

**NARAYANA GRABS
THE LION'S SHARE IN JEE-ADV.2022**

**5 RANKS in OPEN CATEGORY
ONLY FROM NARAYANA
IN TOP 10 AIR**



JEE MAIN (JAN) 2023 (25-01-2023-Session-2)
Memory Based Question Paper
MATHEMATICS



MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\sum_{k=0}^6 {}^{51-k}C_3$ is
 (1) ${}^{52}C_4 - {}^{46}C_4$ (2) ${}^{52}C_4 - {}^{45}C_4$
 (3) ${}^{51}C_4 - {}^{45}C_4$ (4) ${}^{51}C_4 - {}^{46}C_4$

Answer (2)

$$\begin{aligned} \text{Sol. } & {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 \dots \dots \underbrace{{}^{45}C_3 + {}^{45}C_4}_{46C_4} - {}^{45}C_4 \\ & {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + \underbrace{{}^{46}C_3 + {}^{46}C_4}_{47C_4} - {}^{45}C_4 \\ & \vdots \\ & \Rightarrow {}^{52}C_4 - {}^{45}C_4 \end{aligned}$$

2. If $f(x) = 2x^n + \lambda$ and $f(4) = 133$, $f(5) = 255$, then sum of positive integral divisors of $f(3) - f(2)$ is
 (1) 60 (2) 22
 (3) 40 (4) 6

Answer (1)

$$\text{Sol. } 2 \cdot 4^n + \lambda = 133 \quad \dots \text{(i)}$$

$$2 \cdot 5^n + \lambda = 255 \quad \dots \text{(ii)}$$

$$2(5^n - 4^n) = 122$$

$$n = 3$$

$$f(x) = 2x^3 + \lambda$$

$$f(3) - f(2) = 2(3^3 - 2^3)$$

$$= 38$$

$$\text{Divisors} = 1, 2, 19, 38$$

3. If $\left| \frac{z+2i}{z-i} \right| = 2$ is a circle, then centre of the circle is
 (1) (0, 0) (2) (0, 2)
 (3) (2, 0) (4) (-2, 0)

Answer (2)

$$\text{Sol. } (z+2i)(\bar{z}-2i) = 4(z-i)(\bar{z}+i)$$

$$\Rightarrow z\bar{z} + 2i\bar{z} - 2iz + 4 = 4z\bar{z} - 4i\bar{z} + 4iz + 4$$

$$\Rightarrow 3z\bar{z} - 6i\bar{z} + 6iz = 0$$

$$\Rightarrow z\bar{z} - 2i\bar{z} + 2iz = 0$$

$$\therefore \text{Centre} = 2i \text{ i.e., } (0, 2)$$

4. If $\frac{dy}{dt} + \alpha \cdot y = \gamma \cdot e^{-\beta t}$, then $\lim_{t \rightarrow \infty} y(t)$,

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\alpha \neq \beta$, is equal to

- (1) 0 (2) 1
 (3) Does not exist (4) $\alpha\beta$

Answer (1)

$$\text{Sol. } \frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$$

Integrating factor (I.F.) = $e^{\alpha t}$

Solution of D.E.

$$ye^{\alpha t} = \gamma \int e^{(\alpha-\beta)t} dt$$

$$\Rightarrow ye^{\alpha t} = \frac{\gamma}{\alpha - \beta} \cdot e^{(\alpha-\beta)t} + C$$

$$\Rightarrow y(t) = \frac{\gamma}{\alpha - \beta} \cdot \alpha^{-\beta t} + C \cdot e^{-\alpha t}$$

$$\lim_{t \rightarrow \infty} y(t) = 0$$

5. If $(p \rightarrow q) \nabla (p \Delta q)$ is tautology, then operator ∇, Δ denotes

- | | |
|--|---|
| (1) $\Delta \rightarrow \text{OR}$
$\nabla \rightarrow \text{AND}$ | (2) $\Delta \rightarrow \text{AND}$
$\nabla \rightarrow \text{OR}$ |
| (3) $\Delta \rightarrow \text{AND}$
$\nabla \rightarrow \text{AND}$ | (4) $\Delta \rightarrow \text{OR}$
$\nabla \rightarrow \text{OR}$ |

Answer (4)

$$\text{Sol. } (p \rightarrow q) \nabla (p \Delta q) \equiv T$$

Only if ∇ is OR and Δ is OR

6. Number of numbers between 5000 and 10000 by using the digits 1, 3, 5, 7, 9 without repetition is equal to

- (1) 120 (2) 72
 (3) 12 (4) 6

Answer (2)

11. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$ such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$ then $\vec{a} - 6\vec{b}$ equals
- $3(\hat{i} + \hat{j} + \hat{k})$
 - $\hat{i} + \hat{j} + \hat{k}$
 - $2(\hat{i} + \hat{j} + \hat{k})$
 - $4(\hat{i} + \hat{j} + \hat{k})$

Answer (1)

Sol. $(\vec{a} \times \vec{b}) = \hat{i} - \hat{j}$

$$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = (-\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} - \hat{j})$$

$$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\Rightarrow 3\vec{b} = -2\hat{i} - 2\hat{j} - \hat{k}$$

$$\therefore \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$= 3(\hat{i} + \hat{j} + \hat{k})$$

12. $16 \int_{-1}^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to
- $\frac{11}{12} + \ln 4$
 - $\frac{11}{12} - \ln 4$
 - $\frac{11}{6} - \ln 4$
 - $\frac{11}{6} + \ln 4$

Answer (3)

Sol. $I = \int \frac{dx}{x^3(x^2+2)^2}$

$$= \frac{1}{4} \int \frac{x}{x^2+2} dx + \frac{1}{4} \int \frac{x}{(x^2+2)^2} dx - \frac{1}{4} \int \frac{1}{x} dx + \frac{1}{4} \int \frac{1}{x^3} dx$$

$$= \frac{\ln(x^2+2)}{8} - \frac{\ln x}{4} - \frac{1}{8(x^2+2)} - \frac{1}{8x^3}$$

$$16 \int_{-1}^2 \frac{dx}{x^3(x+2)^2} = 2\ln 6 - 2\ln 3 - 4\ln 2 + \frac{11}{6}$$

13. If $A = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$

If $M = A^T B A$, then the matrix $AM^{2023}A^T$ is

- $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & -2023i \\ 0 & -1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Answer (1)

Sol. $AA^T = I$

$$M = A^T B A$$

$$AM^{2023}A^T$$

$$= A \underbrace{(A^T B A)(A^T B A)(A^T B A) \dots (A^T B A)}_{2023 \text{ times}} A^T$$

$$= B^{2023}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

14. ?

15. ?

16. ?

17. ?

18. ?

19. ?

20. ?

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the remainder when $(2023)^{2023}$ is divided by 35.

Answer (7)

$$\text{Sol. } 2023 \equiv -7 \pmod{35}$$

$$(2023)^2 \equiv 14 \pmod{35}$$

$$(2023)^4 \equiv -14 \pmod{35}$$

$$(2023)^{16} \equiv -14 \pmod{35}$$

$$(2023)^{2020} \equiv -14 \pmod{35}$$

$$\text{and } (2023)^3 \equiv 7 \pmod{35}$$

$$\therefore (2023)^{2023} \equiv 7 \pmod{35}$$

$$\therefore \text{remainder} = 7$$

22. $\int_{1/3}^3 |\ln x| dx = \frac{m}{n} \ln\left(\frac{n^2}{e}\right)$ then value of $m^2 + n^2 - 5$

Answer (20)

$$\text{Sol. } \int_{1/3}^1 -\ln x dx + \int_1^3 \ln x dx$$

$$= -(x \ln x - x) \Big|_{1/3}^1 + (x \ln x - x) \Big|_1^3$$

$$= \frac{2}{3} + \frac{1}{3} \ln \frac{1}{3} + 3 \ln 3 - 2$$

$$= \frac{4}{3} (\ln 9 - \ln e)$$

$$= \frac{4}{3} \ln \left(\frac{3^2}{e} \right)$$

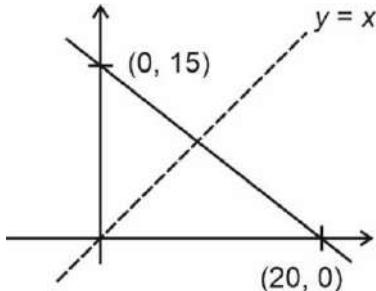
$$m = 4, n = 3$$

$$m^2 + n^2 - 5 = 20$$

23. A triangle is formed with x-axis, y-axis & line $3x + 4y = 60$. A point $P(a, b)$ lies strictly inside the triangle such that a is a positive integer and b is a multiple of 'a'. Find such number of points (a, b) .

Answer (31)

Sol.



as b is multiple of a the required points will lie on line $y = kx$ ($k \in \mathbb{Z}$)

$$\therefore 3x + 4kx = 60$$

$$\Rightarrow x = \frac{60}{3+4k}$$

If $k = 1$ 8 integral values

$k = 2$ 5 integral values

$k = 3$ 3 integral values

$k = 4$ 3 integral values

$k = 5$ 2 integral values

$k = 6$ 2 integral values

$k = 7$ 1 integral values

$k = 8$ 1 integral values

$\vdots \vdots$

$k = 14$ 1 integral values

\therefore Total 31 points

24. If $a, b, \frac{1}{18}$ are in G.P. and $\frac{1}{10}, \frac{1}{a}, \frac{1}{b}$ are in A.P. then
 find value of $a + 180b$

Answer (20.00)

$$\text{Sol. } b^2 = \frac{a}{18}, \frac{2}{a} = \frac{1}{10} + \frac{1}{b}$$

$$\Rightarrow a = \frac{20b}{10+b}$$

$$\text{OR } 18b^2 = \frac{20b}{10+b}$$

$b = 0$ (rejected)

$$\text{OR } 9b = \frac{10}{10+b}$$

$$90b + 9b^2 = 10$$

$$\Rightarrow 9b^2 + 90b - 10 = 0$$

$$a + 180b = 18b^2 + 180b$$

$$= 20$$

25. In a city, 25% of the population is smoker and a smoker has 27 times more chance of being diagnosed with lung cancer. A person is selected at random and found to be diagnosed with lung cancer. If the probability of him being smoker is $\frac{K}{40}$, find the value of K .

Answer (36.00)

$$\text{Sol. Probability of a person being smoker} = \frac{1}{4}$$

$$\text{Probability of a person being non smoker} = \frac{3}{4}$$

$$P\left(\frac{\text{Person is smoker}}{\text{Person diagnosed with cancer}}\right) = \frac{\frac{1}{4} \cdot 27P}{\frac{1}{4} \cdot 27P + \frac{3}{4}P}$$

(P is probability that a non-smoker is diagnosed with cancer)

$$= \frac{27}{30} = \frac{9}{10} = \frac{36}{40}$$

26.

27.

28.

29.

30.