1. The range of $y = \frac{x^2+2x+1}{x^2-8x+12}$ is, for $x \in (2, 6)$

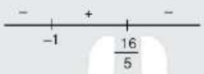
(1) $(-\infty, \infty)$ (2) $\left(-\infty, \frac{21}{4}\right]$ (3) $(1, \infty)$ (4) $\left[\frac{21}{4}, \infty\right)$

Ans. (2)

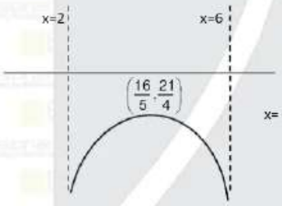
Sol. $y = \frac{x^2+2x+1}{x^2-8x+12} = \frac{(x+1)^2}{(x-2)(x-6)}$ ... (1)

$\Rightarrow \frac{dy}{dx} = \frac{-2(x+1)(5x-16)}{(x-2)^2(x-6)^2}$

Number line of $\frac{dy}{dx}$ =



So Graph of $y = \frac{(x+1)^2}{(x-2(x-6)}$ for $x \in (2, 6)$



$x = \frac{16}{5}$ is maxima point

at $x = \frac{16}{5}$ Þ $y = \frac{21}{4}$ so range = $\left(-\infty, \frac{21}{4}\right]$

2. If the foci of hyperbola with eccentricity $\sqrt{2}$ are at $(1 \pm \sqrt{2}, 0)$ then the length of latusrectum is

(1) $\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Ans. (2)

Sol. $S(1+\sqrt{2}, 0)$ $S'(1-\sqrt{2}, 0)$

$SS' = 2ae = 2\sqrt{2} \Rightarrow ae = \sqrt{2} \Rightarrow a\sqrt{2} = \sqrt{2} \Rightarrow a = 1$

$L.R. = \frac{2b^2}{a} = \frac{2a^2(e^2-1)}{a} \Rightarrow 2a(e^2-1) = 2$

3. If $\int_0^a \frac{x}{\sqrt{a+x}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then the value of a is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(2)**

$$\int_0^a \frac{x\left(\sqrt{a+x}+\sqrt{x}\right)}{a}dx = \frac{1}{a}\left[\int_0^a x\sqrt{a+x}\,dx + \int_0^a x^{3/2}dx\right]$$

Put $a + x = t^2 \Rightarrow dx = 2tdt$

$$\frac{1}{a}\int_{\sqrt{a}}^{\sqrt{2a}}\left(t^2-a\right)2t^2dt + \frac{2a^{3/2}}{5}$$

$$\frac{1}{a}\left[\frac{2t^5}{5}-\frac{2at^3}{3}\right]_{\sqrt{a}}^{\sqrt{2a}} + \frac{2a^{3/2}}{5}$$

$$\frac{4a\sqrt{2a}+10a\sqrt{a}}{15} = \frac{16+20\sqrt{2}}{15} \Rightarrow a = 2$$

4. If a_n is n^{th} term and s_n is sum of n terms of an A.P. such that $a_7 = 6$, $S_n = 0$ and a_1a_4 is minimum then the value of $n! + a_n(n + 2)$ is

(1) 140 (2) 139 (3) 145 (4) 141

Ans. **(4)**

Sol. $a_7 = 6$

$\Rightarrow a + 6d = 6$

$\Rightarrow a = 6 - 6d$

$a_1a_4 = a(a + 3d)$

$= (6 - 6d)(6 - 6d + 3d)$

$= 18(d - 1)(d - 2)$

$= 18(d^2 - 3d + 2)$

Let $f(d) = 18(d^2 - 3d + 2)$

$f'(d) = 18(2d - 3)$

at $d = \frac{3}{2}$, a_1a_4 is minimum

So, $a = 6 - 6d = -3$

$S_n = 0$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 0$

$\Rightarrow \frac{n}{2}\left[-6+(n-1)\frac{3}{2}\right) = 0]$

$\Rightarrow n = 5$

Now, $a_5 = a + 4d = 3$

Now $n! + a_n(n + 2) = 5! + 3 \times 7$

$= 120 + 21 = 141$

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5. The coefficient of x^{-6} in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$ is

5. The coefficient of x^{-6} in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$ is

(1) 5140 (2) 5240 (3) 5040 (4) 4940

Ans. (3)

Sol. $T_{r+1} = {}^9C_r\left(\frac{5}{2x^2}\right)^r\cdot\left(\frac{4x}{5}\right)^{9-r}$

$= {}^9C_r\,\frac{2^{18-3r}}{5^{9-2r}}\,x^{9-3r}$

for coefficient of $x^{-6} \Rightarrow 9-3r = -6 \Rightarrow r = 5$

coff. of $x^{-6} = \frac{{}^9C_5\,.\,2^3}{5^{-1}}$

$= \frac{9.8.7.5}{4.3.2.1}.(8\times 5)$

$= 5040$

6. If $\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{11}{21}$ then the value of n^2+n+15 is.

(1) 45 (2) 50 (3) 35 (4) 57

Ans. (1)

Sol. $\frac{\left(\frac{(2n+1)!}{(n+2)!}\right)}{\frac{(2n-1)!}{(n-1)!}} = \frac{11}{21}$

$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$

$\Rightarrow 42(2n+1) = 11\,(n+1)\,(n+2)$

$\Rightarrow 11n^2 - 51n - 20 = 0$

$\Rightarrow (n-5)\,(11n+4) = 0$

$\Rightarrow n = 5$

$\Rightarrow n^2 + n + 15 = 45$

7. If $z = \frac{i-1}{\cos\pi/6 + i\sin\pi/6}$, then z is equal to

(1) $\sqrt{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$ (2) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{7\pi}{12}-i\sin\frac{7\pi}{12}\right)$ (4) $\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$

Ans. (1)

Sol. $Z = \frac{i-1}{\cos\pi/6 + i\sin\pi/6} = \frac{\sqrt{2}\,e^{i3\pi/4}}{e^{i\pi/6}} = \sqrt{2}\,e^{i7\pi/12}$

$= \sqrt{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$

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8. Sum of series $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + \ldots + 15.29^2$ is

8. Sum of series $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + ... + 15.29^2$ is

Ans. **(6952)**

Sol. $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + ... + 15.29^2 = !! 5952$

$$\sum_{r=1}^{15}(2r-1)^2 - \sum 2r(r-1)^2$$

$$1 + \sum_{r=1}^{7}(2r+1)\,(4r+1)^2 - 2r(4r-1)^2$$

$$1 + \sum_{r=1}^{7}2r(16r + (4r+1)^2$$

$$1 + 48\sum_{r=1}^{7}r^2 + 8\sum r + \sum 1$$

$$1 + 48 \times \frac{7\times8\times15}{6} + 8\times\frac{7\times8}{2} + 7$$

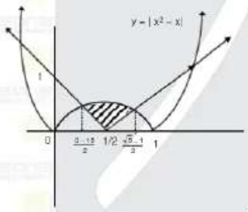
$$1 + 56 \times 120 + 56 \times 4 + 7$$

$$8 + 56 \times 124 = 6944 + 8 = 6952$$

9. If area of curve $|2x - 1| \le y \le |x^2 - x|$ is A then value of $(6A + 11)^2$ is.

Ans. **125**

Sol. $y \ge |2x - 1|$ $y \le |x^2 - x|$



Both curve are symmetric about $x = \frac{1}{2}$ Hence

$$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}\left(\left(x - x^2\right) - (1 - 2x)\right)dx$$

$$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}\left(-x^2 + 3x - 1\right)dx = 2\left(\frac{-x^3}{3} + \frac{3}{2}x^2 - x\right)_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$$

on solving $6A + = 5\sqrt{5}$

$(6A + 11)^2 = 125$ Ans.

10. The equation $e^{4x} + 3e^{3x} + 13e^{2x} - 8e^x + 1 = 0$ has

(1) Two real root both are positive (2) No real roots

10. The equation $e^{4x} + 3e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0$ has

(1) Two real root both are positive
(2) No real roots
(3) Four real roots of which two are negative
(4) Two real roots both are negative

Ans. (4)

Sol. Let $e^x = t$

$f(t) = t^4 + 3t^3 + 13t^2 - 8t + 1$

$f'(t) = 4t^3 + 9t^2 + 26t - 8$

$f''(t) = 12t^2 + 18t + 26 > 0 \ \forall\ t$

$\Rightarrow f'(t) = 0$ has at most one real root

$\because f'(0) < 0$ and $f'\left(\frac{1}{2}\right) > 0$

then real root of $f'(t) = 0$ lies between 0 and $\frac{1}{2}$

$\because f\left(\frac{1}{4}\right) < 0$ and $f\left(\frac{1}{2}\right) > 0$ and $f(0) > 0$

$\Rightarrow f(t) = 0$ has at most two real roots

Let α, β be real roots of $f(t) = 0$

for roots of given equation $\Rightarrow e^x = \alpha, \beta$

$\Rightarrow x = \ell n\alpha, \ell n\beta < 0$

given equation has two negative real roots

11. If $a, b \in I$ and relation R_1 is defined as $a^2 - b^2 \in I$ and relation R_2 is defined as $2 + \frac{a}{b} > 0$ then.

(1) R_1 is symmetric but R_2 is not
(2) R_2 is symmetric but R_1 is not
(3) R_1 and R_2 are both symmetric
(4) R_1 and R_2 are both transitive

Ans. (1)

Sol. $a R_1 b \Rightarrow a^2 - b^2 \in I$ always true Hence equivalence

$a R_2 b \Rightarrow 2 + \frac{a}{b} > 0$ not symmetric when $a = 2, b = 1$

When $a = -8$, $b = 6$, $c = 3$

Clearly $(-8, 6) \in R_2$, $(6, 3) \in R_2$ but $(-8, 3) \notin R_2$

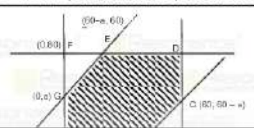
Hence not transitive.

12. Let A be an event that the absolute difference of two real number lying between 0 and 60 is less than a. If the probability of happening of event A is $\frac{11}{36}$ then a is equal to

Ans. 10

Sol. $|x - y| < a \Rightarrow -a < x - y < a$

$\Rightarrow \quad x - y < a$ and $x - y > -a$



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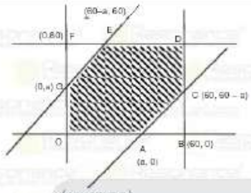

$$P(A) = \frac{ar(OACDEG)}{(OBDF)}$$

$$= \frac{ar(OBDF) - ar(ABC) - ar(EFG)}{ar(OBDF)}$$

$\Rightarrow$ $$\frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$$

$\Rightarrow$ $1100 = 3600 - (60-a)^2$

$\Rightarrow$ $(60-a)^2 = 2500 \Rightarrow 60 - a = 50$

$\Rightarrow$ $a = 10$

13. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{b} \times \vec{c}$ & $\vec{r}.\vec{a} = 0$ then the value of $25.|\vec{r}|^2$

Ans. 202.75

Sol. $\vec{r} \times \vec{b} = \vec{b} \times \vec{c} = -\vec{c} \times \vec{b}$

$\Rightarrow \left(\vec{r} + \vec{c}\right) \times \vec{b} = 0 \Rightarrow \vec{r} + \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \lambda\vec{b} - \vec{c}$

$\vec{r}.\vec{a} = \lambda\left(\vec{b}.\vec{a}\right) - \vec{c}.\vec{a} = 0$

$\lambda = \frac{\vec{c}.\vec{a}}{\vec{b}.\vec{a}} = \frac{-1}{-10} = \frac{1}{10}$

$\vec{r} = \frac{1}{10}\vec{b} - \vec{c} = \frac{1}{10}\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$\vec{r} = \frac{-9}{10}\hat{i} - \frac{21}{10}\hat{j} - \frac{17}{10}\hat{k}$

$10\vec{r} = -9\hat{i} - 21\hat{j} - 17\hat{k}$

$100|\vec{r}|^2 = 811$

$25|\vec{r}|^2 = \frac{811}{4} = 202.75$