

PART : MATHEMATICS

1. Number of 7 digit odd number, formed by using the digits 1,2,2,2,3,3,5 only is:
 (1) 300 (2) 240 (3) 180 (4) 360

Ans. (2)

Sol. ----- $1 = \frac{6!}{23!} = 60$

----- $3 = \frac{6!}{3!} = 120$

----- $5 = \frac{6!}{3!2!} = 60$

Total = $60 + 120 + 60 = 240$

2. Let $S_1 : 1 + 5 + 9 + 13 + \dots$

$S_2 : 1 + 6 + 11 + 16 + \dots$ are two given series then their 8^{th} common term from starting is

Ans. (141)

Sol. $S_1 : 1 + 5 + 9 + 13 + \dots$ A.P.

$S_2 : 1 + 6 + 11 + 16 + \dots$ A.P.

First common term = 1

common difference of common AP is $\text{LCM}(d_1, d_2) = \text{LCM}(4, 5) = 20$

$\therefore 1a = 1 + (8-1)20 = 140 + 1 = 141$ Ans

3. If 50^{th} root of x is 12 and 50^{th} root of y is 18, then the remainder when $(x+y)$ is divided by 25, is
 (1) 21 (2) 23 (3) 22 (4) 24

Ans. (2)

Sol. $(x)^{1/50} = 12$ and $(y)^{1/50} = 18$

$\Rightarrow x = 12^{50} \Rightarrow y = 18^{50}$

$x + y = 12^{50} + 18^{50}$

$= (2^2 \times 3^2)^{50} + (3^2 \times 2)^{50}$

$= 6^{50}(2^{20} + 3^{20})$

$= (5+1)^{20}(4^{20} + 9^{20})$

$= (25k+1)[(5-1)^{20} + (10-1)^{20}]$

$= (25k+1)(25k-1+25l-1)$

$= (25k+1)(25t+23)$

$= 25r + 23$

so remainder is 23, when $x+y$ is divided by 25.

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4. If $|a| = 1$, $|b| = 2$, $\bar{a} \cdot \bar{b} = 4$ and $\bar{c} = 2(\bar{a} \times \bar{b}) - 3\bar{b}$ then $\bar{b} \cdot \bar{c} =$

- (1) 0 (2) 12 (3) -12 (4) -6

Ans. (3)

Sol. $\bar{b} \cdot \bar{c} = 0 - 3\bar{b} \cdot \bar{b}$

$$\bar{b} \cdot \bar{c} = -12$$

5. $\int \sqrt{\sec x + 1} dx =$

- (1) $\sqrt{2} \sin^{-1} \left(\sqrt{2} \sin \frac{x}{2} \right) + C$ (2) $2 \sin^{-1} \left(\sqrt{2} \sin \frac{x}{2} \right) + C$
 (3) $2 \sin^{-1} \left(\sqrt{2} \sin \left(\frac{x}{\sqrt{2}} \right) \right) + C$ (4) $2 \cos^{-1} \left(\sqrt{2} \sin \left(\frac{x}{\sqrt{2}} \right) \right) + C$

Ans. (2)

Sol. $\int \sqrt{\sec x + 1} dx = \int \sqrt{\frac{1 + \cos x}{\cos x}} dx$

$$= \int \sqrt{\frac{2 \cos^2 \frac{x}{2}}{1 - 2 \sin^2 \frac{x}{2}}} dx$$

$$= \int \frac{\sqrt{2} \cos \frac{x}{2} dx}{\sqrt{1 - 2 \sin^2 \frac{x}{2}}}$$

$$\text{Let } \sqrt{2} \sin \frac{x}{2} = t$$

$$\therefore \sqrt{2} \cos \frac{x}{2} \cdot \frac{1}{2} dx = dt$$

$$\therefore 1 = 2 \int \frac{dt}{\sqrt{1-t^2}} = 2 \sin^{-1} t + C$$

$$= 2 \sin^{-1} \left(\sqrt{2} \sin \frac{x}{2} \right) + C$$

6. $\lim_{n \rightarrow \infty} \frac{3}{n} \left[4 + \left(2 + \frac{1}{n} \right)^2 + \left(2 + \frac{2}{n} \right)^2 + \dots + \left(3 - \frac{1}{n} \right)^2 \right]$ is

- (1) 19 (2) 21 (3) -19 (4) 0

Ans. (1)

Sol. $\lim_{n \rightarrow \infty} \frac{3}{n} \left[4 + \left(2 + \frac{1}{n} \right)^2 + \left(2 + \frac{2}{n} \right)^2 + \dots + \left(3 - \frac{1}{n} \right)^2 \right]$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[\left(2 + \frac{0}{n} \right)^2 + \left(2 + \frac{1}{n} \right)^2 + \left(2 + \frac{2}{n} \right)^2 + \dots + \left(2 + \frac{n-1}{n} \right)^2 \right]$$

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$$\lim_{n \rightarrow \infty} \frac{3}{n} \left(2 + \frac{r}{n} \right)^2$$

$$\text{Let } \frac{r}{n} = x$$

$$\therefore \frac{1}{n} = dx$$

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$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{3}{n} \left(2 + \frac{r}{n} \right)^2$$

$$\text{Let } \frac{r}{n} = x$$

$$\therefore \frac{1}{n} = dx$$

$$= 3 \int_0^1 (2+x)^2 dx$$

$$= 3 \left(\frac{2+x)^3}{3} \right)_0^1$$

$$= 3^3 - 2^3$$

$$= 27 - 8 = 19$$

7. A common tangent is drawn to the parabola $y^2 = 16x$ and the circle $x^2 + y^2 = 8$. If P and Q are the points of contact of this common tangent then the value of $(PQ)^2$ is ____.

Ans. (72)

Sol. Any tangent for $y^2 = 16x$ is $y = mx + \frac{4}{m}$

it is also a tangent to $x^2 + y^2 = 8$

$$\text{if } \left(\frac{4}{m} \right)^2 = 8(1+m^2)$$

$$\frac{16}{m^2} = 8(1+m^2)$$

$$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow (m^2 + 2)(m^2 - 1) = 0 \Rightarrow m = \pm 1$$

\therefore common tangent is $y = x + 4$

$$\Rightarrow x - y + 4 = 0$$

tangent to $x^2 + y^2 = 8$ is $xx_1 + yy_1 - 8 = 0$

$$\therefore \frac{x_1}{1} = \frac{y_1}{-1} = \frac{-8}{4} \quad \therefore Q(-2, -2)$$

tangent to $y^2 = 16x$ is $yy_1 = 8(x + x_1)$

$$8x - yy_1 + 8x_1 = 0$$

$$\frac{8}{1} = \frac{-y_1}{-1} = \frac{8x_1}{4}$$

$$y_1 = 8, x_1 = 4,$$

$$\therefore P(4, 8)$$

$$\therefore \text{distance} = PQ = \sqrt{(4+2)^2 + (8-2)^2} = \sqrt{36+36} = 6\sqrt{2}$$

$$\therefore PQ^2 = 72$$



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$$8. \text{ If } f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$1 \quad x = 0$$

$$\frac{\sin(x-1)}{x+1} \quad x \neq -1$$

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8. If $f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$g(x) = \begin{cases} \frac{\sin(x+1)}{x+1} & x \neq -1 \\ 4 & x = -1 \end{cases}$

and $h(x) = 2[x] + f(x)$; where $[\]$ is G.I.F
 then $\lim_{x \rightarrow 1} g(h(x-1))$ is equal to _____

(1) $\frac{-\sin 2}{2}$ (2) $\frac{\sin 2}{2}$ (3) 0 (4) Does not exist

Ans. (2)

Sol. L.H.L $\lim_{h \rightarrow 0^+} g(h(1-h-1))$ R.H.L $\lim_{h \rightarrow 0^+} g(h(1+h-1))$

$\lim_{h \rightarrow 0^+} g(2-h+\frac{-h}{1-h})$ $= \lim_{h \rightarrow 0^+} g(2h+f(h))$

$\lim_{h \rightarrow 0^+} g(2-1-1)$ $= \lim_{h \rightarrow 0^+} g(0+1)$

$\lim_{h \rightarrow 0^+} g(-3) = \frac{\sin 2}{2}$ $= \lim_{h \rightarrow 0^+} \frac{\sin 2}{2}$

$= \frac{\sin 2}{2}$

$\therefore \lim_{x \rightarrow 1} g(h(x-1)) = \frac{\sin 2}{2}$

9. Three points $(1,0,2)$, $(1,\lambda,-1)$ & $(2,1,\lambda)$ lie on a plane whose normal vector is perpendicular to the line $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{1}$, then number of value of λ is _____

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (3)

Sol. Plane through $(1,0,2)$ is $a(x-1) + b(y-0) + c(z-2) = 0$ (1)

$A(1,0,2)$, $B(1,\lambda,-1)$, $C(2,1,\lambda)$

$AB = \lambda i - 3k$

$AC = i + j + (\lambda + 2)k$

$AB \times AC = \begin{vmatrix} i & j & k \\ 0 & \lambda & -3 \\ 1 & 1 & (\lambda + 2) \end{vmatrix}$

$= i(\lambda^2 - 2\lambda + 3) - j(0 + 3) + k(0 - \lambda)$

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$= i(\lambda^2 - 2\lambda + 3) - j(3) - \lambda k$

Given line is \perp to normal vector

$\therefore (\lambda^2 - 2\lambda + 3) \cdot 1 + (-3) \cdot 1 + (-\lambda) \cdot 1 = 0$

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$$= i(\lambda^2 - 2\lambda + 3)j - 3k - \lambda k$$

Given line is \perp to normal vector

$$\therefore (\lambda^2 - 2\lambda + 3) \cdot 1 + (-3) \cdot 1 + (-\lambda) \cdot 1 = 0$$

$$\lambda^2 - 2\lambda + 3 - 3 - \lambda = 0$$

$$\lambda - 3\lambda = 0$$

$$\lambda = 0, 3$$

10. The value of $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$

If $a_1 = 1$ and a_i are consecutive natural numbers.

(1) $\frac{\pi}{4} - \cot^{-1} 2021$

(2) $\frac{\pi}{4} - \cot^{-1} 2022$

(3) $\frac{\pi}{4} - \tan^{-1} 2021$

(4) $\frac{\pi}{4} - \tan^{-1} 2022$

Ans. (2)

Sol. a_1, a_2, a_3, \dots natural numbers

$$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1 + a_{2021}a_{2022}}\right)$$

$$\Rightarrow \tan^{-1} a_2 - \tan^{-1} a_1 + \tan^{-1} a_3 - \tan^{-1} a_2 + \dots + \tan^{-1} a_{2022} - \tan^{-1} a_{2021}$$

$$\Rightarrow \tan^{-1} a_{2022} - \tan^{-1} 1$$

$$\Rightarrow \tan^{-1} 2022 - \frac{\pi}{4} \Rightarrow \frac{\pi}{2} - \cot^{-1} 2022 - \frac{\pi}{4} = \frac{\pi}{4} - \cot^{-1} 2022$$

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