JEE-Main-30-01-2023 (Memory Based) [Evening Shift]

Physics

Question: A person covers 4 km by 3 km/h and another 4 km by 5 km/h. Find V_{avg}.

Options:

(a) 4 km/h

(b) 3.75 km/h

(c) 5 km/h

(d) 8 km/h

Answer: (b)

Solution:

$$V_{avg} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{4+4}{\frac{4}{3}+\frac{4}{5}} = \frac{8}{\frac{32}{15}} = \frac{15}{4} = 3.75 km/h$$

Question: A prism has $A = 6°, u_1 = 1.54$ Another inverted prism has $u_2 = 1.72$. Fiind A_2 for dispersion without deviation.

Options:

(a) $4.5°$

(b) $2.5°$

(c) $1.5°$

(d) $5.5°$

Answer: (a)

Solution: $\delta = (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0$

$$0.54 \times 6° - 0.72 \times A_2 = 0$$

$$\Rightarrow A_2 = 4.5°$$

Question: Match the columns

a. Pressure gradient	p. $Kg\, ms^{-1}$
b. Impulse	q. $Kg\, s^{-2}$
c. Viscosity	r. $Kg\, m^{-2}s^{-2}$
d. Surface tension	s. $Kg\, m^{-1}s^{-1}$

Options:

(a) $a-p, b-r, c-q, d-s$

(b) $a-r, b-p, c-s, d-q$
(c) $a-p, b-s, c-q, d-r$
(d) $a-q, b-r, c-p, d-s$

Answer: (b)

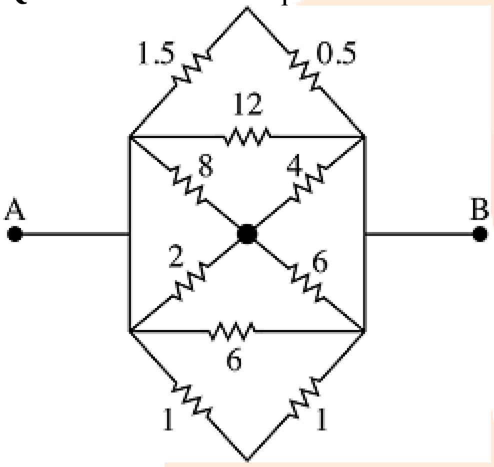
Solution: Pressure gradient $=\dfrac{\Delta P}{\Delta x}=Kg\,m^{-2}s^{-2}$

Impulse $\Delta\vec{P}=Kg\,ms^{-1}$

Viscosity $\dfrac{F}{6\pi rv}=Kgm-s$

Surface tension $=\dfrac{F}{L}$

Question: Find the equivalent resistance between A and B.



Options:

(a) $\dfrac{3}{2}$

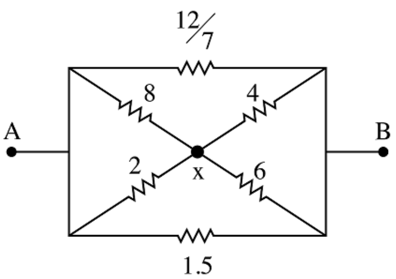
(b) $\dfrac{2}{3}$

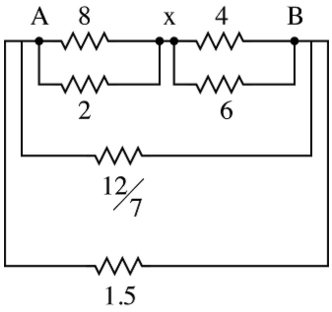
(c) $\dfrac{4}{3}$

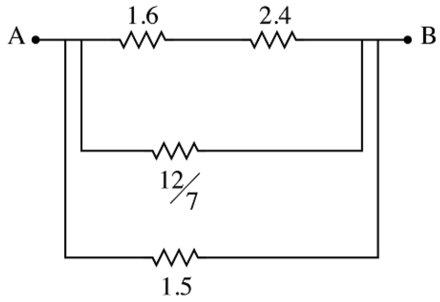
(d) $\dfrac{3}{4}$

Answer: (b)

Solution: The given circuit can be reduced to







$$\frac{1}{R}=\frac{1}{4}+\frac{7}{12}+\frac{2}{3}=\frac{3+7+8}{12}=\frac{3}{2}$$

$$R=\frac{2}{3}$$

Question: A stone of mass 1 kg tied to a string of length 180 cm is whirled in a horizontal circle with angular speed $w=1rad/\sec$. The centripetal acceleration of the stone is about-

Options:

(a) $0.3m/s^2$

(b) $0.9m/s^2$

(c) $1.8m/s^2$

(d) $3.6m/s^2$

Answer: (c)

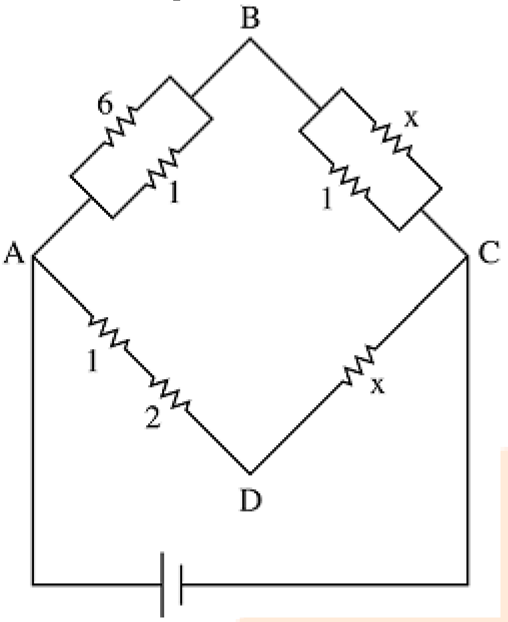
Solution: $l=180\times10^{-2}=1.8m$

$\omega=1rad/s$

$a=\omega^2 l$

$=(1)^2(1.8)=1.8m/s^2$

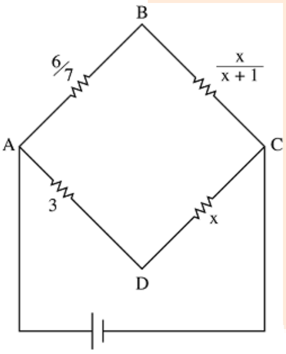
Question: If potential difference across B and D is zero then find the value of x



Options:
(a) 1
(b) 2.5
(c) 5
(d) 7.5

Answer: (b)

Solution: $\because$ PD across B and D is 0, so this must be a balanced WSB



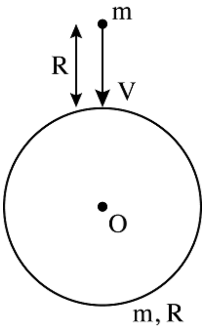
$$\Rightarrow \frac{\frac{6}{7}}{3} = \frac{\frac{x}{x+1}}{x}$$

$$\Rightarrow \frac{2}{7} = \frac{1}{x+1}$$

$$\Rightarrow 2x + 2 = 7$$

$$\Rightarrow x = 2.5$$

Question: An object of mass M is released from distance R from the surface of Earth of mass M. R is the radius of Earth. Find the speed with which it strikes the earth.



Options:

(a) $\sqrt{\frac{GM}{R}}$

(b) $\sqrt{\frac{GM}{2R}}$

(c) $\sqrt{\frac{GM}{4R}}$

(d) $\sqrt{\frac{GM}{8R}}$

Answer: (a)

Solution: By energy conservation

$$-\frac{GMm}{2R} = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{GMm}{2R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM}{R}}$$

Question: A man of mass 10 kg shoots bullets of 0.02 kg at 180 bullets per sec at 100 m/s. Find impulse imparted to gun.

Options:

(a) 320 kg m/s

(b) 200 kg m/s

(c) 360 kg m/s

(d) 180 kg m/s

Answer: (c)

Solution: Impulse $= \Delta P$

$$= n(mv)$$

$$= 180 \times 0.02 \times 100 = 360\, kgm/s$$

Question: A particle performs SHM and its velocity and displacement from equilibrium are related by the equation $4V^2 = 50 - x^2$. Find the time period of SHM

Options:

(a) π seconds

(b) 2π seconds
(c) 4π seconds
(d) 8π seconds

Answer: (c)

Solution: Comparing with $V^2 = \omega^2\left(A^2 - x^2\right)$

$$V^2 = \frac{1}{4}\left(50 - x^2\right)$$

$$\Rightarrow \omega = \frac{1}{2}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{V_2} = 4\pi \text{ seconds}$$

Question: $X_L = 200\Omega, X_C = 100\Omega, R = 100\Omega, V_{rms} = 200\sqrt{2}V$. Find i_{rms}

Options:
(a) 2A
(b) 3A
(c) 5A
(d) 7A

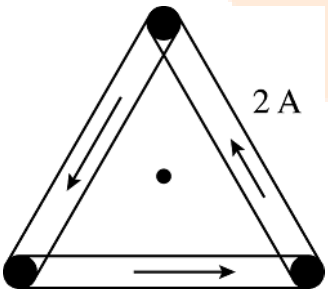
Answer: (a)

Solution:

$$z = \sqrt{100^2 + \left(200 - 100\right)^2} = 100\sqrt{2}$$

$$i_{rm} = \frac{V_{rm}}{z} = \frac{200\sqrt{2}}{100\sqrt{2}} = 2A$$

Question: A current 2A is flowing through the sides of an equilateral triangle loop of side $4\sqrt{3}m$ as shown. Find the magnetic field induction at the centroid of the triangle.



Options:
(a) $3\sqrt{3}\times 10^{-7}T$
(b) $\sqrt{3}\times 10^{-7}T$
(c) $2\sqrt{3}\times 10^{-7}T$
(d) $5\sqrt{3}\times 10^{-7}T$

Answer: (a)

Solution: $B = 3\left[\frac{\mu_0 i}{4\pi r}\left(\sin\alpha + \sin\beta\right)\right]$

$$B=3\left[\frac{\mu_0 i}{4\pi r}\left(\sin 60^\circ+\sin 60^\circ\right)\right]$$

$$B=3\left[\frac{\mu_0 i}{4\pi r}\times\sqrt{3}\right]\ r=\frac{a}{2\sqrt{3}}$$

$$B=3\left[\frac{2\mu_0 i\times 3}{4\pi a}\right]=\frac{9\mu_0 i}{2\pi a}$$

Putting the values, we get

$$B=3\sqrt{3}\times 10^{-7}$$

Question: A faulty scale reads $5^\circ C$ at melting point and 95° at steam point. Find original temperature if this faulty scale reads $41^\circ C$

Options:

(a) $40^\circ C$

(b) $41^\circ C$

(c) $36^\circ C$

(d) $45^\circ C$

Answer: (a)

Solution: $\frac{X^\circ C-0}{100-0}=\frac{41-5}{95-5}$

$$X^\circ C=100\times\frac{36}{90}=40^\circ C$$

Question: A circular coil of 100 turns and area $14\times 10^{-2}m^2$ is initially held in a plane perpendicular to magnetic field B_0. It is rotated about its diameter at an angular velocity of 28 revolutions/min. Find max. EMF induced

Options:

(a) $\frac{11B_0}{15}$

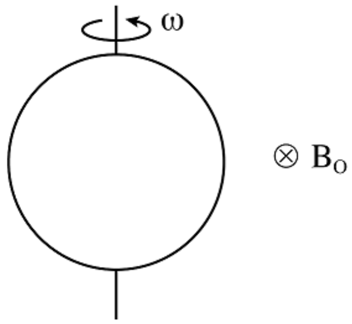
(b) $\frac{22B_0}{15}$

(c) $\frac{44B_0}{15}$

(d) $\frac{20B_0}{15}$

Answer: (b)

Solution:



$\phi = NAB_0 \cos \omega t$

$e = \left|\frac{d\phi}{dt}\right|$

$e = NAB_0 \omega \sin \omega t$

$e_{\max} = NAB_0 \omega$

$= 100 \times 14 \times 10^{-2} \times B_0 \times \frac{28 \times 2\pi}{60}$

$= \frac{2 \times B_0 \times 28 \times 2 \times 22}{60} = \frac{22B_0}{15}$

Question: Statement 1: The efficiency of heat engine is maximum at $-273°C$

Statement 2: Efficiency of heat engine is $\eta = \frac{1 - T_2}{T_1}$

Options:
(a) S1 is correct, S2 is incorrect
(b) S1 and S2 both are correct
(c) $S1$ is incorrect, S2 is correct.
(d) Both are incorrect
Answer: (b)
Solution: Conceptual

JEE-Main-30-01-2023 (Memory Based)
[Evening Shift]

Chemistry

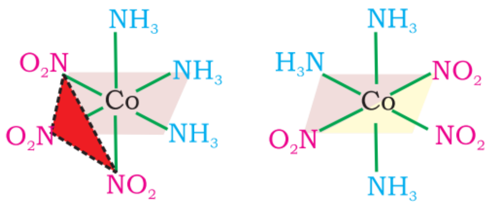
Question: BOD of water is 4 ppm, then which of the following is correct?
Options:
(a) Highly polluted
(b) Slightly polluted
(c) Safe for drink
(d) None of these
Answer: (c)
Solution: Thus, the amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water, is called Biochemical Oxygen Demand (BOD). The amount of BOD in the water is a measure of the amount of organic material in the water, in terms of how much oxygen will be required to break it down biologically. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

Question: Assertion: Antihistamines does not affect secretion of acid in stomach.
Reason: Antiallergic and antacids attack on different receptors
Options:
(a) Assertion and Reason both are correct and is the correct explanation
(b) Both Assertion and Reason are correct
(c) Assertion is incorrect but Reason is correct
(d) Assertion and Reason both are incorrect
Answer: (a)
Solution: NCERT says
Now the question that arises is, "Why do above mentioned antihistamines not affect the secretion of acid in stomach?". The reason is that antiallergic and antacid drugs work on different receptors.

Question: Group 16 H_2E bond dissociation energy.
Options:
(a) Increases down the group
(b) Decreases down the group
(c) First increase then decreases
(d) First decreases then increase
Answer: (b)
Solution: $H_2O = 463\ kJ\ mol^{-1}$, $H_2S = 347\ kJ\ mol^{-1}$, $H_2Se = 276\ kJ\ mol^{-1}$, $H_2Te = 238\ kJ\ mol^{-1}$

Question: Bond angle in Ma_3b_3 type isomer
Options:
(a) 90 & 90
(b) 90 & 120

(c) 120 & 90
(d) 180 & 180
Answer: (a)
Solution:



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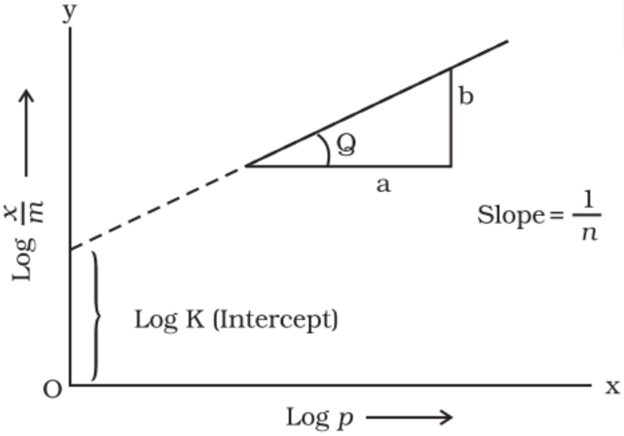
Question: Chloride of which metal is soluble in organic solvent
Options:
(a) Mg
(b) Ca
(c) K
(d) Be
Answer: (d)
Solution: Beryllium halides are essentially covalent and soluble in organic solvents.

Question: Density = 4 g/cm^3, a = 0.5Å, FeO find Z.
Options:
(a) 4
(b) 2
(c) 1
(d) 6
Answer: (a)
Solution: $\text{Density} = \dfrac{Z \times M}{a^3 \times N_A}$

Question: In freundlich isotherm
Slope is 45^o, intercept = 0.6020, Pressure = 0.4 atm find x/m.



Freundlich isotherm

Options:

(a) 1.6
(b) 1.5
(c) 2.6
(d) 1.8
Answer: (a)
Solution:
$\frac{x}{m} = kp^{\frac{1}{n}}$ ∵ Slope = $\frac{1}{n}$ =1 (tan45°)
logk = 0.6020
k = 4
∴ $\frac{x}{m} = 4 \times (0.4)^1 = 1.6$

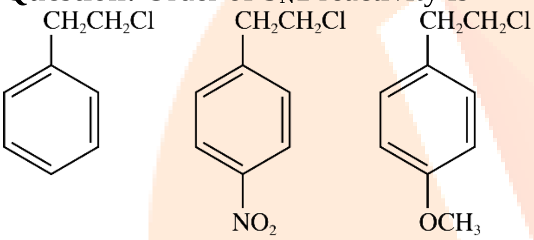
Question: Product Formed on heating lithium nitrate
Options:
(a) Li_2O
(b) $Li(NO_2)$
(c) LiO
(d) Li_3N
Answer: (a)
Solution: Lithium nitrate when heated gives lithium oxide, Li_2O, whereas other alkali metal nitrates decompose to give the corresponding nitrite.
$4LiNO_3 \rightarrow 2Li_2O + 4NO_2 + O_2$

Question: $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$; $\Delta H = -190$ kJ/mol
A. Increasing temperature
B. Increasing pressure
C. Increasing SO_2
D. Increasing O_2
E. Adding catalyst
How many factors are responsible for getting more product?
Options:
(a) 3
(b) 4
(c) 5
(d) 2
Answer: (a)
Solution: For exothermic reaction, increases in temperature retards reaction catalyst has no effect on product formation.

Question: The maximum no of electrons in n = 4 shell
Options:
(a) 72
(b) 50
(c) 16
(d) 32
Answer: (d)
Solution: $2n^2 = 2 \times (4)^2 = 32$

Question: Statement 1: A mixture of chloroform and aniline can be separated by simple distillation.
Statement 2: When separating aniline from a mixture of aniline and water by steam distillation aniline boils below its boiling point.
Options:
(a) Statement I and II are correct
(b) Statement I is correct
(c) Statement II is correct
(d) Both Statement are incorrect
Answer: (a)
Solution:
Distillation: This important method is used to separate (i) volatile liquids from nonvolatile impurities and (ii) the liquids having sufficient difference in their boiling points. Liquids having different boiling points vaporise at different temperatures. The vapours are cooled and the liquids so formed are collected separately. Chloroform (b.p 334 K) and aniline (b.p. 457 K) are easily separated by the technique of distillation.

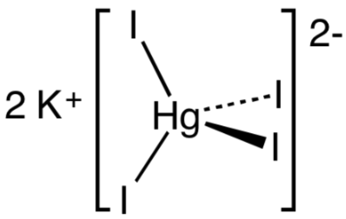
Question: Order of S_N1 reactivity is



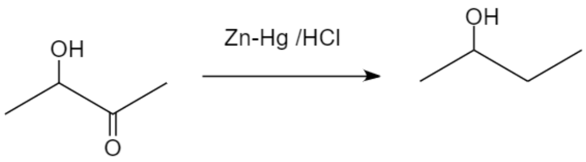
(a) (b) (c)

Options:
(a) c > a > b
(b) c > b > a
(c) a > c > b
(d) a > b > c
Answer: (a)
Solution: S_N1 occur's via carbocation stability.

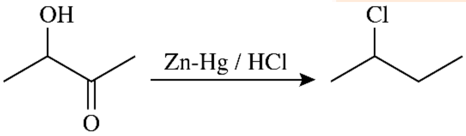
Question: Formula of Nessler's reagent?
Options:
(a) $K_2[HgI_4]$
(b) $K_3[HgI_5]$
(c) $K_4[HgI_6]$
(d) $KI . HgI_2$
Answer: (a)
Solution: K_2HgI_4



Question: Statement-1:

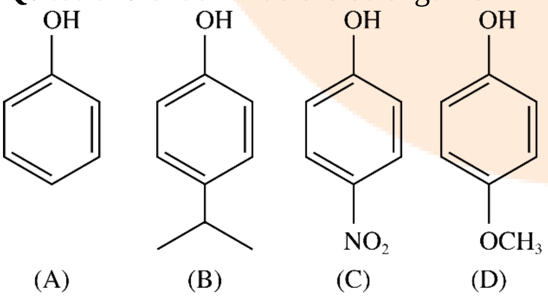


Statement-2: Zn-Hg/HCl will convert C = O into CH_2.
Options:
(a) Statement I is correct
(b) Statement I and II are correct
(c) Statement I is incorrect
(d) Statement II is incorrect
Answer: (c)
Solution:



Question: For a first order reaction half life is 540 s, then calculate time required for 90% decomposition?
Options:
(a) 987 sec
(b) 678 sec
(c) 1281 sec
(d) 1740 sec
Answer: (d)
Solution: 1740 sec

Question: Order of acidic Strength is



Options:
(a) C > A > B > D
(b) C > B > A > D
(c) A > C > B > D
(d) A > B > C > D
Answer: (a)
Solution: C > A > B > D

Question: Lead storage battery have 38% (w/w) H_2SO_4. find the temperature at which the liquid of battery will freeze (i = 2.67; k_f of water = 1.86 K. kg/mol)

Options:

(a) –3.1°C

(b) –31°C

(c) –0.31°C

(d) –0.031°C

Answer: (b)

Solution:

$$m = \frac{38}{98} \times \frac{1000}{62} = 6.254$$

$\Delta T_f = iK_f m = 1.86 \times 2.67 \times 6.25 = 31.059$

or $T_f = -31.059\ ^{o}C$

Question: The option containing the correct match is given as :

List – I	**List – II**
A. $Ni(CO)_4$	**(i)** sp^3
B. $[Ni(CN_4)]^{2-}$	**(ii)** sp^3d^2
C. $[Cu(H_2O)_6]^{2+}$	**(iii)** d^2sp^3
D. $[Fe(CN)_6]^{4-}$	**(iv)** dsp^2

Options:

(a) (A) - (i), (B) - (iv), (C) - (ii), (D) - (iii)

(b) (A) - (iii), (B) - (ii), (C) - (iv), (D) - (i)

(c) (A) - (ii), (B) - (iii), (C) - (iv), (D) - (i)

(d) (A) - (vi), (B) - (ii), (C) - (i), (D) - (iii)

Answer: (a)

Solution:

A. $Ni(CO)_4$	**(i)** sp^3
B. $[Ni(CN_4)]^{2-}$	**(ii)** dsp^2
C. $[Cu(H_2O)_6]^{2+}$	**(iii)** sp^3d^2
D. $[Fe(CN)_6]^{4-}$	**(iv)** d^2sp^3

JEE-Main-30-01-2023 (Memory Based)
[Evening Shift]

Mathematics

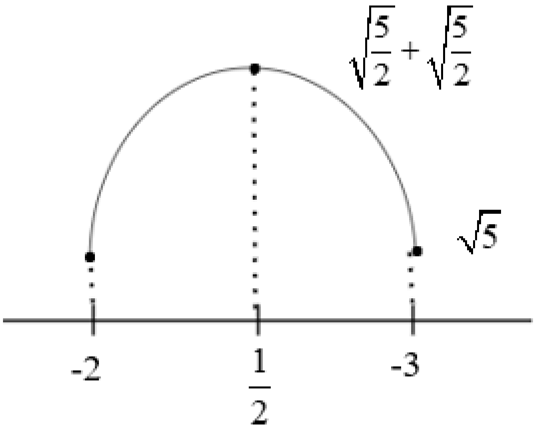
Question: Range of $y=\sqrt{2+x}+\sqrt{3-x}$ is

Answer: $\left[\sqrt{5},\sqrt{10}\right]$

Solution:

$y=\sqrt{2+x}+\sqrt{3-x}$

$y=\sqrt{x-(-2)}+\sqrt{3-x}$



$\therefore$ Minimum value $=\sqrt{5}$

Maximum value $=\sqrt{\frac{5}{2}}+\sqrt{\frac{5}{2}}=\sqrt{10}$

Thus, range $=\left[\sqrt{5},\sqrt{10}\right]$

Question: If $a_1,a_2,a_3,..,a_k$ be an AP with $a_1=1$ and $d=1$, find

$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+...+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$.

Answer: $\frac{\pi}{4}-\cot^{-1}2022$

Solution:

$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+...+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)\quad ....(\text{i})$

Given that $a_1,a_2,a_3,..,a_k$ are in AP with first term = 1 & common difference = 1

$\therefore (\text{i})$ can be written as

$\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+...+\tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$

$= \tan^{-1} a_2 - \tan^{-1} a_1 + \tan^{-1} a_3 - \tan^{-1} a_2 + ... + \tan^{-1} a_{2022} - \tan^{-1} a_{2021}$

$= \tan^{-1} a_{2022} - \tan^{-1} a_1$

$= \tan^{-1} a_{2022} - \tan^{-1} 1$

$= \tan^{-1} a_{2022} - \frac{\pi}{4}$

$= \tan^{-1} 2022 - \frac{\pi}{4}$

$= \frac{\pi}{2} - \cot^{-1} 2022 - \frac{\pi}{4}$

$= \frac{\pi}{4} - \cot^{-1} 2022$

Question: How many 7-digit odd numbers can be formed using the digits $1, 2, 2, 2, 3, 3, 5$?

Answer: 240.00

Solution:

Given digits are $1, 2, 2, 2, 3, 3, 5$

For 7-digit odd numbers, we have the following cases

Case-I:

$\_\ \_\ \_\ \_\ \_\ \_\ \underline{1}$

Number of numbers $= \frac{6!}{3!2!} = 60$

Case-II:

$\_\ \_\ \_\ \_\ \_\ \_\ \underline{3}$

Number of numbers $= \frac{6!}{3!2!} = 60$

Case-III:

$\_\ \_\ \_\ \_\ \_\ \_\ \underline{5}$

Number of numbers $= \frac{6!}{3!} = 120$

$\therefore$ Total number of 7-digit numbers $= 60 + 60 + 120 = 240$

Question: $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a}.\vec{b} = 2$ and $\vec{c} = (\vec{a} \times \vec{b}) - 3\vec{b}$.

Find $\vec{b}.\vec{c}$.

Answer: -48

Solution:

Given that: $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a}.\vec{b} = 2$

And, $\vec{c} = (\vec{a} \times \vec{b}) - 3\vec{b}$

$\vec{b}.\vec{c} = \vec{b} \cdot \left\{\left(\vec{a} \times \vec{b}\right) - 3\vec{b}\right\}$

$= \left(\vec{a} \times \vec{b}\right) \cdot \vec{b} - 3\left|\vec{b}\right|^2$

$= 0 - 3(4)^2$

$= 0 - 48$

$= -48$

Question: Find the 8th common term in the following sequences:
$3, 7, 11, \ldots$ and $1, 6, 11, \ldots.$

Answer: 151.00

Solution:

Given sequences are

$3, 7, 11, 15, \ldots \rightarrow d_1 = 4$

$1, 6, 11, 16, \ldots. \rightarrow d_2 = 5$

$\therefore$ Common difference of the AP of the common terms of above two series is given by LCM $\left(d_1, d_2\right)$

i.e., LCM (4, 5)

Thus, $d = 20$

The first common term is, $a = 11$

$\therefore a_8 = a + 7d = 11 + 7(20) = 11 + 140 = 151$

Question: P is a 3×3 matrix such that $P^T = AP - (a-1)I$, then

Options:

(a) $|adj\ P| = 1$

(b) P is a singular matrix

(c) $|adj\ P| > \frac{1}{2}$

(d) $|adj\ P| > 1$

Answer: (a)

Solution:

$P^T = AP - (a-1)I$

Taking transpose on both sides

$P = aP^T - (a-1)I$

$\Rightarrow P = a\left(aP - (a-1)I\right) - (a-1)I$

$\Rightarrow P = a^2P - a(a-1)I - (a-1)I$

$$\Rightarrow (1-a^2)P = -I(a^2-a+a-1)$$
$$\Rightarrow (1-a^2)P = -I(a^2-1)$$
$$\Rightarrow (1-a^2)(P-I) = 0$$
$$\Rightarrow a^2 = 1 \text{ or } P = I$$
$a^2 = 1$ is neglected as $a \neq 1$ or -1

$\therefore\ P = I$

$|P| = 1$

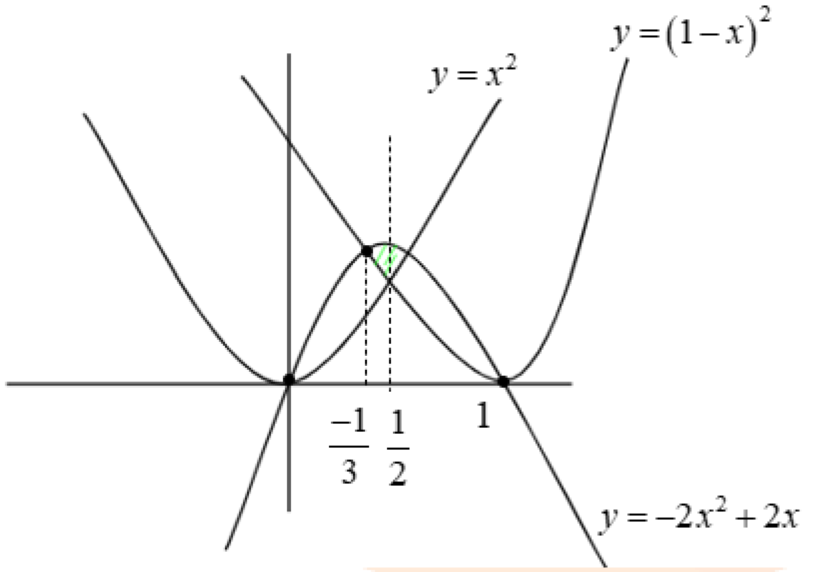
$\therefore |adj\ P| = |P|^2 = 1$

Question: Find the area of the region bounded by $y \geq x^2, y \geq (1-x)^2$ and $y \leq -2x^2 + 2x$.

Answer: $\frac{5}{108}$

Solution:

We have, $y \geq x^2, y \geq (1-x)^2$ and $y \leq -2x^2 + 2x$



Solving $y = (1-x)^2$ & $y = -2x^2 + 2x$, we get

$1 + x^2 - 2x = -2x^2 + 2x$

$3x^2 - 4x + 1 = 0$

$3x^2 - 3x - x + 1 = 0$

$3x(x-1) - 1(x-1) = 0$

$(3x-1)(x-1) = 0$

$3x = 1$ or $x = 1$

$\Rightarrow x = \frac{1}{3}$ or $x = 1$

$\therefore$ Required Area $= 2\int_{\frac{1}{3}}^{\frac{1}{2}}\left[\left(-2x^2+2x\right)-\left(1-x^2\right)\right]dx$

$= 2\int_{\frac{1}{3}}^{\frac{1}{2}}\left(-3x^2+4x-1\right)dx$

$= 2\left[-x^3+2x^2-x\right]_{\frac{1}{3}}^{\frac{1}{2}}$

$= 2\left[\frac{-1}{8}+\frac{1}{2}-\frac{1}{2}-\left(\frac{1}{27}+\frac{2}{9}-\frac{1}{3}\right)\right]$

$= 2\left[\frac{-1}{8}+\frac{4}{27}\right]$

$= \frac{5}{108}$

Question: If $\frac{dy}{dx} = -\frac{\left(x^2+3y^2\right)}{\left(3x^2+y^2\right)}$; $y(1)=0$, then

Answer: $\ln(x+y) = \frac{-2xy}{(x+y)^2}$

Solution:

We have, $\frac{dy}{dx} = -\frac{\left(x^2+3y^2\right)}{\left(3x^2+y^2\right)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\left(1+\frac{3y^2}{x^2}\right)}{3+\frac{y^2}{x^2}} \quad ...(1)$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes

$v + x\frac{dv}{dx} = \frac{-\left(1-3v^2\right)}{3+v^2}$

$x\frac{dv}{dx} = \frac{-1-3v^2-3v-v^3}{3+v^2}$

$\frac{\left(3+v^2\right)dv}{(1+v)^3} = \frac{-dx}{x}$

Integrating both sides, we get

$\int\frac{\left(3+v^2\right)}{\left(1+v^3\right)}dv = -\int\frac{dx}{x} \quad ....(2)$

Put $v+1=t \Rightarrow dv=dt$

Now (2) becomes

$$\int \frac{3+(t-1)^2}{(t)^3}dt=-\ln x+C$$

$$\Rightarrow \int \frac{(4+t^2-2t)}{t^3}dt=-\ln x+C$$

$$\Rightarrow \int \frac{4}{t^3}dt+\int \frac{1}{t}dt-2\int \frac{1}{t^2}dt=-\ln x+C$$

$$\Rightarrow \frac{-2}{t^2}+\ln t+\frac{2}{t}=-\ln x+C$$

$$\Rightarrow \frac{-2}{(1+v)^2}+\ln(1+v)+\frac{2}{(1+v)}=-\ln x+C$$

$$\Rightarrow \frac{-2}{\left(1+\frac{y}{x}\right)^2}+\ln\left(1+\frac{y}{x}\right)+\frac{2}{\left(1+\frac{y}{x}\right)}=-\ln x+C \qquad ...(3)$$

Now given that $y(1)=0$

$$\therefore \frac{-2}{(1+0)^2}+\ln(1+0)+\frac{2}{(1+0)}=-\ln 1+C$$

$$\Rightarrow -2+0+2=0+C$$

$$\Rightarrow C=0$$

Thus, (3) becomes

$$\frac{-2x^2}{(x+y)^2}+\ln(x+y)-\ln x+\frac{2x}{x+y}+\ln x=0$$

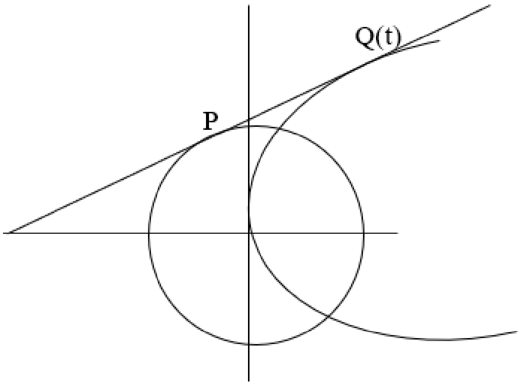
$$\Rightarrow \ln(x+y)=\frac{2x^2-2x(x+y)}{(x+y)^2}$$

$$\Rightarrow \ln(x+y)=\frac{-2xy}{(x+y)^2}$$

Question: Consider the circle $x^2+y^2=8$ and parabola $y^2=16x$. Common tangents are drawn from a point A on x-axis, which touches circle and parabola at P & Q. Then $(PQ)^2=?$

Answer: 72.00

Solution:



We have $x^2 + y^2 = 8$

And parabola, $y^2 = 16x$

Tangent to parabola is given by

$ty = x + 4t^2$

Also, it is tangent to circle

$$\frac{4t^2}{\sqrt{1+t^2}} = 2\sqrt{2}$$

$\Rightarrow 4t^4 = 2 + 2t^2$

$\Rightarrow 2t^4 - t^2 - 1 = 0$

Let $t^2 = u$

$\therefore 2u^2 - u - 1 = 0$

$$\Rightarrow u = \frac{-1}{2}, 1$$

$$\Rightarrow u = 1 \left(\text{as } u = t^2 \neq \frac{-1}{2} \right)$$

$\Rightarrow t^2 = 1$

$\Rightarrow t = \pm 1$

Now, $Q \equiv (4,8)$

$PQ =$ length of tangent of circle from $Q = \sqrt{S_1}$

$= \sqrt{16 + 64 - 8} = \sqrt{72}$

$\therefore (PQ)^2 = 72$

Question: a^3, b^3, c^3 are in AP and $\log_a b, \log_b c, \log_c a$ are in GP.

$a_1 = \dfrac{a+4b+c}{3}, d = \dfrac{a-8b+c}{10}$. Sum of first 20 terms is -444. Find abc.

Answer: 216.00

Solution:

a^3, b^3, c^3 are in AP

$\Rightarrow 2b^3 = a^3 + c^3 \quad \ldots(1)$

$\log_a b, \log_b c, \log_c a$ are in GP

$(\log_b c)^2 = \log_a b \times \log_c a$

$\Rightarrow (\log_b c)^2 = \log_c b$

$\Rightarrow (\log_b c)^3 = 1$

$\Rightarrow \log_b c = 1 \Rightarrow b = c \quad ....(2)$

From (1) & (2)

$a = b = c$

$a_1 = \dfrac{a+4b+c}{3} = \dfrac{6a}{3} = 2a$

$d = \dfrac{a-8b+c}{10} = \dfrac{-3a}{5}$

$S_{20} = \dfrac{20}{2}\left[2(2a)+19\left(\dfrac{-3a}{5}\right)\right]$

$\Rightarrow -444 = 10\left[4a - \dfrac{57a}{5}\right]$

$\Rightarrow -444 = \dfrac{10}{5}[20a - 57a]$

$\Rightarrow -444 = 2[-37a]$

$\Rightarrow 74a = 444$

$\Rightarrow a = 6$

$\therefore abc = a^3 = 6^3 = 216$

Question: If $x = \left(8\sqrt{3}+13\right)^{13}, y = \left(6\sqrt{2}+9\right)^9$, then tell whether $[x]$ and $[y]$ are even or odd.

Answer: $[x]$ is even, $[y]$ is odd

Solution:

$x = \left(8\sqrt{3}+13\right)^{13}$

We know, $R = I + f$

Let $R = \left(8\sqrt{3}+13\right)^{13}$

and $G = \left(8\sqrt{3}-13\right)^{13}$

$R - G = 2k$

$\Rightarrow f = G$

$I + f - G = 2k$

$I = 2k$

I is even

i.e. $[x]$ is even

Now, $y = \left(6\sqrt{2}+9\right)^9$

Let $R = \left(6\sqrt{2}+9\right)^9$

$G=\left(9-6\sqrt{2}\right)^9$

$R+G=2k$

$\Rightarrow f+G=1$

Thus, $I+f+G=2k$

$I+1=2k$

$\Rightarrow I=2k-1$

$\Rightarrow I$ is odd

$\Rightarrow [y]$ is odd.

Question: $f(x)=\begin{cases}\frac{x}{|x|} & ; \quad x\neq 0\\ 1 & ; \quad x=0\end{cases}$, $g(x)=\begin{cases}\frac{\sin(x+1)}{x+1} & ; \quad x\neq -1\\ 1 & ; \quad x=-1\end{cases}$, $h(x)=2[x]-f[x]$, then $\lim\limits_{x\to 1} g(h(x-1))=?$

Answer: 1.00

Solution:

$$f(x)=\begin{cases}\frac{x}{|x|} & ; \quad x\neq 0\\ 1 & ; \quad x=0\end{cases},\ g(x)=\begin{cases}\frac{\sin(x+1)}{x+1} & ; \quad x\neq -1\\ 1 & ; \quad x=-1\end{cases},\ h(x)=2[x]-f[x],$$

$\lim\limits_{x\to 1^+} g(h(x-1))=g(-1)=1$

$\lim\limits_{x\to 1^-} g(h(x-1))=g(-1)=1$

$\therefore \lim\limits_{x\to 1} g(h(x-1))=1$

Question: The curve $ax^2+2bx+cy=0$ and $dx^2+2ex+fy=0$ intersect at $y=1$, where a,b,c are in GP. Find the relation in d,e,f,a,b,c.

Answer: $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in AP

Solution:

Given curves $ax^2+2bx+cy=0$ and $dx^2+2ex+fy=0$

Let the curves intersect at $(\alpha,1)$

$\therefore \quad ax^2+2bx+cy=0 \quad ....(\text{i})$

$dx^2+2ex+fy=0 \quad ....(\text{ii})$

α is the common root

Now, given that a,b,c are in GP

$\Rightarrow b^2=ac \quad ...(\text{iii})$

From (i)

$x=\frac{-2b\pm\sqrt{4b^2-4ac}}{2a}$

$x=\frac{-b}{a}$

$\therefore$ Common root , $\alpha=\frac{-b}{a}$

Substituting in (ii), we get

$d\left(\frac{b^2}{a^2}\right)-2e\left(\frac{b}{a}\right)+f=0$

$d\left(\frac{ac}{a^2}\right)-2e\left(\frac{\sqrt{ac}}{a}\right)+f=0$

$d\frac{c}{a}-2e\sqrt{\frac{c}{a}}+f=0$

$\Rightarrow\frac{d}{a}-\frac{2e}{\sqrt{ac}}+\frac{f}{c}=0$

$\Rightarrow\frac{d}{a}+\frac{f}{c}=\frac{2e}{b}$

$\Rightarrow\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in AP

Question: If the points $(2,k,0),(1,k,-1)$ and $(1,1,2)$ lie in a plane which is parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$. Then $\frac{2k^2+1}{(k-1)(k-2)}=?$

Answer: 18.00

Solution:

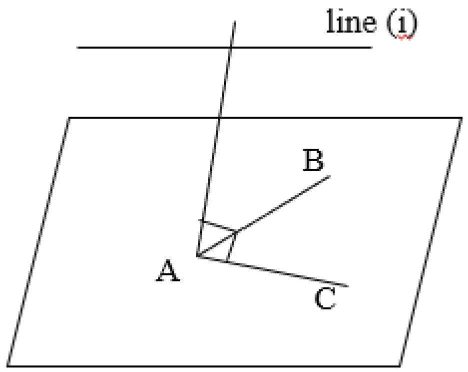
Let $A(2,k,0),B(1,k,-1)$ and $C(1,1,2)$ lie in the plane which is parallel to the line

$\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$ ...(i)

Now $\overrightarrow{AB}$ & $\overrightarrow{AC}$ will lie in the same plane

Since the plane is parallel to line (i)

$\therefore\overrightarrow{AB}\times\overrightarrow{AC}$ will be parallel to line (i)



$\Rightarrow \left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=0$

$\begin{vmatrix} -1 & 0 & -1 \\ -1 & 1-k & 2 \\ 1 & 1 & -1 \end{vmatrix}=0$

$\Rightarrow -1(k-1-2)-1(-1+k-1)=0$

$\Rightarrow 3-k+2-k=0$

$\Rightarrow 2k=5$

$\Rightarrow k=\frac{5}{2}$

$\therefore \ \frac{2k^2+1}{(k-1)(k-2)}=\frac{2\left(\frac{25}{4}\right)+1}{\left(\frac{3}{2}\right)\times\left(\frac{1}{2}\right)}=\frac{27\times 4}{2\times 3}=18$

Question: Line is parallel to the line $x+y-2z-2=0$ and $x-3y+2z=0$, passes through (5, 2, 3). If α is the distance of this line from (4, 3, 8), then $3\alpha^2=?$

Answer: 56.00

Solution:

Required lines is passing through (5, 2, 3)

And is parallel to the lines $x+y-2z-2=0$ and $x-3y+2z=0$

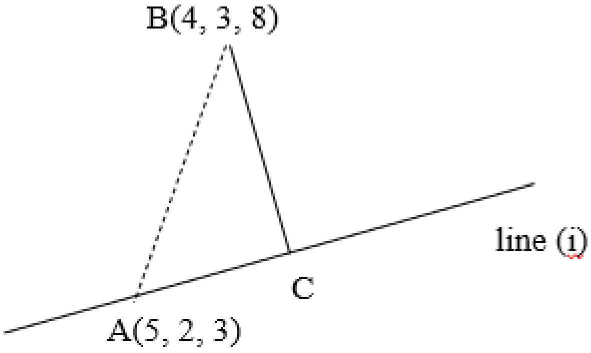
Now line of intersection of $x+y-2z-2=0$ and $x-3y+2z=0$ is given by

$\left(\hat{i}+\hat{j}-2\hat{k}\right)\times\left(\hat{i}-3\hat{j}+2\hat{k}\right)=\hat{i}+\hat{j}+\hat{k}$

Thus, the equation of the required line is

$\frac{x-5}{1}=\frac{y-2}{1}=\frac{z-3}{1} \quad \ldots(\text{i})$

Now, $\overrightarrow{AB}=\hat{i}-\hat{j}-5\hat{k}$



$$\Rightarrow \left|\overrightarrow{AB}\right| = \sqrt{1^2 + 1^2 + 25} = \sqrt{27}$$

And $\left|\overrightarrow{AC}\right| = \left|\left(\hat{i} - \hat{j} - 5\hat{k}\right) \cdot \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right|$

$$= \left|\frac{1}{\sqrt{3}}(-5)\right| = \left|\frac{-5}{\sqrt{3}}\right|$$

$$\left|\overrightarrow{AC}\right| = \frac{5}{\sqrt{3}}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{27 - \frac{25}{3}} = \sqrt{\frac{56}{3}}$$

$$\Rightarrow \alpha = \sqrt{\frac{56}{3}}$$

$$\Rightarrow 3\alpha^2 = 56$$

Question: $\lim\limits_{n\to\infty} \frac{3}{n}\left(4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \left(2 + \frac{3}{n}\right)^2 + \ldots. + \left(3 - \frac{1}{n}\right)^2\right) = ?$

Answer: 19.00

Solution:

$$\lim_{n\to\infty} \frac{3}{n}\left(4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \left(2 + \frac{3}{n}\right)^2 + \ldots. + \left(3 - \frac{1}{n}\right)^2\right)$$

$$3\left\{\frac{4}{n} + \left[\left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 \ldots. + \left(2 + \frac{n-1}{n}\right)^2\right]\frac{1}{n}\right\}$$

$$= 3\left[\frac{4}{n} + \sum_{r=1}^{n-1}\left(2 + \frac{r}{n}\right)^2 \cdot \frac{1}{n}\right] \quad \left[\begin{array}{l} \frac{r}{n} = x \\ \frac{1}{n} = dx \end{array}\right]$$

$$= 3\left[\int_0^1 (2 + x)^2\, dx\right]$$

$$= 3\frac{(x+2)^3}{3}\Bigg|_0^1 = 27 - 8 = 19$$

Question: p: I have fever; q: I will not take medicine; r: I will take rest. "If I have fever then I will take medicine and I will take rest" is given by

Answer: $p \to \sim q \wedge r$

Solution:

Given, p: I have fever

q: I will not take medicine

r: I will take rest

So, "If I have fever then I will take medicine and I will take rest" is written as

$p \to \sim q \wedge r$

Question: The 50th root of x and y is 12 and 18 respectively. Find the remainder when $x+y$ is divided by 5.

Answer: 3.00

Solution:

Given that

$x^{\frac{1}{50}} = 12 \Rightarrow x = 12^{50}$

And $y^{\frac{1}{50}} = 18 \Rightarrow y = 18^{50}$

Now $12^{50} = (144)^{25} = (145-1)^{25}$

On expanding we will get remainder as -1

Similarly $18^{50} = (324)^{25} = (325-1)^{25}$

On expanding we will get remainder as -1

Thus when $x+y$ is divided by 5, we will get remainder as -2 i.e. 3

Question: $p \in [0,10]$, $q =$ maximum value of p such that $x^2 - px - \frac{5}{4}p = 0$ has rational roots. Find area bounded by $0 \le x \le q,\ 0 \le y \le (x-q)^2$.

Answer: 243.00

Solution:

Given that q = maximum value of p such that $x^2 - px - \frac{5}{4}p = 0$ has rational roots, where $p \in [0,10]$

Now, $x^2 - px - \frac{5}{4}p = 0$ has rational roots

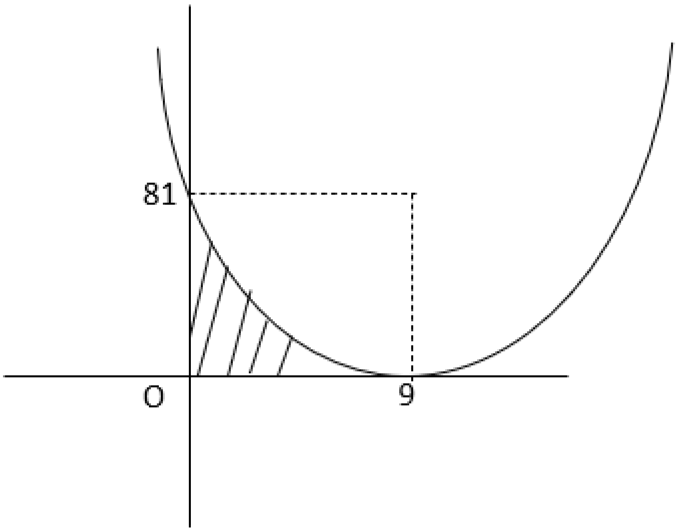
$\Rightarrow p^2 - 5p$ should be perfect square

$\Rightarrow p(p-5)$ should be perfect square

Now $p \in [0,10]$

$\Rightarrow p = 9$

$\therefore q = 9$



$\therefore$ Required area $= \frac{1}{3} \times 9 \times 81 = 243$

Question: Consider $A = \{2, 4, 6, ..., 100\}$ and $B = \{1, 3, 5, ..., 99\}$. $a + b$ leaves remainder 2 when divided by 23, where $a \in A,\ b \in B$. Number of ordered pairs (a, b) are

Answer: 108.00

Solution:

Given, $A = \{2, 4, 6, ..., 100\}$ and $B = \{1, 3, 5, ..., 99\}$

$a + b$ leaves remainder 2 when divided by 23

$a + b = 23k + 2$

$k = 1 \Rightarrow a + b = 25 \rightarrow 12$ pairs

$k = 3 \Rightarrow a + b = 71 \rightarrow 35$ pairs

$k = 5 \Rightarrow a + b = 117 \rightarrow 42$ pairs

$k = 7 \Rightarrow a + b = 163 \rightarrow 19$ pairs

Number of ordered pairs (a, b) are

$12 + 35 + 42 + 19 = 108$