PART : MATHEMATICS

1. Number of 7 digit odd number, formed by using the digits 1,2,2,2,3,3,5 only is:

(1) 300 (2) 240 (3) 180 (4) 360

Ans. **(2)**

Sol. $\_\_\_\_\_\_\,1 = \frac{6!}{2!3!} = 60$

$\_\_\_\_\_\_\,3 = \frac{6!}{3!} = 120$

$\_\_\_\_\_\_\,5 = \frac{6!}{3!2!} = 60$

Total = 60 + 120 + 60 = 240

2. Let $S_1 : 1 + 5 + 9 + 13 + ...$

$S_2 : 1 + 6 + 11 + 16 + ...$ are two given series then their 8^{th} common term from starting is

Ans. **(141)**

Sol. $S_1 : 1 + 5 + 9 + 13 + ...$ A.P.

$S_2 : 1 + 6 + 11 + 16 + ...$ A.P.

First common term = 1

common difference of common AP is LCM (d_1, d_2) = LCM (4, 5) = 20

$\therefore t_8 = 1 + (8-1)20 = 140 + 1 = 141$ Ans

3. If 50^{th} root of x is 12 and 50^{th} root of y is 18, then the remainder when(x+y) is divided by 25, is

(1) 21 (2) 23 (3) 22 (4) 24

Ans. **(2)**

Sol. $(x)^{\frac{1}{50}} = 12$ and $(y)^{\frac{1}{50}} = 18$

$\Rightarrow x = 12^{50} \Rightarrow y = 18^{50}$

$x + y = 12^{50} + 18^{50}$

$= (2^2 \times 3^1)^{50} + (3^2 \times 2)^{50}$

$= 6^{50}(2^{50} + 3^{50})$

$= (5+1)^{50}(4^{25} + 9^{25})$

$= (25\lambda+1)[(5-1)^{25} + (10-1)^{25}]$

$= (25\lambda+1)(25k-1+25\mu-1)$

$= (25\lambda+1)(25t + 23)$

$= 25\ell + 23$

so remainder is 23, when x+y is divided by 25.

4. If $|a| = 1$; $|b| = 2$; $\vec{a}.\vec{b} = 4$ and $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$ then $\vec{b}.\vec{c}$ =

(1) 0 (2) 12 (3) −12 (4) −6

Ans. (3)

Sol. $\vec{b}.\vec{c} = 0 - 3\vec{b}.\vec{b}$

$\vec{b}.\vec{c} = -12$

5. $\int \sqrt{\sec x + 1}\, dx =$

(1) $\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\frac{x}{2}\right) + C$ (2) $2\sin^{-1}\left(\sqrt{2}\sin\frac{x}{2}\right) + C$

(3) $2\sin^{-1}\left(\sqrt{2}\sin\left(\frac{x}{\sqrt{2}}\right)\right) + C$ (4) $2\cos^{-1}\left(\sqrt{2}\sin\left(\frac{x}{\sqrt{2}}\right)\right) + C$

Ans. (2)

Sol. $\int \sqrt{\sec x + 1}\, dx = \int \sqrt{\frac{1+\cos x}{\cos x}}\, dx$

$= \int \sqrt{\frac{2\cos^2\frac{x}{2}}{1 - 2\sin^2\frac{x}{2}}}\, dx$

$= \int \frac{\sqrt{2}\cos\frac{x}{2}\, dx}{\sqrt{1 - 2\sin^2\frac{x}{2}}}$

Let $\sqrt{2}\sin\frac{x}{2} = t$

$\therefore \sqrt{2}\cos\frac{x}{2}.\frac{1}{2}dx = dt$

$\therefore I = 2\int \frac{dt}{\sqrt{1-t^2}} = 2\sin^{-1}t + C$

$= 2\sin^{-1}\left(\sqrt{2}\sin\frac{x}{2}\right) + C$

6. $\lim\limits_{n\to\infty}\frac{3}{n}\left[4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \ldots + \left(3-\frac{1}{n}\right)^2\right]$ is

(1) 19 (2) 21 (3) −19 (4) 0

Ans. (1)

Sol. $\lim\limits_{n\to\infty}\frac{3}{n}\left[4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \ldots + \left(3-\frac{1}{n}\right)^2\right]$

$\lim\limits_{n\to\infty}\frac{3}{n}\left[\left(2+\frac{0}{n}\right)^2 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \ldots + \left(2+\frac{n-1}{n}\right)^2\right]$

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$\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n-1}\frac{3}{n}\left(2+\frac{r}{n}\right)^2$

Let $\frac{r}{n} = x$

$\therefore \frac{1}{n} = dx$

$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{3}{n}\left(2+\frac{r}{n}\right)^2$

Let $\frac{r}{n}=x$

$\therefore \frac{1}{n}=dx$

$=3\int_0^1 (2+x)^2 dx$

$=3\left(\frac{(2+x)^3}{3}\right)_0^1$

$=3^3-2^3$

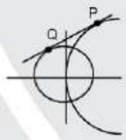
$=27-8=19$

7. A common tangent is drawn to the parabola $y^2 = 16x$ and the circle $x^2 + y^2 = 8$. If P and Q are the points of contact of this common tangent then the value of $(PQ)^2$ is____.

Ans. (72)

Sol. Any tangent for $y^2 = 16x$ is $y = mx + \frac{4}{m}$

it is also a tangent to $x^2 + y^2 = 8$



if $\left(\frac{4}{m}\right)^2 = 8(1+m^2)$

$\frac{16}{m^2} = 8(1+m^2)$

$\Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow (m^2 + 2)(m^2 - 1) = 0 \Rightarrow m = \pm 1$

$\therefore$ common tangent is $y = x + 4$

$\Rightarrow x - y + 4 = 0$

tangent to $x^2 + y^2 = 8$ is $xx_1 + yy_1 - 8 = 0$

$\therefore \frac{x_1}{1} = \frac{y_1}{-1} = \frac{-8}{4} \quad \therefore Q(-2, 2)$

tangent to $y^2 = 16x$ is $yy_1 = 8(x + x_1)$

$8x - yy_1 + 8x_1 = 0$

$\frac{8}{1} = \frac{-y_1}{-1} = \frac{8x_1}{4}$

$y_1 = 8,\ x_1 = 4,$

$\therefore P(4, 8)$

$\therefore$ distance $= PQ = \sqrt{(4+2)^2+(8-2)^2} = \sqrt{36+36} = 6\sqrt{2}$

$\therefore PQ^2 = 72$

8. If $f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$g(x) = \begin{cases} \frac{\sin(x+1)}{x+1} & x \neq -1 \end{cases}$

8. If $f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$g(x) = \begin{cases} \frac{\sin(x+1)}{x+1} & x \neq -1 \\ 4 & x = -1 \end{cases}$

and $h(x) = 2[x] + f(x)$; where [.] is G.I.F

then $\lim_{x \to 1} g(h(x-1))$ is equal to___

(1) $\frac{-\sin 2}{2}$ (2) $\frac{\sin 2}{2}$ (3) 0 (4) Does not exist

Ans. **(2)**

Sol.

L.H.L

$\lim_{h \to 0^+} g(h(1-h-1))$

$\lim_{h \to 0^+} g\left(2[-h] + \frac{-h}{|-h|}\right)$

$\lim_{h \to 0^+} g(2(-1)-1)$

$\lim_{h \to 0^+} g(-3) = \frac{\sin 2}{2}$

R.H.L

$= \lim_{h \to 0^+} g(h(1+h-1))$

$= \lim_{h \to 0^+} g(2[h] + f(h))$

$= \lim_{h \to 0^+} g(0+1)$

$= \lim_{h \to 0^+} \frac{\sin 2}{2}$

$= \frac{\sin 2}{2}$

$\therefore \lim_{x \to 1} g(h(x-1)) = \frac{\sin 2}{2}$

9. Three points (1,0,2), (1,λ,–1) & (2,1,λ) lie on a plane whose normal vector is perpendicular to the line $\frac{x-1}{1} = \frac{y+1/2}{1} = \frac{z+1}{1}$, then number of value of λ is

(1) 0 (2) 1 (3) 2 (4) 3

Ans. **(3)**

Sol. Plane through (1,0,2) is $a(x-1) + b(y-0) + c(z-2) = 0$ ....(1)

A(1,0,2), B(1,λ,–1), C(2,1,λ)

$\overrightarrow{AB} = \lambda i - 3k$

$\overrightarrow{AC} = i + j + (\lambda+2)k$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & \lambda & -3 \\ 1 & 1 & (\lambda-2) \end{vmatrix}$

$= i(\lambda^2 - 2\lambda + 3) - j(0+3) + k(0-\lambda)$

$= i(\lambda^2 - 2\lambda + 3) i - 3j - \lambda k$

Given line is ⊥ to normal vector

$\therefore (\lambda^2 - 2\lambda + 3) \cdot 1 + (-3) \cdot 1 + (-\lambda) \cdot 1 = 0$

$= \hat{i}(\lambda^2 - 2\lambda + 3)\hat{i} - 3\hat{j} - \lambda\hat{k}$

Given line is ⊥ to normal vector

$\therefore (\lambda^2 - 2\lambda + 3)\cdot 1 + (-3)\cdot 1 + (-\lambda)\cdot 1 = 0$

$\lambda^2 - 2\lambda + 3 - 3 - \lambda = 0$

$\lambda - -3\lambda = 0$

$\lambda = 0, 3$

10. The value of $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$

If $a_1 = 1$ and a_i are consecutive natural numbers.

(1) $\frac{\pi}{4} - \cot^{-1}2021$ (2) $\frac{\pi}{4} - \cot^{-1}2022$

(3) $\frac{\pi}{4} - \tan^{-1}2021$ (4) $\frac{\pi}{4} - \tan^{-1}2022$

Ans. (2)

Sol. $a_1, a_2, a_3,$ natural numbers

$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1+a_{2021}a_{2022}}\right)$

$\Rightarrow \tan^{-1}a_2 - \tan^{-1}a_1 + \tan^{-1}a_3 - \tan^{-1}a_2 + ... + \tan^{-1}a_{2022} - \tan^{-1}a_{2021}$

$\Rightarrow \tan^{-1}a_{2022} - \tan^{-1}1$

$\Rightarrow \tan^{-1}2022 - \frac{\pi}{4} \Rightarrow \frac{\pi}{2} - \cot^{-1}2022 - \frac{\pi}{4} = \frac{\pi}{4} - \cot^{-1}2022$