s^2

(c) 100 m/s^2

(d) 200 m/s^2

Answer: (b)

Solution:

$V_1 = \sqrt{2gh} = \sqrt{2\times10\times9.8} = 14 \text{ ms}^{-1}$

$V_2 = \sqrt{2\times10\times5} = 10 \text{ ms}^{-1}$

$\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\left(10\hat{i}\right)-\left(-14\right)\hat{j}}{0.2} = \frac{240}{2}\hat{j} = 120\hat{j}$

$= 120 \text{ ms}^{-1}$

Question: Match quantities with correct dimensions.

Column A	Column B
1. Latent Heat	(p) $MLA^{-1}T^{-3}$
2. Electric field	(q) $ML^{-2}T^{-2}$
3. Pressure Gradient	(r) $ML^2A^{-1}T^{-3}$
4. Electric Potential	(s) L^2T^{-2}

Options:

(a) 1-s, 2-p, 3-q, 4-r

(b) 1-p, 2-s, 3-q, 4-r

(c) 1-s, 2-p, 3-r, 4-q

(d) 1-q, 2-p, 3-s, 4-r

Answer: (a)

Solution:

$L = \frac{Q}{m} = \frac{ML^2T^{-2}}{M}$

$E = \frac{F}{q} = \frac{MLT^{-2}}{AT}$

$\frac{dP}{dx} = \frac{ML^{-1}T^{-2}}{L} = ML^2T^{-2}$

$V = \frac{P.E.}{2} = \frac{ML^2T^{-2}}{AT}$

Question: A nucleus $^{236}X_{92}$ undergoes two α decays and one β^- decay. Find atomic number and mass number.

Options:

(a) Z = 88, Z = 227

(b) Z = 90, Z = 225

(c) Z = 89, Z = 228

(d) Z = 81, Z = 210

Answer: (c)

Solution:

2α decays means A-8 and Z-4

236 – 8 = 228

1β decay means Z + 1

∴ Z = 94 – 4 + 1 = 89

Question: A wire has been made into a circle and placed in constant magnetic field. If the radius of wire starts decreasing at rate of 2 cm/s when wire starts shrinking, find the EMF induced in the wire when radius of wire is 10 cm. Magnetic field strength in the region is 0.8 T.

Options:

(a) 0.01 V

(b) 0.1 V

(c) 1 V

(d) 10 V

Answer: (a)

Solution:

Flux via circular wire is given by

$\phi = \mathrm{B}.\pi \mathrm{r}^2$

So emf induced $|\varepsilon| = \left|\frac{d\phi}{dt}\right| = B.2\pi r\left|\frac{dr}{dt}\right|$

$= 0.8 \times 2\pi \times \frac{10}{100} \times \frac{2}{100}$

$= \frac{3.2\pi}{1000} \approx \frac{10}{1000} = \frac{1}{100}\,\mathrm{V}$

Question: Two antenna's has height 80 m, what is the range to which signal can be transmitted?

Options:

(a) 28 km

(b) 10 km

(c) 64 km

(d) 51 km

Answer: (c)

Solution:

$d = 2\sqrt{2Rh}$

$= 2\sqrt{2 \times 6400 \times 80 \times 10^{-3}}$

$= \sqrt{8 \times 80 \times 6400 \times 10^{-3}}$

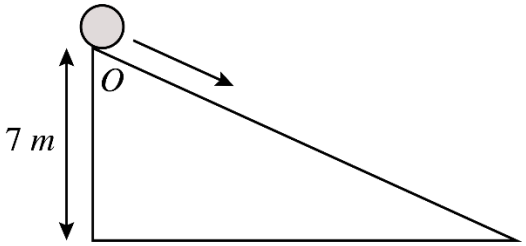
$= 64$ km

Question: Cuboid of dimensions 2L × 2L × L, A charge q is placed at centre for of S with area $4L^2$ flux through opposite side is?

Solution:

$q / 6\varepsilon_0$

Question: A solid sphere is released from point 0 at the top of an incline as shown. Find the value of velocity of centre of mass of sphere at the bottom most point of the incline after it reaches there doing pure rolling ($g = 10\ m/s^2$)



Options:

(a) 3 m/s

(b) 7 m/s

(c) 10 m/s

(d) 0.7 m/s

Answer: (c)

Solution:

$$V=\sqrt{\frac{2gh}{1+\frac{I}{MR^2}}}$$

$$h=\frac{1}{2}mv^2+\frac{1}{2}I\left(\frac{v^2}{R^2}\right)$$

$$v=\sqrt{\frac{2gh}{1+\frac{I}{MR^2}}}=\sqrt{\frac{2\times10\times7}{1+\frac{2}{5}}}$$

$$=\sqrt{\frac{20\times7\times5}{7}}=10\text{ms}^{-1}$$

Question: KE of a solid sphere is 4220 J on a horizontal plane. Find the velocity of COM of the sphere

Options:

(a) $40\sqrt{7}$

(b) $40\sqrt{3}$

(c) $30\sqrt{7}$

(d) $30\sqrt{3}$

Answer: (a)

Solution:

$$mgh=\frac{1}{2}mv^2+\frac{1}{2}I\left(\frac{V^2}{R^2}\right)$$

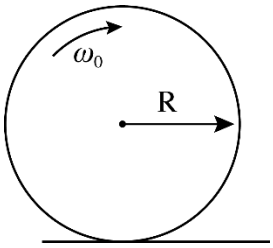
$$V=\sqrt{\frac{2gh}{1+\frac{I}{MR^2}}}=\sqrt{\frac{2\times10\times7}{1+\frac{2}{5}}}=\sqrt{\frac{20\times7\times5}{7}}=10\text{ ms}^{-1}$$

$$K=\frac{1}{2}mV^2+\frac{1}{2}\left[\frac{2}{5}MR^2\right]\times\frac{V^2}{R^2}$$

$$K=\frac{7}{10}MV^2=2240$$

$$V^2 = \frac{112 \times 100}{7} = 40 \text{ms}^{-1}$$

Question: A disk of radius R is given ω_0 angular speed and placed gently on a rough horizontal surface. Find the velocity of centre of disk when pure rolling starts.



Options:

(a) $\frac{R\omega_0}{3}$

(b) $R\omega_0$

(c) $\frac{R\omega_0}{4}$

(d) $2R\omega_0$

Answer: (a)

Solution:

$L_i = L_f$

$$\frac{MR^2}{2}\omega_0 = MRV + \frac{MRV}{2}$$

$$0 + \left[\frac{MR^2}{2}\omega_0\right] = (RMV) + \left[\frac{MR^2}{2}\omega\right](CLW)$$

$$\frac{MR^2}{2}\omega_0 = \frac{3}{2}MRV$$

$$V = \frac{R\omega_0}{3}$$

$\vec{L} = M\left(\vec{r}_{com} \times \vec{v}_{com}\right) + I\vec{\omega}_{com}$

JEE-Mains-29-01-2023 (Memory Based)
[Morning Shift]

Chemistry

Question: Which of the following is paramagnetic in nature?
Options:
(a) Na_2O_2
(b) KO_2
(c) Both (a) and (b)
(d) None of the above
Answer: (b)
Solution: The molecular electronic configuration of

$$O_2^- = (\sigma_{1s})^2 (\sigma_{1s}^*)^2 (\sigma_{2s})^2 (\sigma_{2s}^*)^2 (\sigma_{2p_z})^2 (\pi_{2p_x})^2 (\pi_{2p_y})^2 (\pi_{2p_x}^*)^2 (\pi_{2p_x}^*)^1$$

$$O_2^{2-} = (\sigma_{1s})^2 (\sigma_{1s}^*)^2 (\sigma_{2s})^2 (\sigma_{2s}^*)^2 (\sigma_{2p_z})^2 (\pi_{2p_x})^2 (\pi_{2p_y})^2 (\pi_{2p_x}^*)^2 (\pi_{2p_x}^*)^2$$

Since O_2^- has unpaired electron in its antibonding molecular orbital, thus it is paramagnetic and oppositely O_2^{2-} is diamagnetic.

Question: Arrange the following (increasing order of PKa)
Phenol, 2,4-dinitrophenol, 2,4,5-trimethyl phenol, 4-nitrophenol, 4-chlorophenol
Options:
(a) Phenol < 2,4-dinitrophenol < 2,4,5-trimethyl phenol < 4-nitrophenol < 4-chlorophenol
(b) 2,4-dinitrophenol < Phenol < 2,4,5-trimethyl phenol < 4-nitrophenol < 4-chlorophenol
(c) 2,4,5-trimethyl phenol < 4-chlorophenol < Phenol < 2,4-dinitrophenol < 4-nitrophenol
(d) 2,4-dinitrophenol < 4-nitrophenol < Phenol < 4-chlorophenol < 2,4,5-trimethyl phenol
Answer: (d)
Solution:

Compound	**Pka value**
Phenol	9.98
2,4-dinitrophenol	4.114
2,4,5-trimethyl phenol	10.57
4-nitrophenol	7.14
4-chlorophenol	10.07

Question: At low pressure Van der Waals equation will be
Options:
(a) $\left[1+\frac{a}{RTV}\right]$
(b) $\left[1-\frac{RTV}{a}\right]$

(c) $\left[1-\frac{a}{RTV}\right]$

(d) $\left[1+\frac{RTV}{a}\right]$

Answer: (c)

Solution:

$$pV+\frac{a}{v}=RT$$

$$\therefore \frac{pV}{RT}+\frac{a}{RTV}=1 \quad \therefore Z=\frac{pV}{RT}$$

$$=\left[1-\frac{a}{RTV}\right]$$

Question: Order of Hydration: Mg^{2+}, K^{+}, Rb^{+}, Cs^{+}, Ca^{2+}

Options:

(a) $Mg^{2+} > Ca^{2+} > K^{+} > Rb^{+} > Cs^{+}$

(b) $Mg^{2+} > Ca^{2+} > Cs^{+} > Rb^{+} > K^{+}$

(c) $Ca^{2+} > Mg^{2+} > Cs^{+} > Rb^{+} > K^{+}$

(d) $Cs^{+} > Mg^{2+} > K^{+} > Rb^{+} > Ca^{2+}$

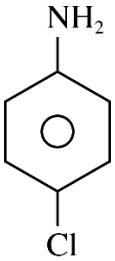
Answer: (a)

Solution: $Mg^{2+} > Ca^{2+} > K^{+} > Rb^{+} > Cs^{+}$

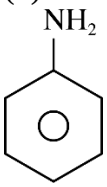
Question: Which compound will give both lassaigne test of nitrogen and halogen?

Options:

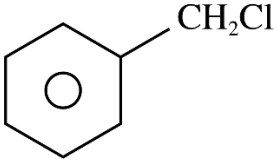
(a)



(b)



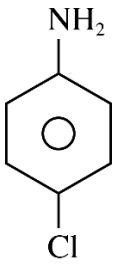
(c)



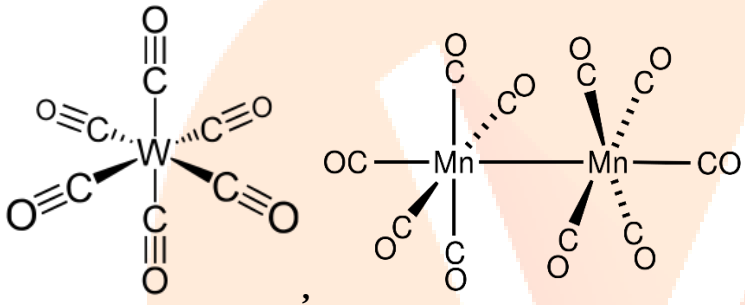
(d) None of these

Answer: (a)

Solution:



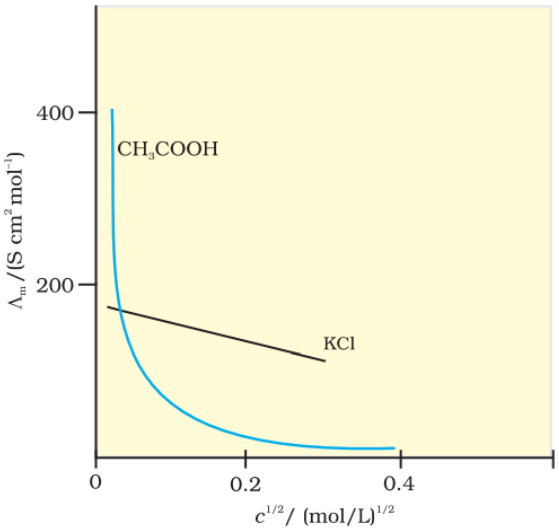
Question: Number of Bridging CO in $W(CO)_6$ and $Mn_2(CO)_{10}$
Options:
(a) 0
(b) 1
(c) 2
(d) 3
Answer: (a)
Solution:



Question: Order of bond dissociation enthalpy of Cl_2, Br_2, I_2, F_2
Options:
(a) $I_2 > Br_2 > Cl_2 > F_2$
(b) $Cl_2 > Br_2 > F_2 > I_2$
(c) $Br_2 > I_2 > F_2 > Cl_2$
(d) $F_2 > Cl_2 > Br_2 > I_2$
Answer: (b)
Solution: Fluorine which is an exception for this order, due to the high electronegativity of fluorine atom it tends to break the bond present between the fluorine molecule hence requiring less energy for the breaking of the bond which results in requiring low bond dissociation enthalpy.

Question: Which of the following ions does not liberate hydrogen gas on reaction with dilute acids?
Options:
(a) Cr^{2+}
(b) Mn^{2+}
(c) Ti^{2+}
(d) V^{2+}
Answer: (b)
Solution: Mn^{2+} does not liberate hydrogen gas on reaction with dilute acids. +2 oxidation state of Mn is very stable due to exactly half filled electronic configuration.

Question: Which of the following is incorrect statement about the following graph?



Options:

(a) λ^om value of weak electrolyte can be determined by extrapolating the graph

(b) λ^om value of weak electrolyte cannot be determined by extrapolating the graph

(c) λ^om value of strong electrolyte can be determined by extrapolating as linear graph is obtained

(d) All of these

Answer: (a)

Solution: For weak electrolyte molar conductivity at infinite dilution cannot be determined experimentally and by extrapolation because graph is not linear, maximum value of molar conductivity at infinite dilution cannot be obtained.

Question: Match the following.

Reaction (Column-I)	**Reagents (Column-II)**
(A) Hofmann Degradation	**(i)** Conc KOH
(B) Clemmensen reaction	**(ii)** NaOH, Br_2
(C) Cannizzaro reaction	**(iii)** Zinc-Hg, HCl
(D) Reimer-Tiemann reaction	**(iv)** $CHCl_3$, NaOH

Options:

(a) A – (ii); B – (i); C – (iii); D – (iv)

(b) A – (iii); B – (i); C – (ii); D – (iv)

(c) A – (iv); B – (iiii); C – (ii); D – (i)

(d) A – (ii); B – (iii); C – (i); D – (iv)

Answer: (d)

Solution:

Reaction (Column-I)	**Reagents (Column-II)**
(A) Hofmann Degradation	**(i)** NaOH, Br_2
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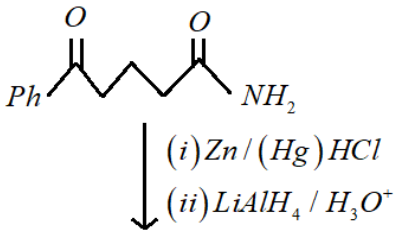
(C) Cannizzaro reaction	**(iii)** Conc KOH
(D) Reimer-Tiemann reaction	**(iv)** $CHCl_3$, NaOH

Question:



Options:

(a) $PhCOCH_2CH_2CH_2CH_2CH_2NH_2$

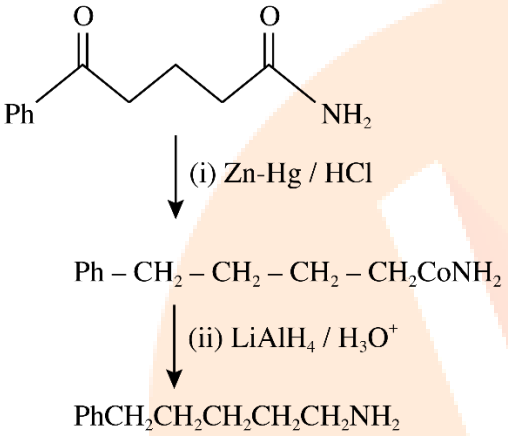
(b) $PhCH_2CH_2CH_2NH_2$

(c) $PhCH_2CH_2CH_2CH_2NH_2$

(d) $PhCH_2CH_2CH_2CH_2CH_2CH_2NH_2$

Answer: (d)

Solution:



Question: Number of millimoles of $Ca(OH)_2$ in 100 mL solution, given pH = 12

Options:

(a) 1

(b) 0.5

(c) 0.05

(d) 2

Answer: (b)

Solution:

pH = 12

pOH = 14 – Ph = 14 – 12 = 2

$\therefore [OH^-] = 10^{-2}$ M

$\therefore [Ca(OH)_2] = 0.5 \times 10^{-2}$ M

Mili moles of $Ca(OH)_2$ in solution

= MV (in mL)

$= 0.5 \times 10^{-2} \times 100 = 0.5$

Question: $K_f = 10^3$, $K_b = 10^2$ find ΔG^o at $\Delta T = 27^oC$

Options:

(a) –700 R

(b) –500 R

(c) –690 R

(d) –400 R

Answer: (c)

Solution:

$\Delta G^o = -2.303\ RT \log K_C$

$= -2.303\ R \times 300 \log \frac{K_F}{K_b}$

$= -2.303\ R \times 300 \log \frac{10^3}{10^2}$

$= -2.303\ R \times 300 \log_{10}$

$= -2.303\ R \times 300 \times 1 = -690\ R$

Question: Which of the following metalist is purified by Mond Process

Options:

(a) Ti

(b) Ge

(c) Ni

(d) Zr

Answer: (c)

Solution:

$$Ni + 4CO \xrightarrow{330-350K} Ni(CO)_4$$

$$Ni(CO)_4 \xrightarrow{450-470K} Ni + CO$$

Question: Which of the following is not odd electron?

$NO, NO_2, NO_2^+, ClO_2, BrCl_3$

Options:

(a) $BrCl_3$

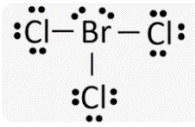
(b) NO

(c) NO_2

(d) NO_2^+ and ClO_2

Answer: (a)

Solution:



Question: Which of the following compound used as Narrow spectrum antibiotics?

Options:

(a) Penicillin

(b) Gentamycin

(c) Erythromycin

(d) All of these

Answer: (d)

Solution: Narrow spectrum antibiotics are used against only limited pathogens. Penicillin, gentamycin and erythromycin are used to treat only certain pathogens such as Staphylococci, Clostridia, Streptococci and hence fall under the class of narrow spectrum antibiotics.

Question: Select the correct statement among the following

Options:

(a) Photochemical smog has high concentration of oxidising agent
(b) Classical smog has high concentration of oxidising agent
(c) Classical smog contains NO_2
(d) None of these
Answer: (a)
Solution:
Smog: The word smog is derived from smoke and fog.
This is the most common example of air pollution that occurs in many cities throughout the world.
There are two types of smog:
(a) Classical smog occurs in cool humid climate. It is a mixture of smoke, fog and sulphur dioxide. Chemically it is a reducing mixture and so it is also called as reducing smog.
(b) Photochemical smog occurs in warm, dry and sunny climate. The main components of the photochemical smog result from the action of sunlight on unsaturated hydrocarbons and nitrogen oxides produced by automobiles and factories. Photochemical smog has high concentration of oxidising agents and is. therefore. called as oxidising smog.
Formation of photochemical smog, when fossil fuels are burnt, a variety of pollutants are emitted into the earth's

Question: In ostwald's process Ammonia is oxidized to give A and oxidised again to give B and forms an oxoacid of nitrogen. B gives positive brown ring test. What are A and B?
Options:
(a) A = NO, B = NO_2
(b) A = NO_2, B = NO
(c) A = HNO_2, B = NO
(d) A = HNO_2, B = NO_2
Answer: (b)
Solution: Ostwald's process for the manufacture of nitric acid.
$4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(g)$
$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$
$4NO_2(g) + 2H_2O(l) + O_2(g) \rightarrow 4HNO_3(aq)$

Question: Which of the following tank use to store hydrogen?
Options:
(a) High pressure tank
(b) Low pressure tank
(c) High temperature tank
(d) Low temperature tank
Answer: (a)
Solution: Hydrogen can be stored physically as either a gas or a liquid. Storage of hydrogen as a gas typically requires high-pressure tanks (350-700 bar [5,000-10,000 psi] tank pressure). Storage of hydrogen as a liquid requires cryogenic temperatures because the boiling point of hydrogen at one atmosphere pressure is -252.8°C.

Question: The compound formed in borax bead test, of $CuSO_4$ in oxidizing flame is _____
Options:
(a) $Cu(BO_2)_2$
(b) Cu_3B_2
(c) $Cu(BO)_2$
(d) $Cu(BO_3)_2$

Answer: (a)

Solution: $CuSO_4(aq) \xrightarrow{\Delta} CuO(s) + SO_3(g)$

$CuO(s) + B_2O_3(s) \rightarrow \underset{\text{Cupric metaborate}}{Cu(BO_2)_2}$

JEE-Main-29-01-2023 (Memory Based)
[Morning Shift]

Mathematics

Question: If $f(x+y)=f(x)+f(y), f(1)=\frac{1}{5}$ and $\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$, then find m.

Answer: 10.00

Solution:

$f(x+y)=f(x)+f(y)$

$f(x)=ax$

$x=1\Rightarrow f(1)=a=\frac{1}{5}$

$f(x)=\frac{1}{5}x$

$\frac{1}{5}\sum\frac{n}{n(m+1)(n+2)}=\frac{1}{12}$

$\frac{1}{5}\left[\sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right]=\frac{1}{12}$

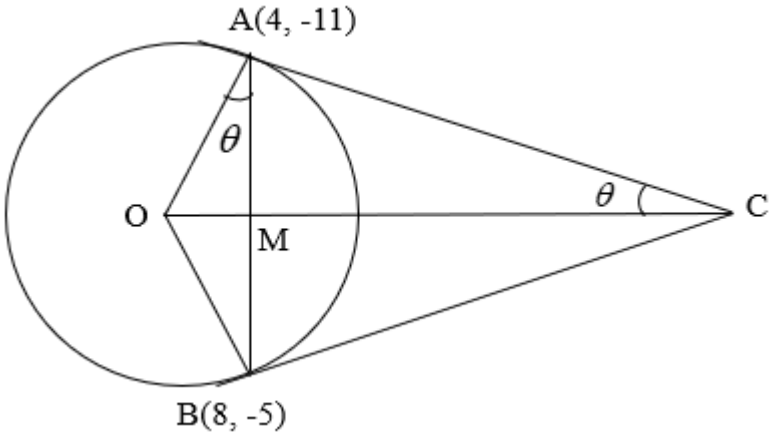
$\frac{1}{2}-\frac{1}{m+2}=\frac{5}{12}$

$\Rightarrow m=10$

Question: Tangent at $A(4,-11)$ and $B(8,-5)$ to $x^2+y^2-3x+10y-15=0$ intersect at C. Find the radius of circle touching AB and having centre at C.

Answer: $\frac{2\sqrt{13}}{3}$

Solution:



Radius $r = \sqrt{\frac{9}{4} + 25 + 15} = \frac{13}{2}$

$$AM = \frac{1}{2} AB$$
$$= \frac{1}{2}\sqrt{16 + 36}$$
$$= \sqrt{13}$$

$$\frac{CM}{AM} = \cot\theta$$
$$CM = \sqrt{13}\cot\theta$$

$$\Delta OAM \Rightarrow OM = \sqrt{\left(\frac{13}{2}\right)^2 - 13} = \sqrt{\frac{169 - 52}{4}} = \frac{\sqrt{117}}{2}$$

$$CM = \sqrt{13}\cot\theta$$
$$CM = \sqrt{13} \times \frac{\sqrt{13}}{\sqrt{117}} \times 2 = \sqrt{13} \times \frac{\sqrt{13}}{\sqrt{13} \times 3} \times 2 = \frac{2\sqrt{13}}{3}$$

Question: Five digit numbers are formed using 1, 2, 3, 5, 7 (repetition is allowed), and these numbers are arranged in descending order. Find the rank of 35337.

Answer: 1436.00

Solution:

Five digit number using 35337

$\underline{7}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^4$

$\underline{5}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^4$

$\underline{3}\ \underline{7}\ \underline{5}\ \underline{5}\ \underline{5} = 5^3$

$\underline{3}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^3$

$\underline{3}\ \underline{5}\ \underline{7}\ \underline{5}\ \underline{5} = 5^2$

$\underline{3}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^2$

$\underline{3}\ \underline{5}\ \underline{3}\ \underline{7}\ \underline{5} = 5$

$\underline{3}\ \underline{5}\ \underline{3}\ \underline{5}\ \underline{5} = 5$

$\underline{3}\ \underline{5}\ \underline{3}\ \underline{3}\ \underline{7} = 1$

So total = $5^4+5^4+5^3+5^3+5^2+5^2+5+5+1=1436$

Question: A function $f(x)$ is such that $f(x+y)=f(x)+f(y)-1,\ \forall x,y\in R$. If $f'(0)=2$, then $|f(-2)|=?$

Answer: 3.00

Solution:

$f(x+y)=f(x)+f(y)-1$

$x=y=0\Rightarrow f(0)=f(0)+f(0)-1\Rightarrow f(0)=1$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x)+f(h)-1-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)-1}{h}$$

$=f'(0)=2$

$f'(x)=2$

$f(x)=2x+c$

$1=0+c\Rightarrow c=1$

$\therefore\ f(x)=2x+1$

$|f(-2)|=|-3|=3$

Question: If the 3 consecutive coefficients in the expansion of $(1+2x)^n$ are in the ratio $2:5:8$ then the middle term is

Answer: ${}^8C_4(2x)^4$

Solution:

${}^nC_{r-1}(2)^{r-1}:{}^nC_r 2^r:{}^nC_{r+1}2^{r+1}::2:5:8$

$$\frac{{}^{n}C_{r}\,2^{r}}{{}^{n}C_{r-1}\,2^{r-1}}=\frac{5}{2}$$

$$2\left(\frac{n-r+1}{r}\right)=\frac{5}{2}$$

Similarly, $2\left(\frac{n-r}{r+1}\right)=\frac{8}{5}$

Comparing $n=8$

Mid term $={}^{8}C_{4}\left(2x\right)^{4}$

Question: If $\frac{dy}{y}=\left(\frac{x+1}{x^{2}}\right)dx;\ y(1)=e,$ then $\lim\limits_{x\to 0^{+}} f\left(x\right)=?$

Answer: 0.00

Solution:

$$\int\frac{dy}{y}=\int\left(\frac{1}{x}+\frac{1}{x^{2}}\right)dx$$

$$\ln|y|=\ln|x|-\frac{1}{x}+c$$

Given $(1,e)$

$$1=0-1+c$$

$$c=2$$

$\therefore$ We have

$$\ln|y|=\ln|x|-\frac{1}{x}+2$$

$$y=e^{\ln|x|-\frac{1}{x}+2}$$

As we take $\lim\limits_{x\to 0^{+}}$

$$y=e^{-\infty}=0$$

Question: α,β are positive numbers. A is a 3×3 matrix such that $A^{2}=3A+\alpha I$ and $A^{4}=21A+\beta I$. Find α,β.

Answer: -1, -8

Solution:

Given, $A^{2}=3A+\alpha I$ and $A^{4}=21A+\beta I$

$$A^{4}=A^{2}\cdot A^{2}$$

$A^4 = (3A+\alpha I)(3A+\alpha I) = 21A+\beta I$

$9A^2 + 3A\alpha I + 3\alpha AI + \alpha^2 I = 21A+\beta I$

$9A^2 + 6\alpha A + \alpha^2 I = 21A+\beta I$

Again using $A^2 = 3A+\alpha I$ in LHS

$\Rightarrow 9(3A+\alpha I) + 6\alpha A + \alpha^2 I = 21A+\beta I$

$\Rightarrow (27+6\alpha)A + (9\alpha+\alpha^2)I = 21A+\beta I$

$\therefore 27+6\alpha = 21 \quad \& \quad 9\alpha+\alpha^2 = \beta$

$6\alpha = -6 \qquad \& \qquad 1-9 = \beta$

$\alpha = -1 \qquad \& \qquad \beta = -8$

Question: Consider a function $f(x) = \dfrac{x^2+2x+1}{x^2+1}$, then which of the following is correct?

Options:

(a) $f(x)$ is one-one for $x \in (0,\infty)$

(b) $f(x)$ is one-one for $x \in (1,\infty)$

(c) $f(x)$ is one-one for $x \in (2,\infty)$ and many-one for $x \in (-\infty, 0]$

(d)

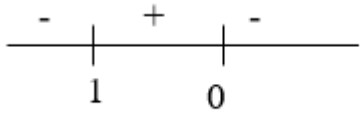
Answer: (c)

Solution:

$$f(x) = \frac{(x^2+1)+2x}{x^2+1} = 1+\frac{2x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)^2 - 2x(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{2-2x^2}{(x^2+1)^2}$$



Question: If real part of the product of z_1 & z_2 is zero i.e., $\mathrm{Re}(z_1 z_2) = 0$ & $\mathrm{Re}(z_1+z_2) = 0$ then $\mathrm{Im}(z_1)$ & $\mathrm{Im}(z_2)$ is

Answer: $\text{Im}(z_1)$ & $\text{Im}(z_2)$ are of opposite signs

Solution:

Given product of z_1 & z_2 is zero

i.e., $\text{Re}(z_1 \cdot z_2) = 0$

$$z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2)$$
$$= (x_1 x_2 - y_1 y_2)$$

$$\text{Re}(z_1 z_2) = x_1 x_2 - y_1 y_2 = 0$$

$$\Rightarrow x_1 x_2 = y_1 y_2 \quad ....(\text{i})$$

$$\text{Re}(z_1 + z_2) = (x_1 + iy_1) + (x_2 + iy_2)$$

$$\text{Re}(z_1 + z_2) = (x_1 + x_2) = 0$$

$$x_2 = -x_1$$

Substitute x_2 in (i)

$$-x_1 \cdot x_1 = y_1 y_2$$
$$\Rightarrow y_1 y_2 = -x_1^2 = -ve$$

$\text{Im}(z_1)$ & $\text{Im}(z_2)$ is one positive and one negative.

Question: If $a_1, a_2, a_3, ..., a_n$ is an increasing GP such that $a_4 \times a_6 = 9$ and $a_5 + a_7 = 12$. Find $a_7 + a_9 = ?$

Answer: 36.00

Solution:

Given $a_5 + a_7 = 12$ and $a_4 \times a_6 = 9$

$$a_5 + a_7 = 12$$
$$ar_4 + ar^6 = 12$$
$$ar^4 (1 + r^2) = 12 \quad ....(1)$$

$$a_4 \cdot a_6 = 9$$
$$ar^3 \cdot ar^5 = 9$$
$$a^2 \cdot r^8 = 9$$
$$a \cdot r^4 = 3$$

Substitute in (1)

$3\left(1+r^2\right)=12$

$r^2=3$

$r=\pm\sqrt{3}$

$a=\dfrac{1}{3}$

Now a_7+a_9

$\Rightarrow ar^6+ar^8=ar^4\left(r^2+r^4\right)$

$=3\times r^2\left(1+r^2\right)$

$=3\times 3(1+3)$

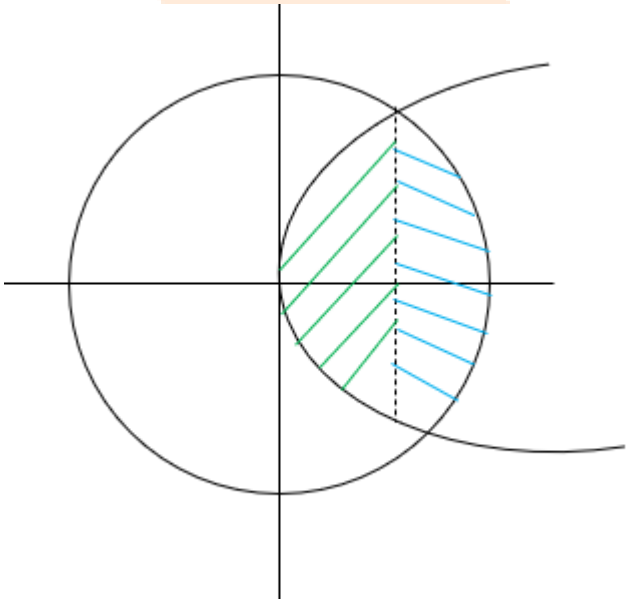
$=9\times 4$

$=36$

Question: Δ is the area between $x^2+y^2\le 21,\ y^2\le 4x$ and $x\ge 1$. Find $\dfrac{1}{2}\left(\Delta-21\sin^{-1}\left(\dfrac{2}{\sqrt{7}}\right)\right)$.

Answer: $\sqrt{3}-\dfrac{4}{3}$

Solution:



$x^2+y^2=21$

$y^2=4x$

$x=3$

$$2\times\int_1^3 2\sqrt{x}dx = 4\left[\frac{2}{3}x^{\frac{3}{2}}\Big|_1^3\right] = \frac{8}{3}\left(3\sqrt{3}-1\right)$$

$$2\int_3^{\sqrt{21}}\sqrt{21-x^2} = 2\left[\frac{1}{2}x\sqrt{21-x^2}+\frac{1}{2}\times 21\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{21}$$

$$= 0+21\frac{\pi}{2}3\times 2\sqrt{3}-21\sin^{-1}\left(\frac{\sqrt{3}}{\sqrt{7}}\right)$$

$$= 21\left(\cos^{-1}\sqrt{\frac{3}{7}}\right)-6\sqrt{3}$$

$$= 21\sin^{-1}\frac{2}{\sqrt{7}}-6\sqrt{3}$$

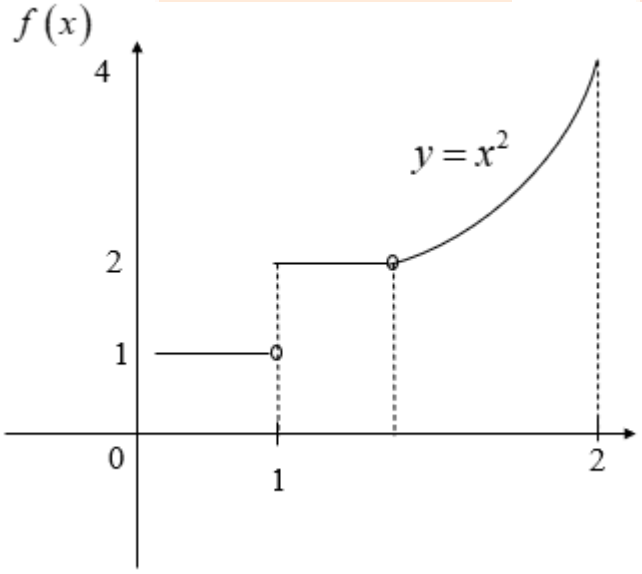
$$\Delta = \frac{8}{3}\left(3\sqrt{3}-1\right)+21\sin^{-1}\frac{2}{\sqrt{7}}-6\sqrt{3}$$

$$\Delta = \sqrt{3}-\frac{4}{3}$$

Question: $\int_0^2 \max\left\{x^2, 1+[x]\right\}dx$ is equal to

Answer: $\frac{5+4\sqrt{2}}{3}$

Solution:



$x^2 = 2 \Rightarrow x = \sqrt{2}$

$$(1\times1)+2\left(\sqrt{2}-1\right)+\int_{\sqrt{2}}^{2}x^2dx$$

$$=\frac{5+4\sqrt{2}}{3}$$

Question: 4 apples are picked one by one without replacing from a bag containing 3 rotten and 7 normal apples. Let x be no. of rotten apples. Find $\bar{x}+v_x$.

Answer: $\frac{6}{5},\frac{14}{25}$

Solution:

X	0	1	2	3
P(X)	$\frac{{}^{7}C_4}{{}^{10}C_4}$	$\frac{{}^{3}C_1\times{}^{7}C_4}{{}^{10}C_4}$	$\frac{{}^{3}C_2\times{}^{7}C_2}{{}^{10}C_4}$	$\frac{{}^{7}C_1}{{}^{10}C_4}$

$$\bar{x}=\sum x_iP_i$$

$$\bar{x}=\frac{6}{5}$$

Variance = $V=\sum x_i^2P_i-\left(\bar{x}\right)^2$

$$V=\frac{14}{25}$$

Question: Domain of $f\left(x\right)=\frac{\log_x\left(x-1\right)}{\log_{x-1}\left(x-4\right)}$ is:

Answer: $x\in\left(4,\infty\right)\setminus\left\{5\right\}$

Solution:

For domain

$x>0,\ x-1>0,\ x\neq1$

& $x-1>0,\ x-1\neq1, x-4>0$

$\log_{x-1}\left(x-4\right)\neq0$

$\Rightarrow x-4\neq1\Rightarrow x\neq5$

$\therefore x\in\left(4,\infty\right)-\left\{5\right\}$

Question: If the coefficient of x^5 in the expansion of $\left(ax^3+\frac{1}{\beta x}\right)^{11}$ and $\left(\alpha x+\frac{1}{\beta x^3}\right)^{11}$ are equal, then the value of $(\alpha+\beta)^2$ is

Answer: 1.00

Solution:

General term of $\left(\alpha x^3+\frac{1}{\beta x}\right)^{11}$ is

$$T_{k+1}={}^{11}C_k\left(\alpha x^3\right)^{11-k}\left(\frac{1}{\beta x}\right)^k$$

$$={}^{11}C_k\alpha^{11-k}\beta^{-k}x^{33-4k}$$

Now for coefficient of x^9, we have

$$33-4k=9$$
$$\Rightarrow 4k=24$$
$$\Rightarrow k=6$$

Similarly, general term of $\left(\alpha x+\frac{1}{\beta x^3}\right)^{11}$ is

$$T_{k+1}={}^{11}C_k\left(\alpha x\right)^{11-k}\left(\frac{1}{\beta x^3}\right)^k$$
$$={}^{11}C_k\alpha^{11-k}\beta^{-k}x^{11-4k}$$

For coefficient of x^{-9}, we have

$$11-4k=9$$
$$\Rightarrow 4k=20$$
$$\Rightarrow k=5$$

$${}^{11}C_6\frac{\alpha^5}{\beta^6}={}^{11}C_5\cdot\frac{\alpha^6}{\beta^5}$$
$$\Rightarrow \alpha\beta=\frac{{}^{11}C_6}{{}^{11}C_5}=1$$
$$\Rightarrow (\alpha\beta)^2=1$$

Question: Consider 3 coplanar vector $\vec{a}=3\hat{i}-4\hat{j}+\lambda\hat{k}$, $\vec{b}=4\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}-4\hat{k}$. Then 9λ is __

Answer: 87.00

Solution:

$$\begin{vmatrix} 3 & -4 & \lambda \\ 4 & 3 & -1 \\ 1 & 3 & -4 \end{vmatrix} = 0$$

$$-27 - 60 + 9\lambda = 0$$

$$9\lambda = 87$$