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PART : MATHEMATICS

1. $\lim\limits_{x\to 0} \dfrac{48\int_0^x \frac{t^3}{1+t^6}dt}{x^4}$ is equal to.

(1) 3 (2) 4 (3) 5 (4) 12

Ans. **(4)**

Sol. $\lim\limits_{x\to 0}\left(\dfrac{48\int_0^x \frac{t^3}{1+t^6}dt}{x^4}\right)$ $\quad(\frac{0}{0}$ form)

using Lebnitz theorem we have

$$\lim_{x\to 0}\frac{48\frac{x^3}{1+x^6}}{4x^3}=\frac{48}{4}=12$$

2. The coefficient of x^{301} in $(1+x)^{500}+x(x+1)^{499}+x^2(1+x)^{498}+\text{-------}(1+x)^0x^{500}$ is.

(1) ${}^{500}C_{200}$ (2) ${}^{500}C_{300}$ (3) ${}^{501}C_{300}$ (4) ${}^{501}C_{200}$

Ans. **(4)**

Sol. $(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}\text{-------}+(1+x)^0x^{500}$

$$=\frac{(1+x)^{500}\left[1-\left(\frac{x}{1+x}\right)^{501}\right]}{1-\frac{x}{1+x}}$$

$$=\frac{(1+x)^{501}\left[1-\left(\frac{x}{1+x}\right)^{501}\right]}{1+x-x}$$

$=(1+x)^{501}-x^{501}$

Now coefficient of x^{301} in $(1+x)^{501}-x^{501}$

$={}^{501}C_{301}={}^{501}C_{200}$

3. If $\tan 15°+\dfrac{1}{\tan 75°}+\dfrac{1}{\tan 105°}+\tan 195°=2\alpha$. Then find $(\alpha+\dfrac{1}{\alpha})$

(1) 2 (2) 3 (3) 4 (4) –4

Ans. **(3)**

Sol. $\Rightarrow \tan 15°+\tan 15°-\tan 15+\tan 15=2\alpha$

$\Rightarrow \alpha=\tan 15°$

Now $\alpha+\dfrac{1}{\alpha}=\tan 15°+\dfrac{1}{\tan 15°}$

$$=\frac{1+\tan^2 15°}{\tan 15°}=\frac{2}{\sin 30°}=4$$

4. The number of four digit numbers using the digits 1,2,3,5 (repetition is allowed) and sum of digits is divisible by 15 is

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4. The number of four digit numbers using the digits 1,2,3,5 (repetition is allowed) and sum of digits is divisible by 15 is

Ans. **(12)**

Sol. Number should be formed using 5,5,3,2

$\Rightarrow \frac{4!}{2!} = 12$

5. Let R = {(a,b), (b,c)} be a relation defined on set A {a,b,c}. Minimum number of elements should be added in relation R to make this relation symmetric and transitive is

(1) 2 (2) 3 (3) 5 (4) 7

Ans. **(4)**

Sol. Required elements in sets for symmetric and transitive (a,a) (b,b) (c,c) (b,a) (c,b) (a,c) (c,a) Minimum no. of elements required = 7

6. If mean and variance of 7 varieties are 8 & 16 respectively. If one data 14 is removed the new mean and variance are a and b respectively then the value of $2a - 3b + 5$ is

(1) 16 (2) -16 (3) 6 (4) 0

Ans. **(2)**

Sol. $\sum_{i=1}^{7} x_i = 7 \times 8 \quad = 56$

$\Sigma x_i^2 = 7(16 + 64) = 560$

Where 14 is removed then

Now man a $\Rightarrow 6a = \sum_{i=1}^{7} x_i - 14$

$\Rightarrow 6a + 14 = 56$

$\Rightarrow a = 7$

New variance b $\Rightarrow 6b = \left(\sum_{i=1}^{7} x_i^2 - 196\right) - (7)^2.6$

$\Rightarrow 6b = 560 - 196 - 294$

$\Rightarrow 6b = 560 - 490 = 70$

$\Rightarrow 3b = 35$

Hence $2a - 3b + 5 = 14 - 35 + 5$

$= -16$

7. Let a line meets the OX & OY axes at A and B respectively in first quadrant such that OA = a, OB = b and perpendicular from origin on it makes an angle $\frac{\pi}{6}$ with y –axis. If area of triangle OAB is $\frac{98\sqrt{3}}{3}$ then the value of $a^2 - b^2$ is

(1) $\frac{392}{3}$ (2) $\frac{395}{3}$ (3) $\frac{389}{3}$ (4) $\frac{403}{3}$

Ans. **(1)**

Sol. Area of $\Delta OAB = \frac{98}{3}\sqrt{3}$

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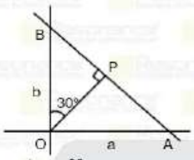
Sol. Area of $\Delta OAB = \frac{98}{3}\sqrt{3}$



$\Rightarrow \frac{1}{2}ab = \frac{98}{3}\sqrt{3} \Rightarrow \sqrt{3}ab - 196$ ....(1)

Now OP = OB cos 30° = OA cos 60°

$\Rightarrow b.\frac{\sqrt{3}}{2} = a.\frac{1}{2}$

$\Rightarrow \sqrt{3}b = a$ ....(2)

by (1) and (2)

$a^2 = 196$

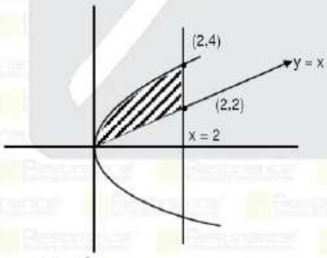
and $b^2 = \frac{a^2}{3}$

so $a^2 - b^2 = a^2 - \frac{a^2}{3} = \frac{2a^2}{3} = \frac{2}{3} \times 196 = \frac{392}{3}$

8. If the area enclosed in the region $\{(x,y) ; y^2 \le 8x, x \le 2 \text{ and } y \ge x\}$ is α then the value of 3α is

Ans. (10.00)

Sol. Area $= \int_0^2 2\sqrt{2}\, x^{1/2}\, dx - \frac{1}{2}(2)(2)$



$\Rightarrow \alpha = \frac{4\sqrt{2}}{3} x^{3/2} \Big|_0^2 - 2$

$\Rightarrow \alpha = \frac{16}{3} - 0 - 2$

$\Rightarrow \alpha = \frac{10}{3} \qquad \Rightarrow 3\alpha = 10$

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9. If $z = 1 + i$ and $z_1 = \frac{i + z(1-i)}{\bar{z}(1-z)}$ then the value of $\frac{12}{\pi}$ arg(z_1) is

Ans. (3)

Sol. $z_1 = \frac{i + (1-i)^2}{\bar{z}(-i)} = \frac{i + 1 + i^2 - 2i}{\bar{z}(-i)}$

$= \frac{-i}{\bar{z}(-i)} = \frac{1}{\bar{z}} = \frac{z}{|z|^2} = \frac{z}{2}$

arg z_1 = arg $z = \frac{\pi}{4}$

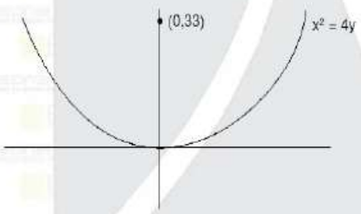
$\Rightarrow \frac{12}{\pi} \arg(z_1) = \frac{12}{\pi} \times \frac{\pi}{4} = 3$

10. Let P(h,k) be any two points on $x^2 = 4y$ which is at shortest distance from Q_1 (0,33) then difference of distance of P(h,k) from directrix of $y^2 = 4(x + y)$ is ?

(1) 2 (2) 4 (3) 6 (4) 8

Ans. (2)

Sol.



For line of $-\frac{2}{h}(X-h)$ shortest distance from (0,33) the line must be normal to the parabola $x^2 = 4y$ at P(h,k) Equation normal at P(h,k) is

$y - k = -\frac{2}{h}(X-h)$

passes (0,33) $\Rightarrow 33 - k = -\frac{2}{h}(-h)$

$\Rightarrow 33 - k = 2$

and $h = \pm 2\sqrt{31}$

Now directrix of y^2 4x + 4y is $x + 2 = 0$

Distance of directrix from $(2\sqrt{31}, 31)$ is $d_1 = 2 + 2\sqrt{31}$

distance of directrix from $(-2\sqrt{31}, 31)$ is $d_2 = 2\sqrt{31} - 2$

$|d_1 - d_2| - 4$

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11. If $a_n = \frac{-2}{4n^2 - 16n + 15}$, and $a_1 + a_2 + \cdots\cdots + a_{25} = \frac{m}{n}$ where m and n are co-prime, then the value of $m + n = \cdots\cdots$

Ans. (191)

Sol. $a_1 + a_2 + \cdots\cdots + a_{25} = \sum_{n=1}^{25} a_n$

$= \Sigma \frac{-2}{4n^2 - 16n + 15} = \Sigma \frac{-2}{(2n-5)(2n-3)}$

$= \sum_{n=1}^{25}\left(\frac{1}{2n-3} - \frac{1}{2n-5}\right)$

$= \left[\left(\frac{1}{-1} - \frac{1}{-3}\right) + \left(\frac{1}{1} - \frac{1}{-1}\right) + \left(\frac{1}{3} - \frac{1}{1}\right) + \cdots\cdots\cdots\right.$

$= \frac{1}{2(25)-3} + \frac{1}{3} = \frac{50}{141}$

$\Rightarrow m = 50, n = 141$

$\Rightarrow m + n = 191$

12. If $S = \{1,2,3,4,5\}$ then the number of one-one function $f : S \to P(S)$, where $P(S)$ is the power set of S, is

(1) ${}^{30}P_5$ (2) ${}^{32}P_5$ (3) ${}^{31}C_5$ (4) ${}^{30}C_5$

Ans. (2)

Sol. $S = \{1,2,3,4,5\}$

$n(P(S)) = 2^5$

Total number of one-one function from S to P(S)

$= {}^{32}P_5$

13. Numbers on the faces of an unbiased die are the elements of set $\{-2,-1,0,1,2,3\}$. If the die is rolled 5 times then the probability that the product of numbers appearing on die is positive, is

(1) $\frac{5}{27}$ (2) $\frac{11}{54}$ (3) $\frac{65}{324}$ (4) $\frac{67}{324}$

Ans. (3)

Sol. All positive or 3 positive and 2 negative or 1 positive and 4 negative

$\Rightarrow P = \left(\frac{3}{6}\right)^5 + {}^5C_2\left(\frac{2}{6}\right)^2\left(\frac{3}{6}\right)^3 + {}^5C_1\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)^4$

$= \frac{3^5}{6^5} + \frac{10.2^2.3^3}{6^5} + \frac{5.3.2^4}{6^5}$

$= \frac{243+1080+240}{6^5} = \frac{1560}{6^5} = \frac{65}{324}$

14 If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of

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14 If coefficient of x^{15} in expansion of $\left(ax^3+\frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of $\left(ax^{\frac{1}{3}}+\frac{1}{bx^3}\right)^{15}$ then $|ab-5|$ is equal to

Ans. (4)

Sol. $(r+1)^{th}$ term in the expansion of $\left(ax^3+\frac{1}{bx^{\frac{1}{3}}}\right)^{15}$

$$T_{r+1}={}^{15}C_r\left(ax^3\right)^{15-r}\left(\frac{1}{bx^{\frac{1}{3}}}\right)^r$$

$$={}^{15}C_r\,a^{15-r}\,x^{45-3r-\frac{r}{3}}\,b^{-r}$$

for coefficient of x^{15}

$$45-3r-\frac{r}{3}=15$$

$$30=\frac{10r}{3}$$

$r=9$

and $(r+1)^{th}$ term in $\left(ax^{\frac{1}{3}}+\frac{1}{bx^3}\right)^{15}$

$$T_{r+1}={}^{15}C_r\left(ax^{\frac{1}{3}}\right)^{15-r}\left(\frac{1}{bx^3}\right)^r$$

$${}^{15}C_r\,a^{15-r}\,b^{-r}\,x^{\frac{15-r}{3}-3r}$$

for coefficient of x^{-15}

$$\frac{15-r}{3}-3r=-15$$

$15-r-9r=-45$

$60=10r\Rightarrow r=6$

Now ${}^{15}C_9\,a^6\,b^{-9}={}^{15}C_6\,a^9\,b^{-6}$

$1=a^3b^3$

$ab=1$

Now $|ab-5|=|1-5|=4$

15. If Y= Y(x) satisfies the differential equation

$$\frac{dy}{dx}-\left(\frac{3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}\right)y=2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right\}$$ and $y(0)=0$ then value of $y(1)$ is

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15. If Y= Y(x) satisfies the differential equation

$$\frac{dy}{dx}-\left(\frac{3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}\right)y=2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right\}$$ and $y(0)=0$ then value of $y(1)$ is

(1) $e^{\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}-1\right)}$ (2) $e^{\frac{1}{\sqrt{2}}\left(1-\frac{\pi}{4}\right)}$ (3) 1 (4) $e^{\frac{1}{2}\left(1-\frac{\pi}{4}\right)}$

Ans. (2)

Sol. $$\frac{dy}{dx}-\frac{3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}y=2x\exp\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right\}$$

$$\text{I.f.}=e^{-\int\frac{3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}dx}$$

$$=e^{-\int\frac{3x^2\cdot x^3\tan^{-1}x^3}{(1+x^6)^{3/2}}dx}$$

Let $\tan^{-1}x^3=t\Rightarrow\frac{3x^2}{1+x^6}dx=dt$

$$=e^{-\int\frac{x^3}{\sqrt{1+x^6}}\cdot\frac{3x^2\tan^{-1}x^3}{(1+x^6)}dx}$$

$$=e^{-\int\frac{\tan t}{\sec t}\cdot t\,dt}$$

$$=e^{-\int t\sin t\,dt}=e^{(t\cos t-\sin t)}$$

$$=e^{\left(\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}\right)}$$

Hence solutions of the differential equation is

$$y\cdot e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}}-\frac{x^3}{\sqrt{1+x^6}}}=\int 2\times e^{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}}\cdot e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}dx+C$$

$=x^2+C$

Passes through $(0,0)\Rightarrow C=0$

Hence $y(1)=e^{\frac{1}{\sqrt{2}}\left(1-\frac{\pi}{4}\right)}$

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