



PART : MATHEMATICS

1. $|a| = \sqrt{6}$, $|b| = \sqrt{14}$ and $|a \times b| = \sqrt{48}$ then find $(\overline{ab})^6$

Ans. (36)

Sol. $(\overline{ab})^6 + (\overline{a \times b})^6 = |a|^6 |b|^6$

$$(\overline{ab})^6 + 48 = 14 \times 6$$

$$(\overline{ab})^6 = 84 - 48 = 36$$

2. A bag contains 6 balls, 2 balls are drawn randomly if both are black then find probability that bag contains at least 5 black balls.

(1) $\left(\frac{5}{7}\right)$

(2) $\left(\frac{1}{2}\right)$

(3) $\left(\frac{1}{7}\right)$

(4) $\left(\frac{3}{7}\right)$

Ans. (1)

Sol. Total possibility = 2B + 4 others
or
= 3B + 3 others
or
= 4B + 2 others
or
= 5B + 1 others
or
= 6B + 0 others

Now

$$\text{required probability} = \frac{{}^5C_2 + {}^6C_2}{{}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2}$$
$$= \frac{10 + 15}{1 + 3 + 6 + 10 + 15} = \frac{25}{35} = \frac{5}{7}$$

3. Find remainder when 5^{59} is divided by 11

Ans. (9)

Sol. $5^{59} = 625 \times 5^{56}$

$$= 625 \times (3124 + 1)^{14}$$

$$= 625 \times (11\lambda + 1)^{14}$$

$$\text{Remainder} = 625 \text{ or } 9$$

4. Sum of four consecutive terms of a G.P. is 126 and their product is 1296 then find sum of common ratio of all G.P.

Ans. (3)

Sol. G.P. $\rightarrow \frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\text{given : } \frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296 \Rightarrow a^4 = 1296 \Rightarrow a = 6$$

$$\text{and } \frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126 \Rightarrow \frac{1}{r^3} + \frac{1}{r} + r + r^3 = 21$$

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$$\left(r^3 + \frac{1}{r}\right) + \left(r + \frac{1}{r}\right) = 21$$

$$\Rightarrow \left(r + \frac{1}{r}\right)^2 - 3r \cdot \frac{1}{r} \left(r + \frac{1}{r}\right) + \left(r + \frac{1}{r}\right) = 21$$

$$t^2 - 2t - 21 = 0$$

$$(t-3)(t^2+3t+7) = 0,$$

$$t = 3$$

$$r + \frac{1}{r} = 3$$

$$r^2 - 3r + 1 = 0 \quad \left\langle \begin{matrix} r_1 \\ r_2 \end{matrix} \right.$$

$$\text{Sum} = 3$$

5. Find number of four digit number which are divisible by 3 or 11 and less than and equal to 2800.
Ans. (709)

Sol. Number of four digit numbers which are divisible by 3 is 600
 1002..... 2799

Number of four digit numbers which are divisible by 11 is 163

Number of four digit numbers which are divisible by 33 is 54

so required number is = 600 + 163 - 54 = 709

6. For ellipse $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$, $0 < b < 2$ maximum distance of normal at point P from its centre is 1 then e equal to

(1) $\frac{\sqrt{3}}{2}$

(2) $\frac{\sqrt{3}}{3}$

(3) $\frac{\sqrt{3}}{4}$

(4) $\frac{3}{4}$

Ans. (1)

Sol. Normal at $(2\cos\theta, b\sin\theta)$ is

$$2x \sec\theta - y \operatorname{cosec}\theta = 4 - b^2$$

$$\text{Now, } 1 = \left(\frac{4 - b^2}{\sqrt{4 \sec^2\theta + b^2 \operatorname{cosec}^2\theta}} \right)_{\max}$$

$$1 = \frac{4 - b^2}{2 + b}$$

$$2 + b = 4 - b^2$$

$$b^2 + b - 2 = 0$$

$$b = -2 \text{ or } 1$$

$$\text{so } b = 1$$

$$\text{Now } e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

7. If a_1, a_2, \dots, a_n is an A.P such that $a_5 = 2a_7$ and $a_{11} = 18$ then find

$$12 \left[\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \frac{1}{\sqrt{a_{12}} + \sqrt{a_{13}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right]$$

(1) 8

(2) 4

(3) 10

(4) 6

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Ans. (1)

Sol. Given $a + 4d = 2(a + 6d)$

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Ans. (1)

Sol. Given $a + 4d = 2(a + 6d)$

$$a + 8d = 0 \dots\dots(1)$$

$$\text{and } a_{11} = 18 \Rightarrow a + 10d = 18 \dots\dots(2)$$

From (1) and (2)

$$-8d + 10d = 18 \Rightarrow d = 9 \text{ and } a = -72$$

$$a_{10} = a + 9d = -72 + 81 = 9$$

$$a_{19} = a + 17d = -72 + 153 = 81$$

Now

$$12 \left[\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right]$$
$$12 \left[\frac{\sqrt{a_{10}} - \sqrt{a_{11}}}{-d} + \frac{\sqrt{a_{11}} - \sqrt{a_{12}}}{-d} + \dots + \frac{\sqrt{a_{17}} - \sqrt{a_{18}}}{-d} \right]$$
$$12 \left[\frac{\sqrt{a_{10}} - \sqrt{a_{18}}}{-d} \right] = 12 \left[\frac{3-9}{-9} \right] = 12 \times \frac{6}{9} = 8$$

8. Five digit numbers are formed using 0, 2, 3, 4, 7, 9 (repetition is allowed) and arranged in increasing order then the rank of 42937 is

Ans. (3005)

Sol. Number starting with 2 & 3 = 2×6^4

$$\text{Number starting with } 40 = 6^3$$

$$\text{Number starting with } 420, 422, 423, 424$$

$$427 = 5 \times 6^2$$

$$\text{Similarly starting with } 4290 \text{ \& } 4292 = 2 \times 6$$

$$\text{Number starting with } 4293 \text{ before } 42937 = 6 - 1 = 5$$

$$\text{Total} = 2 \times 6^4 + 6^3 + 5 \times 6^2 + 2 \times 6 + 5 = 3005$$

9. If $f(x) + \int_0^x f(t) dt = \sqrt{x+1}$; $f(3) = 2$ and $x \geq 3$ then find $12f(8)$

$$(1) 11$$

$$(2) 17$$

$$(3) 1$$

$$(4) 7$$

Ans. (2)

$$\text{Sol. } f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

$$y \cdot x = \int x \cdot \frac{1}{2\sqrt{x+1}} dx + C \text{ put } x+1 = t^2, dx = 2t dt$$

$$y \cdot x = \frac{1}{2} \int \frac{t^2 - 1}{t} 2t dt + C$$

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$$y \cdot x = \frac{1}{2} \left(\frac{2}{3} \cdot 3^3 - 2t \right) + c$$

$$y \cdot x = \frac{\sqrt{x+1}^3}{3} - \sqrt{x+1} + c$$

$$\text{now } f(3) = 2$$

$$6 = \frac{(2)^3}{3} - 2 + c$$

$$\frac{8}{3} - c = c$$

$$\frac{16}{3} = c$$

$$\therefore f(x) = \frac{1}{x} \left(\frac{\sqrt{x+1}^3}{3} - (\sqrt{x+1}) + \frac{16}{3} \right)$$

$$\therefore 12f(8) = \frac{12}{8} \left(\frac{27}{3} - 3 + \frac{16}{3} \right)$$

$$= \frac{3}{2} \left(6 + \frac{16}{3} \right) = 17$$

10. Find the shortest distance between lines $\frac{x-1}{3} = \frac{y-2}{0} = \frac{z-3}{-1}$ & $\frac{x-1}{1} = \frac{y-1}{1} = \frac{4-z}{1}$
- (1) 3 (2) 2 (3) 1 (4) 4

Ans. (3)

$$\text{Sol. } \frac{x-1}{3} = \frac{y-2}{0} = \frac{z-3}{-1} \text{ \& } \frac{x-1}{1} = \frac{y-1}{1} = \frac{4-z}{1}$$

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \mathbf{i}(1) - \mathbf{j}(-2) + \mathbf{k}(3)$$

$$\text{S.D.} = \frac{|0(1) - 1(-2) - 1(3)|}{\sqrt{1^2 + 2^2 + 3^2}}$$

$$= \frac{|0(1) - 1(-2) - 1(3)|}{\sqrt{14}}$$

$$= 1$$

11. A Circle $x^2 + y^2 - 4x - 8y + 11 = 0$ on the tangent to it at $(3, 2)$ is rolled up by 4 units upwards. If M and N are foot of perpendiculars of centres on x-axis, then area of quadrilateral by centres, M and N is
- (1) 20 (2) 10 (3) 30 (4) 40

Ans. (1)

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Sol. tangent: $x - y - 1 = 0$
since $\angle C, C_2D = 45^\circ$

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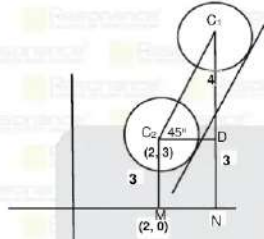
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Sol. tangent; $x - y - 1 = 0$
 since $\angle C_1 C_2 D = 45^\circ$
 $C_2 D = C_1 D = 4$



$$\text{Area} = \frac{1}{2} (3 + 7) \times 4$$

$$= 20 \text{ units.}$$

12. A wire of ℓ length is divided into 2 parts of length ℓ_1 and ℓ_2 . From wire of length ℓ_1 a square is formed with area a_1 and with wire of length ℓ_2 a circle is formed with area a_2 . If $2a_1 + 3a_2$ is minimum then find

$$\frac{\pi \ell_1}{\ell_2}$$

Ans. (6)

Sol. $\ell = 4x + 2\pi r$, where x = side of square and r = radius of circle

$$\text{now let } 2a_1 + 3a_2 = A$$

$$A = 2x^2 + 3\pi r^2$$

$$= 2x^2 + 3\pi \left(\frac{\ell - 4x}{2\pi} \right)^2 = 2x^2 + \frac{3}{4\pi} (\ell - 4x)^2$$

$$\frac{dA}{dx} = 4x - \frac{2 \times 3 \times 4}{4\pi} (\ell - 4x)$$

for maxima minima

$$\frac{dA}{dx} = 0$$

$$\therefore x = \frac{2 \times 3}{4\pi} (\ell - 4\pi)$$

$$4\pi x = 6\ell - 24x$$

$$x = \frac{6\ell}{4\pi + 24}$$

$$\text{now } 4x = \ell_1 = \frac{6\ell}{\pi + 6} \quad \text{also } \ell_2 = 2\pi r$$

$$= \ell - 4x = \ell - \frac{6\ell}{\pi + 6} = \frac{\pi\ell + 6\ell - 6\ell}{\pi + 6} = \frac{\pi\ell}{\pi + 6}$$

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$$\text{now } \frac{\pi/2}{\ell_2} = \frac{\pi+6}{\pi+6} = 6$$

13. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$ then sum of diagonal element of $(A+I)^{11}$ is

- (1) 4097 (2) 4095 (3) 6466 (4) 2048

Ans. (1)

$$\text{Sol. } A^e = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$$\begin{aligned} \text{Now } (A+I)^{11} &= {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \dots + {}^{11}C_{11} I \\ &= A ({}^{11}C_0 + {}^{11}C_1 + \dots + {}^{11}C_{11}) + I \\ &= A (2^{11} - 1) + I \\ \text{Trace of } (A+I)^{11} &= 2^{11} + 4(2^{11} - 1) + 1 + (-3)(2^{11} - 1) + 1 \\ &= 2 \times 2^{11} + 1 \\ &= 2^{12} + 1 \end{aligned}$$

14. If $\sin^{-1}(\alpha/17) + \cos^{-1}(4/5) - \tan^{-1}(\frac{77}{36}) = 0$ then find $\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$

- (1) π (2) 2π (3) 3π (4) 4π

Ans. (1)

$$\begin{aligned} \text{Sol. } \sin^{-1} \frac{\alpha}{17} &= \tan^{-1} \frac{77}{36} - \cos^{-1} \frac{4}{5} \\ \sin^{-1} \frac{\alpha}{17} &= \tan^{-1} \frac{77}{36} - \sin^{-1} \frac{3}{5} \\ \frac{\alpha}{17} &= \sin \left(\tan^{-1} \frac{77}{36} \right) \cos \sin^{-1} \left(\frac{3}{5} \right) - \cos \tan^{-1} \left(\frac{77}{36} \right) \sin \sin^{-1} \left(\frac{3}{5} \right) \\ \frac{\alpha}{17} &= \frac{77}{85} \times \frac{4}{5} - \frac{36}{85} \times \frac{3}{5} \\ &= \frac{308 - 108}{425} = \frac{200}{425} = \frac{8}{17} \\ \alpha &= 8 \end{aligned}$$

$$\begin{aligned} \text{Now } \sin^{-1} \sin \alpha + \cos^{-1} \cos \alpha &= \sin^{-1} \sin 8 + \cos^{-1} \cos 8 \\ &= 8 + 3\pi + 8 - 2\pi \\ &= \pi \end{aligned}$$

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15. If $B = \ln(1-x)$ and $P(x) = \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^{20}}{20} \right)$, then $\int_{-1}^{1} \frac{e^{2B}}{1+x} dx$ equals

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15. If $B = \ln(1-x)$ and $P(x) = \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^{50}}{50}\right)$, then $\int_0^x \frac{t^{50}}{1-t} dt$ equals

- (1) $-(B+P(x))$ (2) $-B+P(x)$
 (3) $B-P(x)$ (4) $B+P(x)$

Ans. (3)

Sol. $\int_0^x t^{50}(1-t)^{-1} dt$

$$= \int_0^x t^{50}(1+t+t^2+\dots+\infty) dt = \left(\frac{t^{51}}{51} + \frac{t^{52}}{52} + \dots\right)_0^x = \frac{x^{51}}{51} + \frac{x^{52}}{52} + \dots + \infty$$

$$\ln(1-x) - P(x) = B - P(x)$$

16. If $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$ then find real roots.

Ans. (3)

Sol. $\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)} = \sqrt{2(2x-1)(x-3)}$

$$x-3=0 \text{ and } \sqrt{x-1} + \sqrt{x+3} = \sqrt{2(2x-1)} \dots\dots\dots(1)$$

$$\text{Squaring both side : } x-1+x+3+2\sqrt{(x-1)(x+3)} = 4x-2$$

$$2\sqrt{(x-1)(x+3)} = 2x-4$$

$$\sqrt{x^2+3x-x-3} = x-2 \dots\dots\dots(2)$$

$$\text{Squaring both side : } x^2+2x-3 = x^2-4x+4$$

$$6x = 7 \Rightarrow x = \frac{7}{6}$$

At $x = \frac{7}{6}$ equation (2) positive = negative

Rejected

\therefore real root $x = 3$

17. Curve $C_1 : |z| = 4$ and C_2 : the locus of $z + \frac{1}{z}$ (where z satisfy the curve C_1) then C_1 & C_2

- (1) Intersect at 2 points (2) Intersect at 4 points
 (3) do not intersect (4) None of these

Ans. (2)

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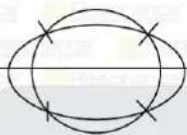
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Sol. $|z| = 4$ is a circle and $z + \frac{1}{z}$ is an ellipse $\frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$



18. $\int_0^{\frac{\pi}{3}} \frac{2+3\sin x}{\sin x(1+\cos x)} dx$

(1) $\frac{10}{3} + \sqrt{3} - \frac{1}{2} \log 3$

(2) $\frac{20}{3} + \sqrt{3} - \log 3$

(3) $\frac{10}{3} - \sqrt{3} + \frac{1}{2} \log 3$

(4) $\frac{20}{3} - \sqrt{3} + \log 3$

Ans. (3)

Sol. $= \int_0^{\frac{\pi}{3}} \frac{2(1-\cos x)}{\sin x(1-\cos^2 x)} dx + \int_0^{\frac{\pi}{3}} \frac{3}{1+\cos x} dx$

$$= \int_0^{\frac{\pi}{3}} \frac{2}{\sin^2 x} dx - 2 \int_0^{\frac{\pi}{3}} \cot x \cdot \operatorname{cosec}^2 x dx + \frac{3}{2} \int_0^{\frac{\pi}{3}} \sec^2 \frac{x}{2} dx$$

$$= 2 \int_0^{\frac{\pi}{3}} \sqrt{1+\cot^2 x} \cdot \operatorname{cosec}^2 x dx - 2 \int_0^{\frac{\pi}{3}} \cot x \cdot \operatorname{cosec}^2 x dx + 3 \left[\tan \frac{x}{2} \right]_0^{\frac{\pi}{3}}$$

Let $\cot x = t$

$$= -2 \int_{\frac{1}{\sqrt{3}}}^0 \sqrt{1+t^2} dt + 2 \int_{\frac{1}{\sqrt{3}}}^0 t dt + 3 \left[1 - \frac{1}{\sqrt{3}} \right]$$

$$= -2 \left[\frac{1}{2} \sqrt{1+t^2} + \frac{1}{2} \log(t + \sqrt{1+t^2}) \right]_{\frac{1}{\sqrt{3}}}^0 + \left[\frac{t^2}{2} \right]_{\frac{1}{\sqrt{3}}}^0 + 3 - \sqrt{3}$$

$$= \frac{2}{3} + \log \sqrt{3} - \frac{1}{3} + 3 - \sqrt{3} = \frac{10}{3} - \sqrt{3} + \frac{1}{2} \log 3$$

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19. The direction ratio of two lines which are parallel are given by $\langle 2, 1, -1 \rangle$ and $\langle \alpha + \beta, 1 + \beta, 2 \rangle$. Then the value of $|2\alpha + 3\beta|$ is _____.

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19. The direction ratio of two lines which are parallel are given by $\langle 2, 1, -1 \rangle$ and $\langle \alpha + \beta, 1 + \beta, 2 \rangle$. Then the value of $|2\alpha + 3\beta|$ is _____.

Ans. (11)

Sol. $\frac{\alpha + \beta}{2} = \frac{1 + \beta}{1} = \frac{2}{-1}$
 $\alpha = -1, \quad \beta = -3$
 $|2\alpha + 3\beta| = 11$

20. Domain of the function $y = \frac{[x]}{1+x^2}$ is $[2, 6]$ then Range of y is

- (1) $\left(\frac{5}{37}, \frac{2}{5}\right]$ (2) $\left(\frac{5}{26}, \frac{2}{5}\right]$ (3) $\left(\frac{4}{26}, \frac{2}{5}\right]$ (4) $\left(\frac{5}{26}, \frac{2}{5}\right]$

Ans. (1)

Sol. $y = \frac{[x]}{1+x^2} = \begin{cases} \frac{2}{1+x^2} & ; 2 \leq x < 3 \\ \frac{3}{1+x^2} & ; 3 \leq x < 4 \\ \frac{4}{1+x^2} & ; 4 \leq x < 5 \\ \frac{5}{1+x^2} & ; 5 \leq x < 6 \end{cases}$

Since, $y = \frac{[x]}{1+x^2}$ is decreasing.

for range, $f(2) = \frac{2}{5} = 0.4$

$f(3) = \frac{3}{10} = 0.3$

$f(4) = \frac{4}{17} = 0.23$

$f(5) = \frac{5}{26} = 0.19$

$f(6) = \frac{5}{37} = 0.13$

So range of $y = \left(\frac{5}{37}, \frac{2}{5}\right]$

21. Let $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ and \vec{b}, \vec{c} are non-zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$

Statement -1 : $|\vec{a} + \lambda \vec{c}| \geq |\vec{a}|$ for $\lambda \in \mathbb{R}$

Statement -2 : \vec{a} & \vec{c} are parallel, then

(1) Both correct

(2) Both wrong

(3) Only S-1 correct

(4) Only S₂ correct

Ans. (3)

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Sol. $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a} + \vec{b} - \vec{c}|^2$

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Sol. $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a} + \vec{b} - \vec{c}|^2$

$$4\vec{a}\vec{c} + 4\vec{a}\vec{c} = 0 \Rightarrow \vec{a}\vec{c} = 0$$

Statement -1 $|\vec{a} + \lambda\vec{c}|^2 \geq |\vec{a}|^2$

$$a^2 + \lambda^2 c^2 + 2\vec{a}\vec{c}\lambda \geq a^2$$

$$\lambda^2 c^2 \geq 0 \text{ which is true}$$

Statement -1 is true

Statement -2 \vec{a} & \vec{c} are perpendicular. So, statement 2 is false

22. $y = f(x)$ is a parabola with focus $(-\frac{1}{2}, 0)$ and directrix $y = -\frac{1}{2}$.

Given that $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$. Then number of solution for x is

Ans. (2)

Sol. Equation of parabola $\sqrt{(x + \frac{1}{2})^2 + y^2} = |y + \frac{1}{2}|$

$$x^2 + \frac{1}{4} + x + y^2 = y^2 + \frac{1}{4} + y$$

$$x^2 + x = y$$

$$\left(x + \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$$

$$Y = f(x) = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$

Now $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$

$$\cos^{-1} \frac{1}{\sqrt{1+f(x)}} + \sin^{-1} \sqrt{f(x)+1} = \frac{\pi}{2}$$

$$\text{So } \sqrt{f(x)+1} = \frac{1}{\sqrt{1+f(x)}}$$

$$f(x) + 1 = 1$$

$$f(x) = 0$$

$$\left(x + \frac{1}{2}\right)^2 = \pm \frac{1}{2}$$

$$x = 0, -1$$

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