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PART : MATHEMATICS

1. Given functional relation be $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x, \forall x \in R - \{0,1\}$, then the value of f(2) is.

(1) $\frac{9}{4}$ (2) $\frac{7}{4}$ (3) 0 (4) None of these

Ans. (1)

Sol. , $x = 2 \Rightarrow f(2) + f(-1) = 3$ ....(1)

, $x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ....(2)

, $x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ ....(3)

(2) .... (3) $\Rightarrow f(2) - f(-1) = \frac{3}{2}$ ....(4)

(1) + (4) $\Rightarrow 2f(2) = \frac{9}{2} \Rightarrow f(2) = \frac{9}{4}$

2. If $y = x^x$ then value of $y''(2) - 2y'(2)$ is

(1) $2 - 4(\ln 2)^2$ (2) $4(\ln 2)^2 - 2$ (3) $(\ln 2)^2 - 4$ (4) $4(\ln 2)^2 + 2$

Ans. (2)

Sol. $y = x^x$

$\ell n\, y = x\, \ell n\, x$

$\frac{1}{y}.y' = 1 + \ell nx$

$y' = y(1 + \ell n\, x)$ ....(1)

$y' = x^x(1 + \ell n\, x)$

at x = 2 we have y = 4

So $y'(2) = 4(1 + \ell n\, 2)$ ....(2)

and $y'' = y'(1 + \ell n\, x) + \frac{y}{x}$

$y''(2) = y'(1 + \ell n\, 2) + 2$

$y''(2) - y'(2) = y'(\ell n\, 2) + 2$

$y''(2) - 2y'(2) = (\ell n\, 2 - 1)\, y'(2) + 2$

$= 4(\ell n\, 2 - 1)(\ell n\, 2 + 1) + 2$

$= 4(\ell n\, 2)^2 - 2$

3. If the term independent of x (coefficient of x^0) in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315 then the value of $|\alpha|$ is-

(1) 2 (2) 1 (3) 6 (4) 5

Ans. (2)

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Sol. $T_{r+1} = {}^{22}C_r \left(\frac{\alpha}{x^3}\right)^r \left(x^{\frac{2}{3}}\right)^{22-r}$

Sol. $T_{r+1} = {}^{22}C_r \left(\frac{\alpha}{x^3}\right)^r \left(x^{\frac{2}{3}}\right)^{22-r}$

$= {}^{22}C_r.\alpha^r.x^{\frac{44-11r}{3}}$

for $x^0 \Rightarrow r = 4$

coefficient of $x^0 = {}^{22}C_4.\alpha^4 = 7315$

$\Rightarrow \frac{22.21.20.19}{4.3.2}\alpha^4 = 7315$

$\Rightarrow (11\times7\times5\times19)\alpha^4 = 7315$

$\Rightarrow \alpha^4 = 1$

$\Rightarrow |\alpha| = 1$

4. The number of integral solution of equation $x + y + z = 21$ where $x \geq 1$, $y \geq 3$, $z \geq 5$

(1) ${}^{14}C_7$ (2) ${}^{14}C_3$ (3) ${}^{14}C_2$ (4) None of these

Ans. (3)

Sol. $x \geq 1, y \geq 3, z \geq 5 \Rightarrow x-1 \geq 0, y-3 \geq 0, z-5 \geq 0$

Let $x - 1 = X$, $y - 3 = Y$, $z - 5 = Z$

So $x + y + z = 21$

$X + 1 + Y + 3 + Z + 5 = 21$

$X + Y + Z = 12 \quad X \geq 0, Y \geq 0, Z \geq 0$

No. of integral solution $= {}^{12+3-1}C_2 = {}^{14}C_2$

5. The value of $\sum_{n=1}^{\infty} \frac{(2n^2+2n+4)}{(2n)!}$ is

(1) $3e + \frac{3}{2e} - 4$ (2) $e + \frac{3}{2e} - 4$ (3) $3e + \frac{3}{e} + 4$ (4) $e + \frac{1}{2e} - 4$

Ans. (1)

Sol. $\sum_{n=1}^{\infty} \frac{2n^2+2n+4)}{2n!}$

$= \frac{1}{2}\sum_{n=1}^{\infty} \frac{2n(2n-1)+6n+8}{2n!}$

$= \frac{1}{2}\sum_{n=1}^{\infty}\left\{\frac{1}{(2n-2)!} + \frac{3}{(2n-1)!} + \frac{8}{2n!}\right\}$

$= \frac{1}{2}\left\{\left(\frac{e+e^{-1}}{2}\right) + 3\left(\frac{e-e^{-1}}{2}\right) + 8\left(\frac{e+e^{-1}}{2} - 1\right)\right\}$

$= \frac{1}{2}\{6e + 3e^{-1} - 8\}$

$= 3e + \frac{3}{2e} - 4$

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6. If a matrix $A = \frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$ then

(1) $A^{30} - A^{25} = I - A$ (2) $A^{30} = A^{25}$ (3) $A^{30} - A^{25} = A - I$ (4) $A^{30} - A^{25} = 2I - A$

Ans. (1)

Sol. $A = \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix}$ ....(1)

$\because A(\alpha) . A(\beta) = A(\alpha + \beta)$

So by above.

$$A^{30}\left(\frac{\pi}{3}\right) = \begin{bmatrix} \cos 30\left(\frac{\pi}{3}\right) & -\sin 30\left(\frac{\pi}{3}\right) \\ \sin 30\left(\frac{\pi}{3}\right) & \cos 30\left(\frac{\pi}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{25}\left(\frac{\pi}{3}\right) = \begin{bmatrix} \cos 25\left(\frac{\pi}{3}\right) & -\sin 25\left(\frac{\pi}{3}\right) \\ \sin 25\left(\frac{\pi}{3}\right) & \cos 25\left(\frac{\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = A$$

$A^{30} - A^{25} = I - A$

7. The solution set of equation $2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$ is 'S' then

(1) Number of elements in S which are smaller than $\frac{1}{2}$ is 1

(2) Number of elements in S which are greater than $\frac{1}{2}$ is 1

(3) Number of elements in S which are smaller than $-\frac{5}{2}$ is 1

(4) Number of elements in S which are greater than 1 is 1

Ans. (1)

Sol. Put $x = \tan\theta$ $\theta \in \left(0, \frac{\pi}{4}\right)$

$$2\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$2\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right] = \cos^{-1}[\cos(2\theta)]$$

$$\Rightarrow 2\left(\frac{\pi}{4} - \theta\right) = 2\theta \Rightarrow \theta = \frac{\pi}{8}$$

$$\Rightarrow x = \tan\frac{\pi}{8}$$

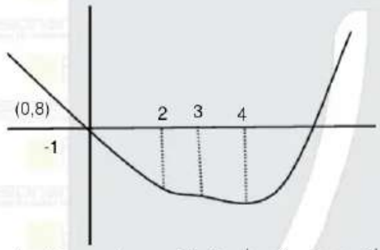
8. The sum of absolute maximum and absolute minimum value of function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in interval $[-1, 3]$ is

8. The sum of absolute maximum and absolute minimum value of function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in interval $[-1, 3]$ is

Ans. (24)

Sol. $f(x) = \begin{cases} x^2 - 5x + 6 - 3x + 2 & , \quad x \in (-\infty, 2) \cup [3, \infty) \\ -(x^2 - 5x + 6) - 3x + 2 & , \quad x \in [2, 3] \end{cases}$

$\Rightarrow f(x) = \begin{cases} x^2 - 8x + 8 & , \quad x \in (-\infty, 2) \cup [3, \infty) \\ -x^2 + 2x - 4 & , \quad x \in [2, 3] \end{cases}$



absolute maximum $= |f(-1)| = |(-1)^2 - 8(-1) + 8| = 17$

absolute minimum $= |f(3)| = 7$

sum $= 17 + 7 = 24$

9. Three different Arithmetic progression have first term 2, 3, 7, and common differences 3, 4, 5 respectively if the last terms of sequences are 359, 239 and 197 respectively then the sum of all common terms of the three progression is.

Ana. (321)

Sol. $S_1 = \{2,5,8,11,14,\_\_\_\_\_,359\}$

$S_2 = \{3,7,11,15,\_\_\_\_\_,239\}$

$S_3 = \{7,12,17,22,\_\_\_\_\_\ 197\}$

common AP

$S = \{47,107,167\}$

Hence sum of common AP = 321

10. $9 = x_1 < x_2 < x_3 < \ldots\ldots < x_7$ are in arithmetic progression with common difference d. If the standard deviation of the seven terms is 4 and $\bar{x}$ is the mean then the value of $\bar{x} + x_6$ is

Ans. (34)

10. $9 = x_1 < x_2 < x_3 < \ldots\ldots\ldots < x_7$ are in arithmetic progression with common difference d. If the standard deviation of the seven terms is 4 and $\bar{x}$ is the mean then the value of $\bar{x} + x_6$ is

Ans. (34)

Sol. Mean $\Rightarrow \bar{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a+6d]}{7} = a + 3d = x_4$

Variance $= \frac{\sum_{i=1}^{7}(x_i - \bar{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7}(x_i - x_4)^2}{7} = 16$

$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$

$= 4d^2 = 16 \quad \Rightarrow \quad d = 2$

$\Rightarrow \bar{x} = 9 + 3(2) = 15$

& $x_6 = a + 5d = 9 + 5(2) = 19 \Rightarrow \bar{x} + x_6 = 34$

11. Which of the following is tautology

(1) $p \to (\sim p \wedge q)$ (2) $p \to (p \vee q)$ (3) $p \to (\sim p \vee q)$ (4) $p \to (\sim p \wedge \sim q)$

Ans. (2)

Sol. A) $p \to (\sim p \wedge q) \equiv \sim p \vee (\sim p \wedge q) \neq T$

B) $p \to (p \vee q) = \sim p \vee (p \vee q) = (\sim p \vee p) \vee q$

$\equiv T \vee q$

$\equiv t \vee q$

C) $p \to (\sim p \vee q) \equiv \sim p \vee (\sim p \vee q) \neq t$ (for $p = T, q = F$)

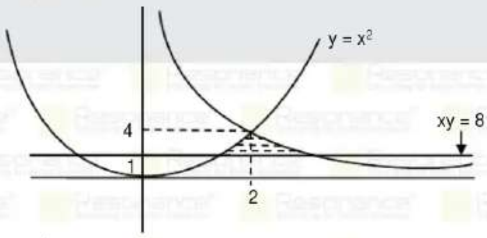
D) $p \to (\sim p \wedge q) = \sim p \vee (\sim p \wedge \sim q) \neq t$ (for $p = T$)

12. The area bounded by curves $xy < 8$, $y < x^2$ and $y > 1$ is

(1) $4\ln 2 - \frac{14}{3}$ (2) $4\ln 2 + \frac{20}{3}$ (3) $8\ln 4 - \frac{14}{3}$ (4) $8\ln 4 - \frac{20}{3}$

Ans. (3)

Sol. $A = \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right) dt$



$= \left[8\ln y - \frac{y^{3/2}}{3/2}\right]_1^4 = 8(\ln 4 - \ln 1) - \frac{2}{3}(4^{3/2} - 1)$

$= 8\ln 4 - \frac{16}{3} + \frac{2}{3} = 8\ln 4 - \frac{14}{3}$ Ans. C

$= 8\ln 4 - \frac{16}{3} + \frac{2}{3} = 8\ln 4 - \frac{14}{3}$ Ans. C

13. The value of $\int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$ is

(1) $\frac{\pi^2}{6\sqrt{3}}$ (2) $\frac{\pi^2}{12}$ (3) $\frac{\pi^2}{3\sqrt{3}}$ (4) $\frac{\pi^2}{12\sqrt{3}}$

Ans. (1)

Sol. $I = \int_{-\pi/4}^{\pi/4} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$ ....(1)

replacing x by – x

$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{-x+\frac{\pi}{4}}{2-\cos 2x} dx$ ....(2)

(1) + (2) $\Rightarrow$ $2I = \int_{-\pi/4}^{\pi/4} \frac{\frac{\pi}{2}}{2-\cos 2x} dx$

$I = \frac{\pi}{4} \cdot \int_{-\pi/4}^{\pi/4} \frac{dx}{2-\cos 2x}$

$= \frac{\pi}{4} \cdot 2\int_0^{\pi/4} \frac{dx}{2-\cos 2x} = \frac{\pi}{2} \cdot \int_0^{\pi/4} \frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}}$

$I = \frac{\pi}{2} \cdot \int_0^{\pi/4} \frac{\sec^2 x}{1-3\tan^2 x} dx$ Let tan x = t

Þ $\sec^2 x\, dx = dt$

$= \frac{\pi}{2}\int_0^1 \frac{dt}{1+3t^2}$

$= \frac{\pi}{2} \cdot \frac{1}{\sqrt{3}} \left(\tan^{-1}\sqrt{3}t\right)_0^1 = \frac{\pi}{2\sqrt{3}} \left(\tan^{-1}\sqrt{3} - \tan^{-1} 0\right)$

$I = \frac{\pi^2}{6\sqrt{3}}$

14. For some value of α, β system of equations

$\alpha x + y + z = 1$

$x + \alpha y + z = 1$

$x + y + \alpha z = \beta$ has infinitely many solutions, then -

(1) $\alpha = 1, \beta = 1$ (2) $\alpha = 1, \beta = -1$

(3) $\alpha = -1, \beta = -1$ (4) $\alpha = -1, \beta = 1$

Ans. (1)

Ans. (1)

Sol. $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = \alpha(\alpha^2 - 1) - (\alpha - 1) + (1 - \alpha) = \alpha^3 - 3\alpha + 2 = (\alpha - 1)(\alpha^2 + \alpha - 2)$

$(\alpha - 1)(\alpha - 1)(\alpha + 2)$

for $\alpha = 1$ system of equations is

$x + y + z = 1$

$x + y + z = 1$

$x + y + z = \beta \Rightarrow \beta = 1$

15. If (x_0, y_0) is a point on curve $3x^2 - 4y^2 = 36$ which is nearest to line $3x + 2y = 1$ then the value of $\sqrt{2}|x_0 - y_0|$ is

Ans. (9)

Sol. equation of tangent at (x_0, y_0) is $T = 0$

$\Rightarrow 3xx_0 - 4yy_0 = -36$

it should be parallel to $3x + 2y = 1$

$\frac{3x_0}{4y_0} = \frac{-3}{2} \Rightarrow \quad x_0 = -2y_0$

$\because (x_0, y_0)$ lies on curve $\Rightarrow 3x_0^2 - 4y_0^2 = 36$

$\Rightarrow 3(-2y_0)^2 - 4y_0^2 = 36$

$\Rightarrow y_0^2 = \frac{9}{2} \Rightarrow y_0 = \pm\frac{3}{\sqrt{2}}$

$\Rightarrow x_0 = \mp 3\sqrt{2}$

Point $\Rightarrow \left(3\sqrt{2}, -\frac{3}{\sqrt{2}}\right)$ or $\left(-3\sqrt{2}, \frac{3}{\sqrt{2}}\right)$

$\Rightarrow \sqrt{2}|x_0 - y_0| = \sqrt{2}\left|3\sqrt{2} - \left(-\frac{3}{\sqrt{2}}\right)\right| = 9$

16. How many six digit numbers are formed by digits 4, 5 and 9 which are divisible by 6 is.

Ans. (81)

Sol. Unit digit must be 4 since number should be divisible by 2.

Four out of remaining five places, each has 3 options and remaining one place will have only one option so total number of six digit numbers = 3.3.3.3.1 = 81

17. A vector $(\vec{r})$ satisfy $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$

17. A vector $(\vec{r})$ satisfy $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}.\vec{b}=0$

Where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$\vec{b}=2\hat{i}-\hat{j}+\hat{k}$

$\vec{c}=3\hat{i}+\hat{k}$

then value of $|\vec{r}|$ is

(1) $\frac{5}{2}$ (2) $\frac{5}{\sqrt{3}}$ (3) $\frac{3\sqrt{3}}{2}$ (4) $\frac{5\sqrt{3}}{2}$

Ans. (4)

Sol. $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$

$\Rightarrow (\vec{r}-\vec{c})\times\vec{a}=0$

$\Rightarrow (\vec{r}-\vec{c})=\lambda\vec{a}$

$\Rightarrow \vec{r}=\vec{c}+\lambda\vec{a}$

$\Rightarrow \vec{r}.\vec{b}=\vec{c}.\vec{b}+\lambda\vec{a}.\vec{b}$

$\Rightarrow 0=(6+1)+\lambda(2-1+1)$

$\Rightarrow \lambda=-\frac{7}{2}$

Hence $\vec{r}=\vec{c}-\frac{7}{2}\vec{a}$

$=(3\hat{i}+\hat{k})-\frac{7}{2}(\hat{i}+\hat{j}+\hat{k})$

$=-\frac{1}{2}\hat{i}-\frac{7}{2}\hat{j}-\frac{5}{2}\hat{k}$

$\Rightarrow |\vec{r}|=\sqrt{\frac{1}{4}+\frac{49}{4}+\frac{25}{4}}=\frac{5\sqrt{3}}{2}$

18. Two unbiased dice are thrown simultaneously. A is the event such that the number on the first die is less than second die. B is the event such that number on the first die is even and number on the second die is odd. C is the event such that first die shows odd number and second die shows even number then

(1) $n((A\cup B)\cap C)=6$ (2) A and B are mutually exclusive events

(3) A & B independent event (4) $n(A)=18$, $n(B)=6$, $n(C)=6$

Ans. (1)

Sol. A (I < II) B (EO) C(OE)

$n(A)=15$ $n(B)=9$ $n(C)=9$

$n(A\cap B)=3$ $n(A\cap C)=6$ $n(B\cap C)=0$

$n(A\cap B\cap C)=0$

$n((A\cup B)\cap C)=n(A\cap C)+n(B\cap C)-n(A\cap B\cap C)=6$

19. If $\frac{dy}{dx}=\frac{x^2+3y^2}{3x^2+y^2}$, satisfies y (1) = 0, then

19. If $\frac{dy}{dx}=\frac{x^2+3y^2}{3x^2+y^2}$, satisfies y (1) = 0, then

(1) $\frac{2x^2}{(x-y)^2}=\ell n|x-y|+\frac{2x}{x-y}$ (2) $\frac{2x}{(x-y)^2}=\ell n|x-y|+1$

(3) $\frac{2x^2}{(x-y)^2}=\ell n|x-y|+\frac{y}{x-y}$ (4) $\frac{2x}{(x-y)^2}=\ell n|x-y|+\frac{y}{x-y}$

Ans. (1)

Sol. $\frac{dy}{dx}=\frac{x^2+3y^2}{3x^2+y^2}$ is a homogeneous equation

Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx}=\frac{1+3v^2}{3+v^2}$

$\Rightarrow \frac{3+v^2}{1+3v^2-3v-v^3}dv=\frac{dx}{x}$

$\Rightarrow \int\frac{3+v^2}{-v^3+3v^2-3v+1}dv=\int\frac{dx}{x}$

$\Rightarrow \int\frac{3+v^2}{(1-v)^3}dv=\int\frac{dx}{x}$

$\Rightarrow \int\left\{\frac{1}{1-v}-\frac{2}{(1-v)^2}+\frac{4}{(1-v)^3}\right\}dv=\int\frac{dx}{x}$

$\Rightarrow -\ell n|1-v|-\frac{2}{(1-v)}+\frac{2}{(1-v)^2}=\ell nx+\ell nc$

$\Rightarrow -\ell n|x-y|-\frac{2x}{x-y}+\frac{2x^2}{(x-y)^2}=\ell nc$

Since y (1) = 0

$\Rightarrow \ell nc = 0$

Hence $\frac{2x^2}{(x-y)^2}=\ell n|x-y|+\frac{2x}{x-y}$