MATHEMATICS

1. $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \tan 195^\circ + \frac{1}{\tan 105^\circ} = 2a$, find value of $a + \frac{1}{a}$.

(1) $2+\sqrt{3}$ (2) 4 (3) $4-\sqrt{3}$ (4) $4+\sqrt{3}$

Ans. (2)

Sol. $\tan 15^\circ + \tan 15^\circ + \tan 15^\circ - \tan 15^\circ = 2a$

$\Rightarrow \quad a = \tan 15^\circ$

$\therefore \quad a + \frac{1}{a} = \tan 15^\circ + \cot 15^\circ$

$= 2\,\text{cosec}\, 30^\circ$

$= 4$

2. How many 4 digit numbers which are divisible by 15 can be formed using digits 1, 5, 3 & 2 (Repetition of digits allowed)

Sol.

			5

Last digit must be 5 and sum of digits is divisible by 3 for divisible by 15

Case-I : $(1, 1, 2) \rightarrow \frac{3!}{2} = 3$

Case-II : $(1, 3, 3), (1, 5, 1), (2, 2, 3) \rightarrow \frac{3!}{2} \times 3 = 9$

Case-III : $(2, 3, 5) \rightarrow 3! = 6$

Case-IV : $(3, 5, 5) \rightarrow \frac{3!}{2!} = 3$

Total = 21

3. Find coefficient of x^{301} in the series : $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + \ldots + x^{500}$.

(1) ${}^{501}C_{301}$ (2) ${}^{500}C_{301}$ (3) ${}^{501}C_{300}$ (4) ${}^{501}C_{302}$

Ans. **(1)**

Sol. $$\frac{(1+x)^{500}\left[\left(\frac{x}{1+x}\right)^{501} - 1\right]}{\left(\frac{x}{1+x} - 1\right)}$$

$= (1 + x)^{501}\left(-\left(\frac{x}{1+x}\right)^{501} + 1\right)$

$= (1 + x)^{501} - x^{501}$

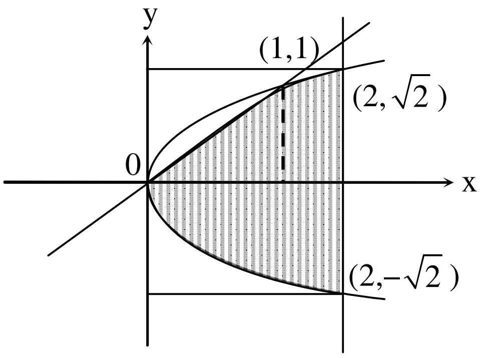
$\because$ Coefficient of $x^{301} = {}^{501}C_{301}$

4. Larger area bounded by curves : $y^2 = x$, $x = y$ and $x = 2$

(1) $\frac{8\sqrt{2}}{3} - \frac{1}{6}$ (2) $\frac{8\sqrt{2}}{3} + \frac{1}{6}$ (3) $\frac{4\sqrt{2}}{3} - \frac{1}{6}$ (4) $\frac{4\sqrt{2}}{3} + \frac{1}{6}$

Ans. **(1)**

Sol.



$$\frac{2}{3}(2\sqrt{2}\times 2) - \int_0^1 (\sqrt{x} - x)dx$$

$$= \frac{2}{3}(4\sqrt{2}) - \left(\frac{2}{3}x^{3/2}\right)_0^1 + \left(\frac{x^2}{2}\right)_0^1$$

$$= \frac{8\sqrt{2}}{3} - \frac{2}{3} + \frac{1}{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{1}{6}$$

5. Evaluate : $\lim_{x\to 0}\left(\frac{48\int_0^x \frac{t^3}{1+t^6}dt}{x^4}\right)$

(1) 16 (2) 4 (3) 24 (4) 12

Ans. **(4)**

Sol. $\lim_{x\to 0}\frac{48\cdot\frac{x^3}{1+x^6}}{4x^3}$

$= 12$

6. A special dice has number 0, –1, –2, 2, 3, 4 on its faces. If it is tossed 5 times then find the probability that product of numbers on dice is positive

(1) $\frac{343}{2592}$ (2) $\frac{521}{2592}$ (3) $\frac{719}{2592}$ (4) $\frac{443}{2592}$

Ans. **(2)**

Sol. (5 positive) + (3 positive, 2 negative) + (1positive, 4negative)

$$= \left(\frac{3}{6}\right)^5 + {}^5C_3\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2 + {}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4$$

$$= \frac{3^5 + 10\times 24\times 4 + 5\times 3\times 16}{6^5} = \frac{1563}{6^5}$$

$$= \frac{521}{2592}$$

7. The value of $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}\,dx$, is (where [·] is G.I.F)

(1) $e^8 + e$ (2) $e^7 - 1$ (3) $e^8 - e$ (4) $e^7 - e$

Ans. **(3)**

Sol. Let $x^3 = t$

$$\Rightarrow I = \left(\frac{e-1}{e}\right), \int_1^8 e^{1+[t]}\,dt$$

$$= \left(\frac{e-1}{e}\right)(e)\ (e + e^2 \ldots\ldots + e^7)$$

$$= (e-1)\ \frac{e(e^7-1)}{(e-1)}$$

$$= e^8 - e$$

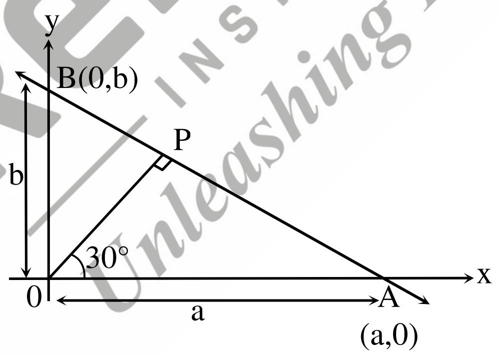
8. In the figure given below : Ar(ΔOAB) = $\frac{\sqrt{3}}{3}$ sq. unit. Find value of $b^2 - a^2$

(1) $\frac{2}{3}$

(2) $\frac{4}{3}$

(3) $\frac{8}{3}$

(4) $\frac{1}{3}$



Ans. **(2)**

Sol. $\frac{1}{2}ab = \frac{1}{\sqrt{3}} \Rightarrow ab = \frac{2}{\sqrt{3}}$

$\tan 60° = \frac{b}{a} \Rightarrow b = \sqrt{3}\,a$

$\therefore\ a^2 = \frac{2}{3}$

Now $a^2 - b^2 = a^2 - 3a^2 = -2a^2 = \frac{4}{3}$

9. Let P is a point on parabola $y^2 = \frac{x}{4}$ which is at a minimum distance from point (0, 33). Find distance of point P from directrix of parabola $y^2 = 4(x + y)$

Ans. **(6)**

Sol. $N : y + tx = \frac{2}{16}t + \frac{1}{16}t^3$

Pass (0, 33)

$\Rightarrow 33 = \frac{t}{8} + \frac{t^3}{16}$

$t^3 + 2t - 528 = 0$

$(t - 8)(t^2 + 8t + 66) = 0$

$t = 8$

$\text{Point} \equiv \left(\frac{1}{16}.64, 2.\frac{1}{16}.8\right) \equiv (4, 1)$

Parabola : $y^2 - 4y + 4 = 4x + 4$

$(y - 2)^2 = 4(x + 1) \rightarrow$ dirername : $x = -2$

distance of point = 6 units

10. If f(x) be a differentiable function such that $5f(x + y) = f(x) f(y)$; $\forall x, y \in R$ & $f(3) = 320$ then $\sum_{n=0}^{n=5} f(n) =$

(1) 6800 (2) 6825 (3) 7024 (4) 5240

Ans. **(2)**

Sol. Given $5f(x + y) = f(x) f(y)$; $\forall x, y \in R$ & $f(3) = 320$

at $x = 0$, $y = 0$, $5f(0) = f^2(0) \Rightarrow f(0) = 5$

$x = 1, y = 1 \quad 5f(2) = f^2(1)$

$x = 1, y = 2 \quad 5f(3) = f(1)f(2) \Rightarrow 5(320) = \frac{f^3(1)}{5} \Rightarrow f(1) = 20$

$f(2) = 80 = 5 \cdot 2^4$

$x = 2, y = 2 \quad 5f(4) = f^2(2) = 80 \times 80$

$f(4) = 1280 = 5 \cdot 2^8$

$x = 2, y = 3 \quad 5f(5) = f(2) f(3) = (80)(320)$

$f(5) = 5120 = 5 \cdot 2^{10}$

$\sum_{n=0}^{n=5} f(n) = 5[1 + 2^2 + 2^4 + \ldots.. + 2^{10}] = 6825$

11. Find number of normals of given polynomial $3x^5 - 25x^3 + 40x + 11 = 0$, which are parallel to line $x - 20y + 10 = 0$. **[Data may be different]**

(1) 0 (2) 2 (3) 3 (4) 4

Ans. **(4)**

Sol. $f(x) = 3x^5 - 25x^3 + 40x + 11$

$f'(x) = 15x^4 - 75x^2 + 40$

slope of normal at (x_1, y_1)

$$\frac{-1}{15x_1^4 - 75x_1^2 + 40} = \frac{1}{20}$$

$$15x_1^4 - 75x_1^2 + 40 = -20$$

$$15x_1^4 - 75x_1^2 + 60 = 0$$

$$x_1^4 - 5x_1^2 + 4 = 0 \Rightarrow (x_1^2 - 1)(x_1^2 - 4) = 0$$

$x_1 = \pm 1, \pm 2$

Number of normals = 4

12. The line L_1 is perpendicular to the plane $2x + 2y - z = 6$ and passes through P(1, 1, 1). The minimum distance between L_1 and the line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \mu(2\hat{i} - \hat{j} + 3\hat{k})$ is

(Data may be different)

(1) $\frac{2\sqrt{5}}{5}$ (2) $\frac{4\sqrt{5}}{5}$ (3) $\frac{1}{\sqrt{5}}$ (4) $\frac{3}{\sqrt{5}}$

Ans. **(2)**

Sol. L_1 is $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-1}{-1}$ and L is $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$

Hence minimum distance is

$$D = \left|\frac{[\vec{a} - \vec{b}\ \vec{c}\ \vec{d}]}{\vec{c} \times \vec{d}}\right|$$

Now, $$[\vec{a} - \vec{b}\ \vec{c}\ \vec{d}] = \begin{vmatrix} 0 & 1 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 3 \end{vmatrix} = -8 - 12 = -20$$

and $\vec{c} \times \vec{d} = 5\hat{i} - 8\hat{j} - 6\hat{k}$

$$\therefore \quad \Delta = \frac{20}{\sqrt{25 + 64 + 36}} = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

13. If the value of x satisfying the equation $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1$ is $\sin^{-1}\left(\frac{\sqrt{\alpha}-1}{\beta}\right)$, then the value of $\alpha + \beta$ is

(1) 5 (2) 7 (3) 9 (4) 8

Ans. (2)

Sol. $\log_{\cos x}\left(\frac{\cos x}{\sin x}\right) + 4\log_{\sin x}\left(\frac{\sin x}{\cos x}\right) = 1$

Let $\log_{\cos x}\sin x = t$

$\therefore\ 1 - t + 4\left(1 - \frac{1}{t}\right) = 1$

$\Rightarrow t - t^2 + 4t - 4 = t$

$\Rightarrow t = 2$

$\Rightarrow \sin x = \cos^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

$\Rightarrow x = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

14. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of $\left(ax^{1/3} + \frac{1}{bx^3}\right)^{15}$, then $|ab - 5|$ is equal to____ .

(1) 4 (2) 3 (3) 6 (4) 0

Ans. (1)

Sol. Coefficient of $x^{15} = {}^{15}C_r (ax^3)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$45 - 3r - \frac{r}{3} = 15 \Rightarrow \frac{10r}{3} = 30 \Rightarrow r = 9$

Coefficient x^{15} in $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15} = {}^{15}C_9 \frac{a^6}{b^9}$

For coefficient of x^{-15} in $\left(ax^{1/3} + \frac{1}{bx^3}\right)^{15}$

$\Rightarrow {}^{15}C_k\left(ax^{1/3}\right)^{15-k}\left(\frac{1}{bx^3}\right)^k \Rightarrow \frac{15-k}{3} - 3k = -15$

$\therefore\ k = 6$

Now ${}^{15}C_9\left(\frac{a^6}{b^9}\right) = {}^{15}C_6 \frac{a^9}{b^6} \Rightarrow (ab)^3 = 1$

$ab = 1$

$\therefore\ |ab - 5| = 4$

15. Let $A = \begin{bmatrix} p & q \\ m & n \end{bmatrix}$ and 'd' is the determinant of matrix A and $d \neq 0$. If $|A - d \,.\, \text{adj } A| = 0$, then

(1) $1 + d^2 = m^2 + n^2$
(2) $(1 + d)^2 = (p + n)^2$
(3) $(1 - d)^2 = (p - n)^2$
(4) $1 + d^2 = p^2 + n^2$

Ans. **(4)**

Sol. Given $|A - d \text{ adj } A| = 0$

$$\Rightarrow \left|\begin{bmatrix} p & q \\ m & n \end{bmatrix} - d\begin{bmatrix} n & q \\ -m & p \end{bmatrix}\right| = 0$$

$$\Rightarrow \begin{vmatrix} p - nd & q + dq \\ m + md & n - dp \end{vmatrix} = 0$$

$$\Rightarrow (p - nd)(n - dp) - qm(d + 1)^2 = 0$$

$$\Rightarrow pn - dp^2 - n^2d + npd^2 - qm(d + 1)^2 = 0$$

$$\Rightarrow (pn - mq) + d^2 . (pn - mq) - d(p^2 + n^2 + 2qm) = 0$$

$$\Rightarrow d + d^3 - d((p + n)^2 - 2d) = 0$$

$$\Rightarrow 1 + d^2 = (p + n)^2 - 2d$$

$$\Rightarrow (1 + d)^2 = (p + n)^2$$

16. If $a_n = \frac{-2}{4n^2 - 16n + 15}$ and $a_1 + a_2 + \ldots\ldots\ldots + a_{25} = \frac{m}{n}$, where m & n are coprime numbers, then (m + n) is equal to

(1) 182
(2) 192
(3) 191
(4) 201

Ans. **(3)**

Sol. $a_n = \frac{-2}{(2n-3)(2n-5)}$

$$a_n = \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)} = \frac{1}{2n-3} - \frac{1}{2n-5}$$

$a_1 + a_2 + \ldots\ldots + a_{25} =$

$$\left(\left(\frac{1}{-1}\right) - \left(\frac{1}{-3}\right) + \left(\frac{1}{1} - \frac{1}{-1}\right) + \left(\frac{1}{3} - \frac{1}{1}\right) + \ldots + \left(\frac{1}{47} - \frac{1}{45}\right)\right)$$

$$\frac{1}{47} + \frac{1}{3} = \frac{50}{141}$$