44 YEARS OF EXCELLENCE

NARAYANA EDUCATIONAL INSTITUTIONS

NARAYANA GRABS THE LION'S SHARE IN JEE-ADV.2022

5 RANKS in OPEN CATEGORY ONLY FROM NARAYANA IN TOP 10 AIR

JEE MAIN (JAN) 2023 (29-01-2023-Session-1)

Memory Based Question Paper

MATHEMATICS

JEE-Main-29-01-2023 (Memory Based) [Morning Shift]

Mathematics

Question: If $f\left(x+y\right)=f\left(x\right)+f\left(y\right), f\left(1\right)=\frac{1}{5}$ and $\sum_{n=1}^{m}\frac{f\left(n\right)}{n\left(n+1\right)\left(n+2\right)}=\frac{1}{12}$, then find m.

Answer: 10.00

Solution:

$f\left(x+y\right)=f\left(x\right)+f\left(y\right)$

$f\left(x\right)=ax$

$x=1\Rightarrow f\left(1\right)=a=\frac{1}{5}$

$f\left(x\right)=\frac{1}{5}x$

$\frac{1}{5}\sum\frac{n}{n\left(m+1\right)\left(n+2\right)}=\frac{1}{12}$

$\frac{1}{5}\left[\sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)\right]=\frac{1}{12}$

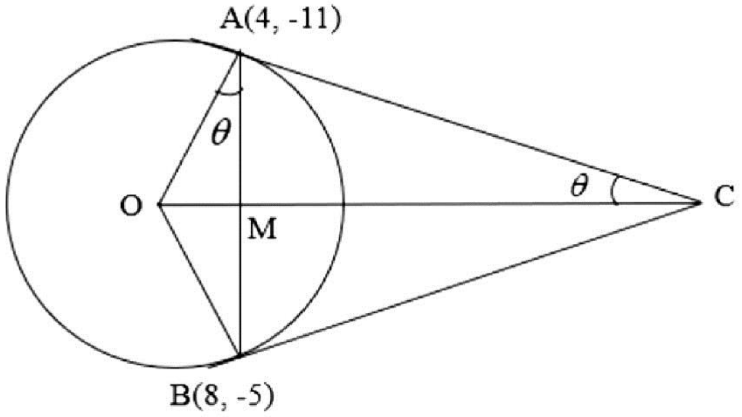
$\frac{1}{2}-\frac{1}{m+2}=\frac{5}{12}$

$\Rightarrow m=10$

Question: Tangent at $A\left(4,-11\right)$ and $B\left(8,-5\right)$ to $x^2+y^2-3x+10y-15=0$ intersect at C. Find the radius of circle touching AB and having centre at C.

Answer: $\frac{2\sqrt{13}}{3}$

Solution:



Radius $r = \sqrt{\frac{9}{4} + 25 + 15} = \frac{13}{2}$

$$AM = \frac{1}{2}AB$$
$$= \frac{1}{2}\sqrt{16+36}$$
$$= \sqrt{13}$$

$$\frac{CM}{AM} = \cot\theta$$
$$CM = \sqrt{13}\cot\theta$$

$$\Delta OAM \Rightarrow OM = \sqrt{\left(\frac{13}{2}\right)^2 - 13} = \sqrt{\frac{169-52}{4}} = \frac{\sqrt{117}}{2}$$

$$CM = \sqrt{13}\cot\theta$$
$$CM = \sqrt{13} \times \frac{\sqrt{13}}{\sqrt{117}} \times 2 = \sqrt{13} \times \frac{\sqrt{13}}{\sqrt{13} \times 3} \times 2 = \frac{2\sqrt{13}}{3}$$

Question: Five digit numbers are formed using 1, 2, 3, 5, 7 (repetition is allowed), and these numbers are arranged in descending order. Find the rank of 35337.

Answer: 1436.00

Solution:

Five digit number using 35337

$\underline{7}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^4$

$\underline{5}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^4$

$\underline{3}\ \underline{7}\ \underline{5}\ \underline{5}\ \underline{5} = 5^3$

$\underline{3}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^3$

$\underline{3}\ \underline{5}\ \underline{7}\ \underline{5}\ \underline{5} = 5^2$

$\underline{3}\ \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} = 5^2$

$\underline{3}\ \underline{5}\ \underline{3}\ \underline{7}\ \underline{5} = 5$

$\underline{3}\ \underline{5}\ \underline{3}\ \underline{5}\ \underline{5} = 5$

$\underline{3}\ \underline{5}\ \underline{3}\ \underline{3}\ \underline{7} = 1$

So total = $5^4 + 5^4 + 5^3 + 5^3 + 5^2 + 5^2 + 5 + 5 + 1 = 1436$

Question: A function $f(x)$ is such that $f(x+y) = f(x) + f(y) - 1,\ \forall x, y \in R$. If $f'(0) = 2$, then $|f(-2)| = ?$

Answer: 3.00

Solution:

$f(x+y) = f(x) + f(y) - 1$

$x = y = 0 \Rightarrow f(0) = f(0) + f(0) - 1 \Rightarrow f(0) = 1$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{f(x) + f(h) - 1 - f(x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{f(h) - 1}{h}$

$= f'(0) = 2$

$f'(x) = 2$

$f(x) = 2x + c$

$1 = 0 + c \Rightarrow c = 1$

$\therefore\ f(x) = 2x + 1$

$|f(-2)| = |-3| = 3$

Question: If the 3 consecutive coefficients in the expansion of $(1+2x)^n$ are in the ratio $2:5:8$ then the middle term is

Answer: ${}^8C_4(2x)^4$

Solution:

${}^nC_{r-1}(2)^{r-1} : {}^nC_r 2^r : {}^nC_{r+1} 2^{r+1} :: 2:5:8$

$$\frac{{}^{n}C_{r}2^{r}}{{}^{n}C_{r-1}2^{r-1}}=\frac{5}{2}$$

$$2\left(\frac{n-r+1}{r}\right)=\frac{5}{2}$$

Similarly, $2\left(\frac{n-r}{r+1}\right)=\frac{8}{5}$

Comparing $n=8$

Mid term $={}^{8}C_{4}\left(2x\right)^{4}$

Question: If $\frac{dy}{y}=\left(\frac{x+1}{x^{2}}\right)dx;\ y\left(1\right)=e,$ then $\lim\limits_{x\to 0^{+}} f\left(x\right)=?$

Answer: 0.00

Solution:

$$\int\frac{dy}{y}=\int\left(\frac{1}{x}+\frac{1}{x^{2}}\right)dx$$

$$\ln\left|y\right|=\ln\left|x\right|-\frac{1}{x}+c$$

Given $\left(1,e\right)$

$$1=0-1+c$$

$$c=2$$

$\therefore$ We have

$$\ln\left|y\right|=\ln\left|x\right|-\frac{1}{x}+2$$

$$y=e^{\ln\left|x\right|-\frac{1}{x}+2}$$

As we take $\lim\limits_{x\to 0^{+}}$

$$y=e^{-\infty}=0$$

Question: α,β are positive numbers. A is a 3×3 matrix such that $A^{2}=3A+\alpha I$ and $A^{4}=21A+\beta I$. Find α,β .

Answer: -1, -8

Solution:

Given, $A^{2}=3A+\alpha I$ and $A^{4}=21A+\beta I$

$$A^{4}=A^{2}\cdot A^{2}$$

$A^4 = (3A+\alpha I)(3A+\alpha I) = 21A + \beta I$

$9A^2 + 3A\alpha I + 3\alpha AI + \alpha^2 I = 21A + \beta I$

$9A^2 + 6\alpha A + \alpha^2 I = 21A + \beta I$

Again using $A^2 = 3A + \alpha I$ in LHS

$\Rightarrow 9(3A+\alpha I) + 6\alpha A + \alpha^2 I = 21A + \beta I$

$\Rightarrow (27+6\alpha)A + (9\alpha + \alpha^2)I = 21A + \beta I$

$\therefore 27 + 6\alpha = 21$ & $9\alpha + \alpha^2 = \beta$

$6\alpha = -6$ & $1 - 9 = \beta$

$\alpha = -1$ & $\beta = -8$

Question: Consider a function $f(x) = \dfrac{x^2+2x+1}{x^2+1}$, then which of the following is correct?

Options:

(a) $f(x)$ is one-one for $x \in (0,\infty)$

(b) $f(x)$ is one-one for $x \in (1,\infty)$

(c) $f(x)$ is one-one for $x \in (2,\infty)$ and many-one for $x \in (-\infty, 0]$

(d)

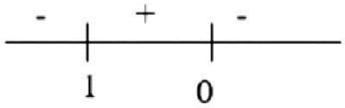
Answer: (c)

Solution:

$$f(x) = \frac{(x^2+1)+2x}{x^2+1} = 1 + \frac{2x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)^2 - 2x(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{2-2x^2}{(x^2+1)^2}$$



Question: If real part of the product of z_1 & z_2 is zero i.e., $\text{Re}(z_1 z_2) = 0$ & $\text{Re}(z_1 + z_2) = 0$ then $\text{Im}(z_1)$ & $\text{Im}(z_2)$ is

Answer: $\text{Im}(z_1)$ & $\text{Im}(z_2)$ are of opposite signs

Solution:

Given product of z_1 & z_2 is zero

i.e., $\text{Re}(z_1 \cdot z_2) = 0$

$$z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2)$$
$$= (x_1x_2 - y_1y_2)$$

$$\text{Re}(z_1z_2) = x_1x_2 - y_1y_2 = 0$$

$$\Rightarrow x_1x_2 = y_1y_2 \quad ....(\text{i})$$

$$\text{Re}(z_1 + z_2) = (x_1 + iy_1) + (x_2 + iy_2)$$

$$\text{Re}(z_1 + z_2) = (x_1 + x_2) = 0$$

$$x_2 = -x_1$$

Substitute x_2 in (i)

$$-x_1 \cdot x_1 = y_1y_2$$
$$\Rightarrow y_1y_2 = -x_1^{\ 2} = -ve$$

$\text{Im}(z_1)$ & $\text{Im}(z_2)$ is one positive and one negative.

Question: If $a_1, a_2, a_3, ..., a_n$ is an increasing GP such that $a_4 \times a_6 = 9$ and $a_5 + a_7 = 12$. Find $a_7 + a_9 = ?$

Answer: 36.00

Solution:

Given $a_5 + a_7 = 12$ and $a_4 \times a_6 = 9$

$$a_5 + a_7 = 12$$
$$ar_4 + ar^6 = 12$$
$$ar^4(1 + r^2) = 12 \quad ....(1)$$

$$a_4 \cdot a_6 = 9$$
$$ar^3 \cdot ar^5 = 9$$
$$a^2 \cdot r^8 = 9$$
$$a \cdot r^4 = 3$$

Substitute in (1)

$3\left(1+r^2\right)=12$

$r^2=3$

$r=\pm\sqrt{3}$

$a=\frac{1}{3}$

Now a_7+a_9

$\Rightarrow ar^6+ar^8=ar^4\left(r^2+r^4\right)$

$=3\times r^2\left(1+r^2\right)$

$=3\times 3(1+3)$

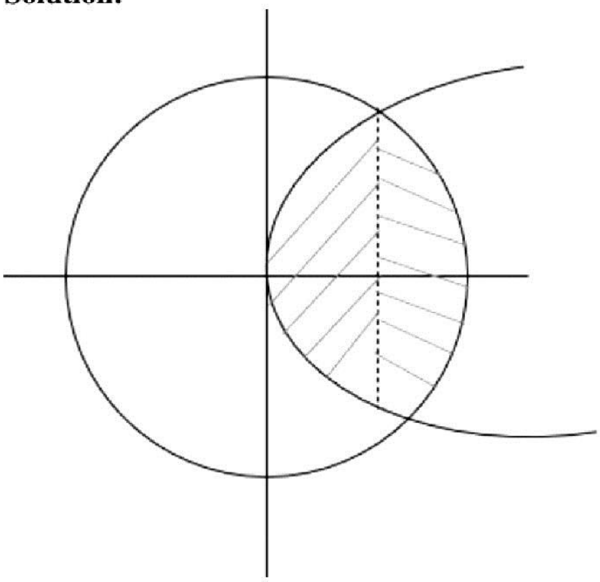
$=9\times 4$

$=36$

Question: Δ is the area between $x^2+y^2\le 21,\ y^2\le 4x$ and $x\ge 1$. Find $\frac{1}{2}\left(\Delta-21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)$.

Answer: $\sqrt{3}-\frac{4}{3}$

Solution:



$x^2+y^2=21$

$y^2=4x$

$x=3$

$$2\times\int_1^3 2\sqrt{x}dx = 4\left[\frac{2}{3}x^{\frac{3}{2}}\Big|_1^3\right] = \frac{8}{3}\left(3\sqrt{3}-1\right)$$

$$2\int_3^{\sqrt{21}}\sqrt{21-x^2} = 2\left[\frac{1}{2}x\sqrt{21-x^2}+\frac{1}{2}\times 21\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{21}$$

$$= 0+21\frac{\pi}{2}3\times 2\sqrt{3}-21\sin^{-1}\left(\frac{\sqrt{3}}{\sqrt{7}}\right)$$

$$= 21\left(\cos^{-1}\sqrt{\frac{3}{7}}\right)-6\sqrt{3}$$

$$= 21\sin^{-1}\frac{2}{\sqrt{7}}-6\sqrt{3}$$

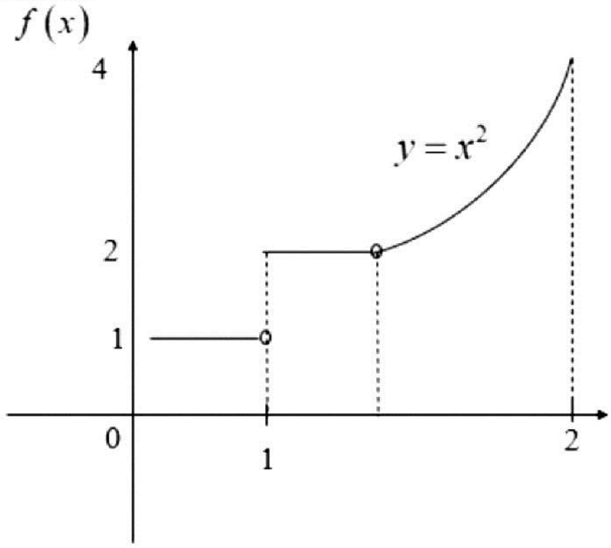
$$\Delta = \frac{8}{3}\left(3\sqrt{3}-1\right)+21\sin^{-1}\frac{2}{\sqrt{7}}-6\sqrt{3}$$

$$\Delta = \sqrt{3}-\frac{4}{3}$$

Question: $\int_0^2 \max\left\{x^2, 1+[x]\right\}dx$ is equal to

Answer: $\frac{5+4\sqrt{2}}{3}$

Solution:



$$x^2 = 2 \Rightarrow x = \sqrt{2}$$

$$\left(1\times1\right)+2\left(\sqrt{2}-1\right)+\int_{\sqrt{2}}^{2}x^2dx$$

$$=\frac{5+4\sqrt{2}}{3}$$

Question: 4 apples are picked one by one without replacing from a bag containing 3 rotten and 7 normal apples. Let x be no. of rotten apples. Find $\overline{x}+v_x$.

Answer: $\frac{6}{5},\frac{14}{25}$

Solution:

X	0	1	2	3
P(X)	$\frac{{}^{7}C_4}{{}^{10}C_4}$	$\frac{{}^{3}C_1\times{}^{7}C_4}{{}^{10}C_4}$	$\frac{{}^{3}C_2\times{}^{7}C_2}{{}^{10}C_4}$	$\frac{{}^{7}C_1}{{}^{10}C_4}$

$$\overline{x}=\sum x_iP_i$$
$$\overline{x}=\frac{6}{5}$$

$$\text{Variance}=V=\sum x_i^2P_i-\left(\overline{x}\right)^2$$

$$V=\frac{14}{25}$$

Question: Domain of $f\left(x\right)=\frac{\log_x\left(x-1\right)}{\log_{x-1}\left(x-4\right)}$ is:

Answer: $x\in\left(4,\infty\right)\setminus\left\{5\right\}$

Solution:

For domain

$x>0,\ x-1>0,\ x\neq1$

& $x-1>0,\ x-1\neq1, x-4>0$

$\log_{x-1}\left(x-4\right)\neq0$

$\Rightarrow x-4\neq1\Rightarrow x\neq5$

$\therefore\ x\in\left(4,\infty\right)-\left\{5\right\}$

Question: If the coefficient of x^5 in the expansion of $\left(ax^3+\frac{1}{\beta x}\right)^{11}$ and $\left(\alpha x+\frac{1}{\beta x^3}\right)^{11}$ are equal, then the value of $(\alpha+\beta)^2$ is

Answer: 1.00

Solution:

General term of $\left(\alpha x^3+\frac{1}{\beta x}\right)^{11}$ is

$$T_{k+1}={}^{11}C_k\left(\alpha x^3\right)^{11-k}\left(\frac{1}{\beta x}\right)^k$$

$$={}^{11}C_k\alpha^{11-k}\beta^{-k}x^{33-4k}$$

Now for coefficient of x^9, we have

$33-4k=9$

$\Rightarrow 4k=24$

$\Rightarrow k=6$

Similarly, general term of $\left(\alpha x+\frac{1}{\beta x^3}\right)^{11}$ is

$$T_{k+1}={}^{11}C_k\left(\alpha x\right)^{11-k}\left(\frac{1}{\beta x^3}\right)^k$$

$$={}^{11}C_k\alpha^{11-k}\beta^{-k}x^{11-4k}$$

For coefficient of x^{-9}, we have

$11-4k=9$

$\Rightarrow 4k=20$

$\Rightarrow k=5$

$${}^{11}C_6\frac{\alpha^5}{\beta^6}={}^{11}C_5.\frac{\alpha^6}{\beta^5}$$

$$\Rightarrow \alpha\beta=\frac{{}^{11}C_6}{{}^{11}C_5}=1$$

$$\Rightarrow (\alpha\beta)^2=1$$

Question: Consider 3 coplanar vector $\vec{a}=3\hat{i}-4\hat{j}+\lambda\hat{k}, \vec{b}=4\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}-4\hat{k}$. Then 9λ is __

Answer: 87.00

Solution:

$$\begin{vmatrix} 3 & -4 & \lambda \\ 4 & 3 & -1 \\ 1 & 3 & -4 \end{vmatrix} = 0$$

$$-27 - 60 + 9\lambda = 0$$

$$9\lambda = 87$$