

**NARAYANA GRABS
THE LION'S SHARE IN JEE-ADV.2022**

5 RANKS in OPEN CATEGORY
ONLY FROM NARAYANA
IN TOP 10 AIR



JEE MAIN (JAN) 2023 (25-01-2023-Session-2)

Memory Based Question Paper
MATHEMATICS



MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\sum_{k=0}^6 {}^{51-k}C_3$ is

- (1) ${}^{52}C_4 - {}^{46}C_4$ (2) ${}^{52}C_4 - {}^{45}C_4$
(3) ${}^{51}C_4 - {}^{45}C_4$ (4) ${}^{51}C_4 - {}^{46}C_4$

Answer (2)

Sol. ${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \dots + \underbrace{{}^{45}C_3 + {}^{45}C_4 - {}^{45}C_4}_{{}^{46}C_4}$
 ${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + \underbrace{{}^{46}C_3 + {}^{46}C_4 - {}^{45}C_4}_{{}^{47}C_4}$

⋮

$\Rightarrow {}^{52}C_4 - {}^{45}C_4$

2. If $f(x) = 2x^n + \lambda$ and $f(4) = 133$, $f(5) = 255$, then sum of positive integral divisors of $f(3) - f(2)$ is

- (1) 60 (2) 22
(3) 40 (4) 6

Answer (1)

Sol. $2 \cdot 4^n + \lambda = 133$... (i)

$2 \cdot 5^n + \lambda = 255$... (ii)

$2(5^n - 4^n) = 122$

$n = 3$

$f(x) = 2x^3 + \lambda$

$f(3) - f(2) = 2(3^3 - 2^3)$

$= 38$

Divisors = 1, 2, 19, 38

3. If $\left| \frac{z+2i}{z-i} \right| = 2$ is a circle, then centre of the circle is

- (1) (0, 0) (2) (0, 2)
(3) (2, 0) (4) (-2, 0)

Answer (2)

Sol. $(z+2i)(\bar{z}-2i) = 4(z-i)(\bar{z}+i)$

$\Rightarrow \bar{z}\bar{z} + 2i\bar{z} - 2iz + 4 = 4z\bar{z} - 4i\bar{z} + 4iz + 4$

$\Rightarrow 3z\bar{z} - 6i\bar{z} + 6iz = 0$

$\Rightarrow z\bar{z} - 2i\bar{z} + 2iz = 0$

\therefore Centre = $2i$ i.e., (0, 2)

4. If $\frac{dy}{dt} + \alpha \cdot y = \gamma \cdot e^{-\beta t}$, then $\lim_{t \rightarrow \infty} y(t)$,

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\alpha \neq \beta$, is equal to

- (1) 0 (2) 1
(3) Does not exist (4) $\alpha\beta$

Answer (1)

Sol. $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

Integrating factor (I.F.) = $e^{\alpha t}$

Solution of D.E.

$ye^{\alpha t} = \gamma \int e^{(\alpha-\beta)t} dt$

$\Rightarrow ye^{\alpha t} = \frac{\gamma}{\alpha-\beta} \cdot e^{(\alpha-\beta)t} + C$

$\Rightarrow y(t) = \frac{\gamma}{\alpha-\beta} \cdot \alpha^{-\beta t} + C \cdot e^{-\alpha t}$

$\lim_{t \rightarrow \infty} y(t) = 0$

5. If $(p \rightarrow q) \nabla (p \Delta q)$ is tautology, then operator ∇ , Δ denotes

- (1) $\Delta \rightarrow$ OR (2) $\Delta \rightarrow$ AND
 $\nabla \rightarrow$ AND $\nabla \rightarrow$ OR
(3) $\Delta \rightarrow$ AND (4) $\Delta \rightarrow$ OR
 $\nabla \rightarrow$ AND $\nabla \rightarrow$ OR

Answer (4)

Sol. $(p \rightarrow q) \nabla (p \Delta q) \equiv T$

Only if ∇ is OR and Δ is OR

6. Number of numbers between 5000 and 10000 by using the digits 1,3,5,7,9 without repetition is equal to

- (1) 120 (2) 72
(3) 12 (4) 6

Answer (2)

11. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$ such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$ then $\vec{a} - 6\vec{b}$ equals

- (1) $3(\hat{i} + \hat{j} + \hat{k})$
- (2) $\hat{i} + \hat{j} + \hat{k}$
- (3) $2(\hat{i} + \hat{j} + \hat{k})$
- (4) $4(\hat{i} + \hat{j} + \hat{k})$

Answer (1)

Sol. $(\vec{a} \times \vec{b}) = \hat{i} - \hat{j}$

$$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = (-\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} - \hat{j})$$

$$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\Rightarrow 3\vec{b} = -2\hat{i} - 2\hat{j} - \hat{k}$$

$$\therefore \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k} \\ = 3(\hat{i} + \hat{j} + \hat{k})$$

12. $16 \int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

- (1) $\frac{11}{12} + \ln 4$
- (2) $\frac{11}{12} - \ln 4$
- (3) $\frac{11}{6} - \ln 4$
- (4) $\frac{11}{6} + \ln 4$

Answer (3)

Sol. $I = \int \frac{dx}{x^3(x^2+2)^2}$

$$= \frac{1}{4} \int \frac{x}{x^2+2} dx + \frac{1}{4} \int \frac{x}{(x^2+2)^2} dx - \frac{1}{4} \int \frac{1}{x} dx + \frac{1}{4} \int \frac{1}{x^3} dx$$

$$= \frac{\ln(x^2+2)}{8} - \frac{\ln x}{4} - \frac{1}{8(x^2+2)} - \frac{1}{8x^3}$$

$$16 \int_1^2 \frac{dx}{x^3(x^2+2)^2} = 2\ln 6 - 2\ln 3 - 4\ln 2 + \frac{11}{6}$$

13. If $A = \begin{bmatrix} 3 & 1 \\ \sqrt{10} & \sqrt{10} \\ -1 & 3 \\ \sqrt{10} & \sqrt{10} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$

If $M = A^T B A$, then the matrix $A M^{2023} A^T$ is

$$(1) \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$$(2) \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$$

$$(3) \begin{bmatrix} 1 & -2023i \\ 0 & -1 \end{bmatrix}$$

$$(4) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Answer (1)

Sol. $A A^T = I$

$$M = A^T B A$$

$$A M^{2023} A^T$$

$$= A \underbrace{(A^T B A)(A^T B A)(A^T B A) \dots (A^T B A)}_{2023 \text{ times}} A^T$$

$$= B^{2023}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

14. ?

15. ?

16. ?

17. ?

18. ?

19. ?

20. ?

24. If $a, b, \frac{1}{18}$ are in G.P. and $\frac{1}{10}, \frac{1}{a}, \frac{1}{b}$ are in A.P. then

find value of $a + 180b$

Answer (20.00)

Sol. $b^2 = \frac{a}{18}, \frac{2}{a} = \frac{1}{10} + \frac{1}{b}$

$$\Rightarrow a = \frac{20b}{10+b}$$

OR $18b^2 = \frac{20b}{10+b}$

$b = 0$ (rejected)

OR $9b = \frac{10}{10+b}$

$90b + 9b^2 = 10$

$\Rightarrow 9b^2 + 90b - 10 = 0$

$a + 180b = 18b^2 + 180b$

$= 20$

25. In a city, 25% of the population is smoker and a smoker has 27 times more than chance of being diagnosed with lung cancer. A person is selected at random and found to be diagnosed with lung cancer. If the probability of him being smoker is $\frac{K}{40}$,

find the value of K .

Answer (36.00)

Sol. Probability of a person being smoker $= \frac{1}{4}$

Probability of a person being non smoker $= \frac{3}{4}$

$$P\left(\frac{\text{Person is smoker}}{\text{Person diagnosed with cancer}}\right) = \frac{\frac{1}{4} \cdot 27P}{\frac{1}{4} \cdot 27P + \frac{3}{4}P}$$

(P is probability that a non-smoker is diagnosed with cancer)

$$= \frac{27}{30} = \frac{9}{10} = \frac{36}{40}$$

26.

27.

28.

29.

30.