PART : MATHEMATICS

1. If $f(x+y) = f(x) + f(y) - 1$ and $f'(0) = 2$, then $|f(-2)|$ is equal to

Ans. 3

Sol. Partial differential w.r.t. x

$f'(x+y) = f'(x)$

put $x=0$

$f'(y) = f'(0)=2$

$\Rightarrow f(y) = 2y + c$

$f(x) = 2x + c$

Now put $x = y = 0 \Rightarrow f(0)=1$

$\Rightarrow f(0) = 0 + c$

$\Rightarrow c = 1$

$\Rightarrow f(x) = 2x + 1$

$\Rightarrow |f(-2)| = 3$

2. If $y(x+1)dx - x^2 dy = 0$ and $y(1) = e$ then find the value of $\lim_{x \to 0^+} y$

Ans. 00.00

Sol. $\frac{x+1}{x^2}dx = \frac{dy}{y}$

$\ln x - \frac{1}{x} = \ln y + c$

$0 - 1 = 1 + c \Rightarrow c = -2$

$\ln x - \frac{1}{x} = \ln y - 2$

$\ln y = \ln x - \frac{1}{x} + 2$

$y = x.e^{2-\frac{1}{x}}$

$\lim_{x \to 0^+} y = 0$

3. $f(x) = \max(x^2, 1+[x])$, where [ ] represents GIF, then $\int_0^2 f(x)dx$ is equal to

(1) $\frac{5}{3} + \frac{4\sqrt{2}}{3}$ (2) $\frac{2}{3} + \frac{4\sqrt{2}}{3}$ (3) $\frac{5}{3} + \frac{-4\sqrt{2}}{3}$ (4) $\frac{2}{3} + \frac{-4\sqrt{2}}{3}$

Ans. (1)

Sol. $\int_0^2 f(x)dx = \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^2 x^2 dx$

$1 + 2(\sqrt{2} - 1) + \frac{8 - 2\sqrt{2}}{3} = \frac{5}{3} + \frac{4\sqrt{2}}{3}$

4. If points A(4, –11) and B(8, –5) lie on a circle $x^2+y^2-3x+10y-15=0$. The tangents drawn at these points A and B intersect at C. then the radius of the circle drawn with centre C and the line joining A & B as its tangent, is

(1) $\frac{\sqrt{52}}{3}$ (2) $\frac{\sqrt{52}}{2}$ (3) $\frac{\sqrt{52}}{9}$ (4) $\frac{\sqrt{52}}{5}$

Ans. (1)

Sol. Equation of line AB is $y+5=\frac{3}{2}(x-8)$

$2y+10=3x-24$

$\Rightarrow 3x-2y-34=0$ .... (1)

let C be (h, k) then equation of AB

$hx+ky-\frac{3}{2}(x+h)+5(y+k)-15=0$

$x\left(h-\frac{3}{2}\right)+y(k+5)-\frac{3}{2}h+5k-15=0$ ..... (2)

comparing (1) and (2)

$\frac{h-\frac{3}{2}}{3}=\frac{k+5}{-2}=\frac{\frac{-3}{2}h+5k-15}{-34}$

$(h, k)=\left(8,\frac{-28}{3}\right)$

equation is $(x-8)^2+\left(y+\frac{28}{3}\right)^2=\frac{52}{9}$

5. If $f(x+y)=f(x)+f(y)\ \forall x,y\in R$, $f(1)=\frac{1}{5}$ and $\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$, then find the value of m

Ans. (m = 10)

Sol. $f(x+y)=f(x)+f(y)\Rightarrow f(x)=ax$

$f(1)=\frac{1}{5}\Rightarrow a.1=\frac{1}{5}\Rightarrow a=\frac{1}{5}$

$f(n)=\frac{n}{5}$

$\therefore \sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}\Rightarrow\sum_{n=1}^{m}\frac{(n)}{5n(n+1)(n+2)}=\frac{1}{12}$

$\left(\sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right)=\frac{5}{12}$

$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots\ldots+\left(\frac{1}{m+1}-\frac{1}{m+2}\right)=\frac{5}{12}$

$\frac{1}{2}-\frac{1}{m+2}=\frac{5}{12}\Rightarrow\frac{1}{2}-\frac{5}{12}=\frac{1}{m+2}$

$\frac{2}{2\times 12}=\frac{1}{m+2}\Rightarrow m+2=12\Rightarrow m=10$

6. If real part of the product of z_1 & z_2 is zero i.e $Re(z_1.z_2) = 0$ & $Re(z_1 + z_2) = 0$, then,

(a) $Im(z_1) > 0$, $Im(z_2) > 0$ (b) $Im(z_1) < 0$, $Im(z_2) > 0$

(c) $Im(z_1) > 0$, $Im(z_2) < 0$ (d) $Im(z_1) < 0$, $Im(z_2) < 0$

Now which is correct option

(1) a and b (2) b and c (3) c and d (d) a and d

Ans. (2)

Sol. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$x_1x_2 - y_1y_2 = 0$ ....(i)

$x_1 + x_2 = 0$ ....(ii)

$x_1^2 + y_1y_2 = 0$

$y_1y_2 = -x_1^2$

$\Rightarrow Im(z_1)$ and $Im(z_2)$ are opposite in sign.

7. 3 rotten apples are mixed with 7 good apples. Now 4 apples are chosen at random without replacement. If random variable x represents number of rotten apples drawn, then the value of $10(\mu + \sigma^2)$, where μ is mean and σ^2 is variance of the probability distribution for the number of rotten apples drawn, is

(1) $\frac{90}{5}$ (2) $\frac{88}{5}$ (3) $\frac{70}{5}$ (4) $\frac{60}{5}$

Ans. (2)

Sol.

x_i	0	1	2	3
p_i	$\frac{35}{210} = \frac{1}{6}$	$\frac{105}{210} = \frac{1}{2}$	$\frac{3 \times 21}{210} = \frac{3}{10}$	$\frac{7}{210} = \frac{1}{30}$

$\mu = \Sigma p_i x_i = \frac{1}{2} \times 1 + \frac{3}{10} \times 2 + \frac{1}{30} \times 3$

$= \frac{1}{2} + \frac{3}{5} + \frac{1}{10} = \frac{5+6+1}{10} = \frac{6}{5}$

$\sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{1}{2} + \frac{3}{10} \times 4 + \frac{1}{30} \times 9 - \frac{36}{25} = \frac{14}{25}$

$10(\mu + \sigma^2) = 10\left(\frac{6}{5} + \frac{14}{25}\right)$

$= 10\left(\frac{44}{25}\right)$

$= \frac{88}{5}$

8. Five digit number is formed by using the digits 1,2,3,5,7 with repetition of digits is allowed. If all such

8. Five digit number is formed by using the digits 1,2,3,5,7 with repetition of digits is allowed. If all such number are written in descending order then what is the position of the number 35337.

Ans. **(1436)**

Sol. Number of numbers starting with 7 $\rightarrow$ 625

Number of numbers starting with 5 $\rightarrow$ 625

Number of numbers starting with 37 $\rightarrow$ 125

Number of numbers starting with 357 $\rightarrow$ 25

Number of numbers starting with 355 $\rightarrow$ 25

Number of numbers starting with 3537 $\rightarrow$ 5

Number of numbers starting with 3535 $\rightarrow$ 5

Number of numbers starting with 35337 $\rightarrow$ 1

1436

The position of the number 35337 is 1436

9. Consider a function $f(x) = \frac{2x^2+x+1}{x^2+1}$, then which of following is correct

(1) f(x) is one-one $\forall x \in (0,2)$

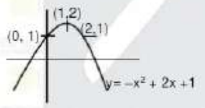
(2) f(x) is many-one $\forall x \in (0,2)$

(3) f(x) is one-one $\forall x \in (0,\infty)$

(4) f(x) one-one $\forall x \in (1,\infty)$

Ans. **(1)**

Sol.



$$f'(x) = \frac{(x^2+1)(4x+1)-(2x^2+x+1)(2x)}{(x^2+1)^2}$$

$$= \frac{4x^3+4x+x^2+1-4x^3-2x^2-2x}{(x^2+1)^2}$$

$$= \frac{-x^2+2x+1}{(x^2+1)^2} > 0 \quad \forall x \in (0,2)$$

So, f(x) is one–one $\forall x \in (0, 2)$

10. In the expansion of $\left(\alpha x - \frac{1}{\beta x}\right)^{11}$, if coefficient of x^9 is equal to coefficient of x^{-9}, then the value of $(\alpha\beta)^2$

Ans. (1)

Sol. $T_{r+1} = {}^{11}C_r (\alpha x)^{11-r}(-\beta x)^{-r} = {}^{11}C_r \alpha^{11-r}.\beta^{-r}.x^{11-2r}.(-1)^{r}$

coefficient of $x^8 = -{}^{11}C_1 \alpha^{10}\beta^{-1}$

coefficient of $x^{-9} = {}^{11}C_{10}\alpha.\beta^{-10}$

$\Rightarrow -{}^{11}C_1.\alpha^{10}.\beta^{-1} - {}^{11}C_{10}\alpha.\beta^{-10}$

$\Rightarrow (\alpha\beta)^9 = -1$

$\Rightarrow (\alpha\beta)^2 = 1$

11. If a_1, a_2, a_3 .... are positive numbers forming G.P. such that $a_5 + a_7 = 12$ and $a_4.a_6 = 9$, then $a_1 a_9 + a_2 a_8 + a_5 + a_7$ is equal to

(1) 10 (2) 20 (3) 30 (4) 15

Ans. (3)

Sol. $a_5 + a_7 = 12$ $a_4.a_6 = 7$

$ar^4 + ar^6 = 12$ $ar^3.ar^5 = 9$

$ar^4(1 + r^2) = 12$ ....(i) $a^2r^8 = 9$

$ar^4 = 3$ ......(ii)

Now, $ar^4(1 + r^2) = 12$ $ar^4 = 3$

$3(1 + r^2) = 12$ $a(\sqrt{3})^4 = 3$

$r = \pm\sqrt{3}$ $\Rightarrow$ $a = \frac{1}{3}$

$\Rightarrow$ $r = \sqrt{3}$

Now, $a_1.a_9 + a_2.a_8 + a_5 + a_7$

$= a.ar^8 + ar.ar^7 + ar^4 + ar^6$

$= a^2r^8 + a^2r^8 + ar^4 + ar^6$

$= \left(\frac{1}{9}\right)(81) + \left(\frac{1}{9}\right)(81) + \left(\frac{1}{3}\right) \times 9 + \left(\frac{1}{3}\right) \times 27$

$= 9 + 9 + 3 + 9 = 30$

12. Let $x_1 x_2 x_3 x_4 x_5 x_6$ be a six digit number such that $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$. If these six digits numbers are arranged in ascending order, then the number at 72th position is

Ans. (245678)

Sol. Number of numbers starting with $1 = {}^8C_5 = 56$

Number of numbers starting with $23 = {}^6C_4 = 15$

Next number at 72nd position is 245678.

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13. Let $A = [a_{ij}]_{n \times n}$ be a matrix and I is an identity matrix of order n

If $A^2 = A + \alpha I$ and

$A^4 = 21A + \beta I$ then scalars α and β is

(1) $\alpha = 110$, $\beta = 10$ (2) $\alpha = 10$, $\beta = 110$ (3) $\alpha = 10$, $\beta = 100$ (4) $\alpha = 100$, $\beta = 10$

Ans. (2)

Sol. $A^2A^2 = (A+\alpha I).(A+\alpha I)$

$A^4 = A^2 + 2\alpha A + \alpha^2 I$

$A^4 = A + \alpha I + 2\alpha A + \alpha^2 I$

$A^4 = (1+2\alpha) A + (\alpha+\alpha^2) I$

{given $A^4 = 21A + \beta I$}

$1+2\alpha = 21$, $\quad \alpha + \alpha^2 = \beta$

$\Rightarrow \alpha = \frac{20}{2}$, $\quad 10 + 100 = \beta$

$\Rightarrow \alpha = 10$, $\quad \beta = 110$

14. If $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2/nx} - (2x+3)}$ then domain of f(x) is

(1) $(0, 1)$ (2) $(2, \infty)$ (C) $(1, \infty)$ (4) $(2, \infty) - \{3\}$

Ans. (4)

Sol. (i) $x - 2 > 0$

$x > 2$

$x \in (2, \infty)$

(ii) $x + 1 > 0$ & $x + 1 \neq 1$

$\Rightarrow x > -1$ $\quad \Rightarrow x \neq 0$

$x \in (-1, 0) \cup (0, \infty)$

(iii) $x > 0$

$x \in (0, \infty)$

(iv) $e^{2/nx} - (2x+3) \Rightarrow x^2 - 2x - 3 \neq 0$

$\Rightarrow x^2 - 3x + x - 3 \neq 0$

$\Rightarrow x(x-3) + 1(x-3) \neq 0$

$\Rightarrow x \neq 3, x \neq -1$

from (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$x \in (2, \infty) - \{3\}$

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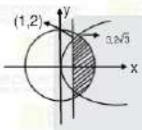
15 The area of region $\{(x,y) : x^2 + y^2 \leq 21, x \geq 1 \ \& \ y^2 \leq 4x\}$ is

15 The area of region $\{(x,y) : x^2+y^2 \le 21, x \ge 1 \ \& \ y^2 \le 4x\}$ is

(1) $8\sqrt{3}-\frac{8}{3}+\frac{21}{2}-\frac{21}{2}\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$
(2) $2\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$
(3) $8\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}$
(4) $8\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{11}}\right)$

Ans. (2)

Sol.



Required area $= 2\left(\int_1^3 2\sqrt{x}\,dx + \int_3^{\sqrt{21}} \sqrt{21-x^2}\,dx\right) = 2\left(2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^3 + \left[\frac{x}{2}\sqrt{21-x^2}+\frac{21}{2}\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{\sqrt{21}}\right)$

$=2\left(\frac{4}{3}(3\sqrt{3}-1)+\frac{21\pi}{4}-\frac{3}{2}\times 2\sqrt{3}-\frac{21}{2}\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)\right) = 8\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-6\sqrt{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

$=2\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

16. Equation $14x^2-31x+3\lambda=0$ having roots α,β and $14x^2-38x+4\lambda=0$ having roots α,γ where $\alpha\ne 0$, then the quadratic equation having roots $\frac{3\alpha}{\beta},\frac{4\alpha}{\gamma}$ is

(1) $49x^2-140x+100=0$
(2) $49x^2-70x+100=0$
(3) $49x^2+140x+100=0$
(4) $49x^2+70x+100=0$

Ans. (1)

Sol. $14\alpha^2-31\alpha+3\lambda=0$ ......(i)

$14\alpha^2-38\alpha+4\lambda=0$ ......(ii)

(i) – (ii)

We get

$7\alpha-\lambda=0$

$\alpha=\frac{\lambda}{7} \Rightarrow \frac{14\lambda^2}{49}-31.\frac{\lambda}{7}+3\lambda=0 \Rightarrow \lambda=0$ or 5

$\because \alpha \ne 0$

$\Rightarrow \lambda=5$ only $\Rightarrow \alpha=\frac{5}{7} \quad \Rightarrow \beta=\frac{3}{2}, \ \gamma=2$

equation having roots $\left(\frac{3\alpha}{\beta},\frac{4\alpha}{\gamma}\right)=\left(\frac{10}{7},\frac{10}{7}\right)$

is $(7x-10)^2-0 \Rightarrow 49x^2-140x+100-0$

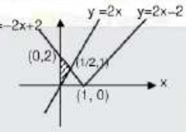
17. If A_1 is the area bounded by $2x \le y \le \sqrt{4(x-1)^2}$ in 1^{st} quadrant & A_2 is the area bounded by

17. If A_1 is the area bounded by $2x \le y \le \sqrt{4(x-1)^2}$ in 1st quadrant & A_2 is the area bounded by $y = \min\left(2x, \sqrt{4(x-1)^2}\right)$ and x-axis. Find $\frac{A_1}{A_2}$

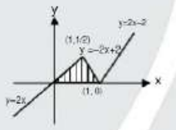
(1) 1/4 (2) 4 (3) 2 (4) 1

Ans. (4)

Sol.



$A_1 = \frac{1}{2}(2)\left(\frac{1}{2}\right) = \frac{1}{2}$



$A_2 = \frac{1}{2}(1)(1) = \frac{1}{2}$ so $\frac{A_1}{A_2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

18. If the coefficients of three consecutive terms in the expansion of $(1+2x)^n$ $(n \in N)$ are in the ratio 2:5:8, then the middle term in the expansion of $(1+2x)^n$ is

(1) ${}^8C_4(2x)^4$ (2) ${}^5C_4(2x)^4$ (3) ${}^{10}C_4(2x)^4$ (4) ${}^6C_4(2x)^4$

Ans. (1)

Sol. Let three consecutive terms, T_r, T_{r+1}, T_{r+2} are

${}^nC_{r-1}(2)^{r-1} : {}^nC_r 2^r : {}^nC_{r+1}2^{r+1} :: 2:5:8$

$\Rightarrow \frac{{}^nC_r(2)^r}{{}^nC_{r-1}2^{r-1}} = \frac{5}{2} \Rightarrow 2\left(\frac{n-r+1}{r}\right) = \frac{5}{2}$ ....(i)

Similarly $2\left(\frac{n-r}{r+1}\right) = \frac{8}{5}$ ....(ii)

from (i) and (ii) n = 8

Hence middle term is 5th term i.e ${}^8C_4(2x)^4$

19. If there are 15 players in a football team and 15 T-shirts are made with their name written on it. If each

19. If there are 15 players in a football team and 15 T-shirts are made with their name written on it. If each player randomly picks up a T-shirt one by one, then the probability that atleast 13 players pick the right T-shirt?

(1) $\frac{106}{15!}$ (2) $1-\left(\frac{106}{15!}\right)$ (3) $\frac{105}{15!}$ (4) $1-\left(\frac{105}{15!}\right)$

Ans. (1)

Sol. Total number of ways of selecting T-shirts = 15!

The number of ways of selecting T- shirts by at least 13 players = Number of ways of selecting T-shirt by 13 players or by 14 players or by 15 players.

$= {}^{15}C_{13} \times (1) + {}^{15}C_{14} \times (0) + {}^{15}C_{15} \times (1)$

$= {}^{15}C_2 + 0 + 1 = 15 \times 7 + 1 = 105 + 1 = 106$

Required probability is equal to $\frac{106}{15!}$

20. If $x = 2$ is a root of $x^2 + px + q = 0$ and $f(x) = \begin{cases} \frac{1-\cos(x^2 - 4px + q^2 + 8q + 16)}{(x-2p)^2} & : x \neq 2p \\ 0 & : x = 2p \end{cases}$

then $\underset{x \to 2p}{\text{Lt}} f(x)$ is

(1) $\frac{-1}{2}$ (2) 0 (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. Put $x = 2 \Rightarrow 4 + p.2 + q = 0$

$x^2 - 2.x.2p + q^2 + 2.q.4 + 4^2$

$= x^2 - 2.x.2p + (q+4)^2$

$= x^2 - 2.x.2p + (-2p)^2 = (x-2p)^2$

$\underset{x \to 2p}{\text{Lt}} f(x) = \underset{x \to 2p}{\text{Lt}} = \frac{1-\cos((x-2p)^2)}{(x-2p)^2}$

Let $x - 2p = \theta$

$\underset{x \to 2p}{\text{Lt}} f(x) = \underset{\theta \to 0}{\text{Lt}} = \frac{1-\cos(\theta)^2}{(\theta)^2}$

$\underset{x \to 2p}{\text{Lt}} f(x) = \underset{\theta \to 0}{\text{Lt}} = \frac{1-\left(1-\frac{(\theta^2)^2}{2!}+\frac{(\theta^2)^4}{4!} \ldots\ldots\right)}{(\theta)^2}$

$\underset{x \to 2p}{\text{Lt}} f(x) = \underset{\theta \to 0}{\text{Lt}} \frac{\theta^2}{2!} - \frac{\theta^4}{4!} \ldots\ldots$

$\underset{x \to 2p}{\text{Lt}} f(x) = 0$

21. If is given that $(((p \wedge q) \vee r) \vee (p \wedge r)) \to ((\sim q) \vee r)$ is fallacy. Then truth value of p, q, r are given by

(1) p : true, q : false, r : false (2) p : false, q : false, r : false

(3) p : true, q : true, r : false (4) p : true, q : false, r : true

Ans. (3)

Sol. $s \to m$ is fallacy

so truth value of s must be T and truth value of m must be F

$\Rightarrow$ m is F $\Rightarrow$ q is T and r is F

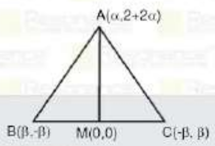
so p must be T

22. In an equilateral ΔABC, point A lies on the line $y - 2x = 2$ and points B and C are lying on the line $y + x = 0$. Points B and C are symmetric w.r.t. origin. The area of ΔABC (in sq. units) is

(1) $4\sqrt{3}$ (2) 8 (3) $\frac{8}{\sqrt{3}}$ (4) $8\sqrt{3}$

Ans. (3)

Sol. Slope of AM $= \frac{2+2\alpha}{\alpha} = 1$



$\Rightarrow \alpha = -2$

$AM = 2\sqrt{2}$

$AB = 2\sqrt{2} \times \frac{2}{\sqrt{3}} = \frac{4\sqrt{2}}{\sqrt{3}}$

area $= \frac{\sqrt{3}}{4}(AB)^2 = \frac{\sqrt{3}}{4} \times \frac{32}{3} = \frac{8}{\sqrt{3}}$

23. A ray along $y = \frac{x}{\sqrt{3}}$ is incident on a surface $x + y = 1$ on xy – plane. The point of intersection of reflected ray with x –axis is

(1) $\left(\frac{1}{\sqrt{3}}, 0\right)$ (2) $\left(1 - \frac{1}{\sqrt{3}}, 0\right)$ (3) $\left(\frac{2}{\sqrt{3}}, 0\right)$ (4) $\left(1 + \frac{1}{\sqrt{3}}, 0\right)$

Ans. (2)

Sol.

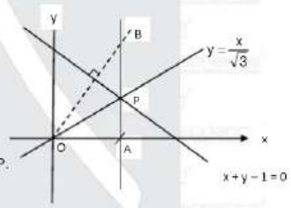


Image of O (0 ,0) in line $x + y - 1 = 0$ lies on reflected ray.

$\frac{x-0}{1} = \frac{y-0}{1} = \frac{-2(0+0-1)}{2} \Rightarrow B(1, 1).$

Also, upon solving we obtain $P = \left(\frac{3-\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$

equation of reflected ray is same as line passing through BP.

slope $= \frac{\frac{\sqrt{3}}{2} - \frac{1}{2} - 1}{\frac{3}{2} - \frac{\sqrt{3}}{2} - 1} = \frac{\sqrt{3}-3}{1-\sqrt{3}} = \sqrt{3}$

Equation of line BP is

$y - 1 = \sqrt{3}(x - 1)$

Put $y = 0 \Rightarrow -\frac{1}{\sqrt{3}} = x - 1$

Required point $\left(1 - \frac{1}{\sqrt{3}}, 0\right)$