

QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

📅 31 JANUARY, 2023

🕒 9:00 AM to 12:00 Noon

SHIFT - 1

Duration : 3 Hours

Maximum Marks : 300

SUBJECT - MATHEMATICS

RESULT JEE ADVANCED 2022

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MATHEMATICS

1. Find the remainder when 5^{99} is divided by 11.

Ans. (9)

Sol. $5^{99} = 5 \cdot (22 + 3)^{49} = 5 (22I + 3^{49})$

Now $3^{49} = 3^4 \cdot (3^5)^9 = 81 \cdot (242 + 1)^9 = 81 \cdot (11I_2 + 1)$

$\therefore 5^{99} = 11I_3 + 5 \times 81 = 11I_3 + 5 \times (77 + 4)$

$\Rightarrow 5^{99} = 11I_4 + 9$

\therefore Remainder is 9

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$, then sum of all diagonal elements of $(A + I)^{11}$.

Ans. (4097)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

$\Rightarrow A^2 = A$

so, $(A + I)^{11} = {}^{11}C_0 I + {}^{11}C_1 A + {}^{11}C_2 A^2 + \dots + {}^{11}C_{11} A^{11}$

$= I + A[{}^{11}C_1 + \dots + {}^{11}C_{11}] = I + A(2^{11} - 1)$

$\therefore \text{tr}(A + I)^{11} = 3 + (2^{11} - 1) \times 2 = 4097$

3. Consider two statements $S_1 : (p \rightarrow q) \vee (\sim q \vee \sim p)$ and $S_2 : (\sim p \vee q) \wedge (\sim q \vee p)$, then

(1) S_1 and S_2 are tautology

(2) S_1 is not tautology and S_2 is contradiction

(3) S_1 is not tautology and S_2 is not contradiction

(4) S_1 is tautology and S_2 is not contradiction

Ans. (4)

Sol. $S_1 : (p \rightarrow q) \vee (\sim q \vee p) \equiv (\sim p \vee q) \vee (\sim q \vee p) \equiv t$

$S_2 : (\sim p \vee q) \wedge (\sim q \vee p) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\Rightarrow S_2$ is neither tautology nor contradiction

4. Range of $f(x) = \frac{[x]}{1+x^2}$; $x \in [2, 6)$ where $[\cdot]$ denotes greatest integer function

(1) $\left(\frac{2}{10}, \frac{2}{5}\right]$

(2) $\left(\frac{4}{26}, \frac{4}{17}\right]$

(3) $\left(\frac{5}{37}, \frac{5}{26}\right]$

(4) $\left(\frac{5}{37}, \frac{2}{5}\right]$

Ans. (4)

Sol. $x \in [2, 3), f = \frac{2}{1+x^2} = \left(\frac{2}{10}, \frac{2}{5}\right]$

$x \in [3, 4), f = \frac{3}{1+x^2} = \left(\frac{3}{17}, \frac{3}{10}\right]$

$x \in [4, 5), f = \frac{4}{1+x^2} = \left(\frac{4}{26}, \frac{4}{17}\right]$

$x \in [5, 6), f = \frac{5}{1+x^2} = \left(\frac{5}{37}, \frac{5}{26}\right]$

So range is $\left(\frac{5}{37}, \frac{5}{26}\right] \cup \left(\frac{4}{26}, \frac{4}{17}\right] \cup \left(\frac{3}{17}, \frac{3}{10}\right] \cup \left(\frac{2}{10}, \frac{2}{5}\right]$

Range is $\left(\frac{5}{37}, \frac{2}{5}\right]$

5. If four consecutive positive terms are in GP whose sum is 126 & product is 1296 then find sum of all possible common ratio of these GP.

Ans. (7)

Sol. Let terms are $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

product $\frac{a}{r^3} \times \frac{a}{r} \times ar \times ar^3 = 1296$.

$a^4 = 1296$

$a = 6$ & $a = -6$

Sum $\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$

Case I : $\frac{1}{r^3} + \frac{1}{r} + r + r^3 = 21$

$\frac{1+r^2+r^4+r^6}{r^3} = 21$

$1+r^2+r^4+r^6 = 21r^3$

$\left(r^3 + \frac{1}{r^3}\right) + \left(r + \frac{1}{r}\right) - 21 = 0$

$\left(r + \frac{1}{r}\right)^3 - 3\left(r + \frac{1}{r}\right) + \left(r + \frac{1}{r}\right) - 21 = 0$

$\left(r + \frac{1}{r}\right)^3 - 2\left(r + \frac{1}{r}\right) - 21 = 0$

$t^3 - 2t - 21 = 0$

$(t-3)(t^2+3t+7) = 0$

$$t = 3$$

$$r + \frac{1}{r} = 3$$

$$r^2 - 3r + 1 = 0$$

$$r = \frac{3 \pm \sqrt{5}}{2}$$

Case II : $r = -6$

$$t^3 - 2t + 21 = 0$$

$$(t + 3)(t^2 - 3t + 7) = 0$$

$$t = -3$$

$$r + \frac{1}{r} = -3$$

$$r^2 + 3r + 1 = 0$$

$$r = \frac{-3 \pm \sqrt{5}}{2}$$

\therefore there are two possible values of common ratio (i.e., r^2) and their sum is 7

6. $f(x) = \frac{x + |x|}{2}$ and $g(x) = \begin{cases} x & ; x < 0 \\ x^2 & ; x \geq 0 \end{cases}$

Find area bounded by $f \circ g(x)$ with line $2y - x = 15$.

(1) 72

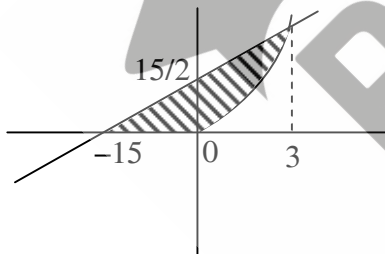
(2) $\frac{123}{4}$

(3) $\frac{172}{5}$

(4) $\frac{72}{5}$

Ans. (1)

Sol.



$$f(x) = \begin{cases} x & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

$$g(x) = \begin{cases} x & ; x < 0 \\ x^2 & ; x \geq 0 \end{cases}$$

$$f(g(x)) = \begin{cases} g(x) & ; g(x) \geq 0 \\ 0 & ; g(x) < 0 \end{cases}$$

$$\Rightarrow f \circ g = \begin{cases} x^2 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$2x^2 - x - 15$$

$$2x^2 - 6x + 5x - 15 = 0$$

$$(2x + 5)(x - 3) = 0$$

$$\text{area} = \frac{1}{2} \times 15 \times \frac{15}{2} + \int_0^3 \left(\frac{x+15}{2} - x^2 \right) dx$$

$$= \frac{225}{4} + \frac{1}{2} \left[\frac{x^2}{2} + 15x - \frac{2x^3}{3} \right]_0^3$$

$$= \frac{225}{4} + \frac{1}{2} \left[\frac{9}{2} + 45 - 18 \right] = \frac{225}{4} + \frac{1}{2} \left[\frac{63}{2} \right] = \frac{288}{4}$$

7. If $\sin^{-1} \alpha + \cos^{-1} \left(\frac{4}{5} \right) - \tan^{-1} \frac{77}{36} = 0$, then $\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$ is

(1) $\pi - \frac{16}{17}$

(2) $\frac{8}{17} - \pi$

(3) $\frac{16}{17}$

(4) $2\pi - \frac{8}{17}$

Ans. (3)

Sol. $\sin^{-1} \alpha = \tan^{-1} \left(\frac{77}{36} \right) - \tan^{-1} \left(\frac{3}{4} \right)$

$$\Rightarrow \sin^{-1} \alpha = \tan^{-1} \frac{8}{15}$$

$$\Rightarrow \sin^{-1} \alpha = \sin^{-1} \frac{8}{17}$$

$$\Rightarrow \alpha = \frac{8}{17}$$

$$\therefore \sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha) = 2\alpha = \frac{16}{17}$$

8. If $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$, $|\vec{a} \times \vec{b}| = \sqrt{46}$ find $(\vec{a} \cdot \vec{b})^2$

(1) 38

(2) 48

(3) $\sqrt{15}$

(4) $2\sqrt{6}$

Ans. (1)

Sol. $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = a^2 b^2$

$$46 + (\vec{a} \cdot \vec{b})^2 = 14 \times 6$$

$$(\vec{a} \cdot \vec{b})^2 = 84 - 46 = 38$$

9. If $\sqrt{x^2 - 9} + \sqrt{x^2 - 4x + 3} = \sqrt{x^2 - \frac{25}{8}x + \frac{3}{8}}$ the number of solution are -

(1) 0

(2) 1

(3) 2

(4) 3

Ans. (2)

Sol. $\sqrt{(x+3)(x-3)} + \sqrt{(x-1)(x-3)} = \frac{\sqrt{(8x-1)(x-3)}}{2\sqrt{2}}$

$\Rightarrow x = 3$

and $\sqrt{x+3} + \sqrt{x-1} = \sqrt{x-\frac{1}{8}}$ gives no sol :

10. $\int_{\pi/3}^{\pi/2} \frac{2}{\sin x(1+\cos x)} dx$ is equal to

(1) $\frac{1}{2}(\ln 3) + \frac{1}{3}$ (2) $\ln 3 + \frac{1}{3}$ (3) $\frac{1}{2}(\ln 3)$ (4) $\frac{1}{3}(\ln 3) + \frac{1}{2}$

Ans. (1)

Sol. $\int_{\pi/3}^{\pi/2} \frac{dx}{2\sin \frac{x}{2} \cos^3 \frac{x}{2}} = \int_{\pi/3}^{\pi/2} \frac{1}{2} \left(\frac{\sec^2 \frac{x}{2} \times \sec^2 \frac{x}{2}}{\tan \frac{x}{2}} \right) dx = \int_{\pi/3}^{\pi/2} \frac{1}{2} \left(\frac{1 + \tan^2 \frac{x}{2}}{\tan \frac{x}{2}} \right) \sec^2 \frac{x}{2} dx$

$\tan \frac{x}{2} = t$

$= \int_{1/\sqrt{3}}^1 \frac{1+t^2}{t} dt = \left[\ln |t| + \frac{t^2}{2} \right]_{1/\sqrt{3}}^1 = \ln(\sqrt{3}) + \frac{1}{3} = \frac{1}{2}(\ln 3) + \frac{1}{3}$

11. If $f(x) + \int_3^x \frac{f(t) \cdot dt}{t} = \sqrt{x+1}$; then find $12f(8)$

Ans. (17)

Sol. Differentiate both side w.r.t. 'x'

$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{1}{2\sqrt{x+1}}$

If $= e^{\int \frac{1}{x} dx} = x$

So solution is $y \cdot x = \int \frac{x}{2\sqrt{x+1}} dx$

$xy = \left| \frac{(t^2-1)}{2t^2} \cdot 2t dt \right| = \frac{t^3}{3} - t + 6$

$yx = \frac{1}{3}(x+1)^{3/2} - (x+1)^{1/2} + c$ {Given $f(3) = 2$ }

$x = 3, y = 2 \Rightarrow 6 = \frac{8}{3} - 2 + c \Rightarrow c = 6 - \frac{2}{3} = \frac{16}{3}$

so $12f(6) = 12 \left(\frac{1}{8} \left(\frac{1}{3} \times 27 - 3 + \frac{16}{3} \right) \right) = \frac{12}{8} \left[6 + \frac{16}{3} \right] = \frac{12}{8} \cdot \frac{34}{3} = 17$

12. $y = f(x)$ is parabola with focus $\left(-\frac{1}{2}, 0\right)$ & directrix $y = -\frac{1}{2}$. Given that

$$\tan^{-1} \sqrt{f(x)} + \sin^{-1} \sqrt{f(x)+1} = \frac{\pi}{2}, \text{ then find number of solution for } x.$$

Ans. (2)

Sol. $\left(x + \frac{1}{2}\right)^2 + y^2 = \left|\frac{y + \frac{1}{2}}{1}\right|^2 \Rightarrow y = x^2 + x$

$$\therefore \tan^{-1} \sqrt{x^2 + x} + \sin^{-1} \left(\sqrt{x^2 + x + 1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} \left(\frac{1}{\sqrt{x^2 + x + 1}}\right) + \sin^{-1} \left(\sqrt{x^2 + x + 1}\right) = \frac{\pi}{2}$$

$$\Rightarrow \sqrt{x^2 + x + 1} = \frac{1}{\sqrt{x^2 + x + 1}}$$

$$x^2 + x + 1 = 1$$

$$x(x + 1) = 0$$

$$x = 0 \text{ or } x = -1 \Rightarrow 2 \text{ solutions}$$

13. $x + y + z = 7$

$$\alpha x + \beta y + 7z = 8$$

$$3x + 2y + z = 2023$$

which of the following is incorrect ?

- (1) If $\alpha = \beta = 7$ then equation has no solution
- (2) If $\alpha - 2\beta + 7 = 0$ and $(\alpha, \beta) \neq (7, 7)$ then equation has unique solution
- (3) If $(\alpha, \beta) = (-1, 4)$, then the equation has unique solution
- (4) There exists (α, β) lying on line $x + 2y = 21$ for which system has no solution

Ans. (2)

Sol. $\Delta = \alpha - 2\beta + 7$

(A) $\alpha = \beta = 7$

equation (1) and (2) are $x + y + z = 7$

and $x + y + z = \frac{8}{7}$

\Rightarrow no solution

(B) $(\alpha, \beta) \neq (7, 7) \Rightarrow D \neq 0 \Rightarrow$ unique solution

(C) $(\alpha, \beta) = (-1, 4) \Rightarrow D \neq 0 \Rightarrow$ unique solution

14. A bag contains 6 balls. If two balls chosen from bag & found to be black, find the probability that bag contains atleast 5 black balls.

- (1) $\frac{5}{7}$ (2) $\frac{3}{7}$ (3) $\frac{6}{7}$ (4) $\frac{4}{7}$

Ans. (1)

Sol. 4 white & 2 black, 3 white & 3 black, 2 white & 4 black, 1 white & 5 black, 0 white & 6 black,

$$\text{probability} = \frac{{}^5C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2} = \frac{10 + 15}{1 + 3 + 6 + 10 + 15} = \frac{25}{35} = \frac{5}{7}$$

15. Find the number of 5 digits numbers which are less than 28000 and are divisible by 3 and 11.

- (1) 6070 (2) 7091 (3) 8070 (4) 7080

Ans. (B)

Sol. 5-digit number div. by 3 : {10002, 10005, , 27999}

⇒ 6000 number

5- digit number div. by 11 : {10010, , 27995}

⇒ 1636

5-digit numbers div. by 33 : {10032, , 27984}

⇒ 545 numbers

∴ total number = 6000 + 1636 – 576 = 7091

16. For the ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$, minimum distance of any normal from centre is 1 then eccentricity of ellipse is

- (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

Ans. (1)

Sol. Minimum distance any normal from centre is a – b

so 2 – b = 1 ⇒ b = 1

so ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$$\text{ecc.} = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

17. Find α for the given data :

x_i	2	4	5	6	7	9
f_i	4	16	α	15	6	4

If variance is 3.01, find α . (Data may vary)

- (1) 2 (2) 3 (3) 5 (4) 4

Ans. (3)

Sol. As we know :

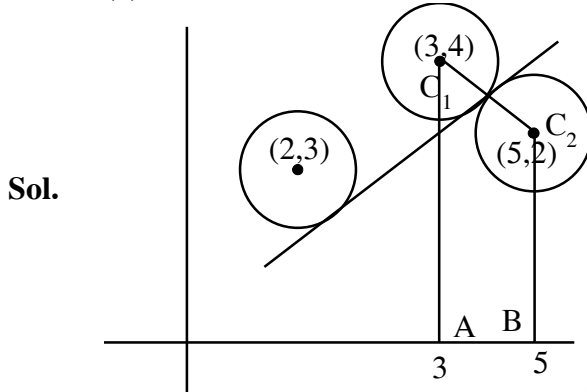
$$\text{variance} = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$$

putting the given data in the formula we get $\alpha = 5$

18. A circle having centre $(2, 3)$ & radius $\sqrt{2}$ rolls 4 unit upwards on tangent at $(3, 2)$. Let it is C_1 circle. Now C_2 is a circle which is image of C_1 about tangent. Area of trapezium made by centre of C_1 & C_2 & their foot on x-axis -

- (1) $10 + 8\sqrt{2}$ (2) $11 + 8\sqrt{2}$ (3) $11 + 6\sqrt{2}$ (4) $10 + 6\sqrt{2}$

Ans. (2)



$$(x - 2)^2 + (y - 3)^2 = 2 \Rightarrow x^2 + y^2 - 4x - 6y + 11 = 0$$

$$\text{Tangent at } (3, 2) \text{ is } x \cdot 3 + y \cdot 2 - 2(x + 3) - 3(y + 2) + 11 = 0$$

$$\Rightarrow x - y - 1 = 0$$

$$\text{New centre of circle } (2 + 4 \cos 45, 3 + 4 \sin 45) = (2 + 2\sqrt{2}, 3 + 2\sqrt{2})$$

Image of $(2 + 2\sqrt{2}, 3 + 2\sqrt{2})$ about tangent is

$$\frac{x - (2 + 2\sqrt{2})}{1} = \frac{y - (3 + 2\sqrt{2})}{-1} = \frac{-2((2 + 2\sqrt{2}) - (3 + 2\sqrt{2}) - 1)}{2} = 2x = 4 + 2\sqrt{2}, y = 5 + 2\sqrt{2}$$

$$\text{Area} = \frac{1}{2} \cdot (4 + 2) \cdot 2 = 6$$

$$\frac{1}{2} (2) (3 + 2\sqrt{2}) (1 + 2\sqrt{2}) = 11 + 8\sqrt{2}$$

19. Let R be a relation over the set $N \times N$ and it is defined by $(a, b) R (c, d) \Rightarrow ad(b - c) = bc(a - d)$. Then relation R is

- (1) Reflexive, symmetric and Transitive
(2) symmetric, Not Reflexive, and Not Transitive
(3) symmetric, Transitive, and Not Reflexive
(4) Transitive, not Reflexive and not symmetric

Ans. (2)

Sol. $((a, b), (a, b)) : ab(b - a) \neq ba(a - b)$

\therefore Not Reflexive

Symmetric

$$((a, b), (c, d)) \quad ad(b - c) = bc(a - d)$$

$$((c, d), (a, b)) \quad cb(d - a) = da(c - b)$$

Hence symmetric

Transitive

$$((2, 3), (3, 2)) \in R \ \& \ ((3, 2), (5, 30)) \in R \Rightarrow ((2, 3), (5, 30)) \notin R$$

Hence Not Transitive

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