01/02/2023 Morning



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# Memory Based Answers & Solutions

Time : 3 hrs. M.M. : 300

# JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

#### **IMPORTANT INSTRUCTIONS:**

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
  - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
  - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.



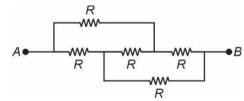
## **PHYSICS**

#### **SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

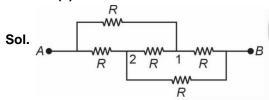
#### Choose the correct answer:

 In the circuit shown find the equivalent resistance between terminals A and B.

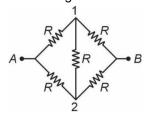


- (1)  $\frac{3R}{2}$
- (2) 2R
- (3) 4R
- (4) R

## Answer (4)



Redrawing the structure.

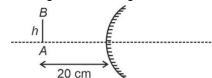


It is wheatstone bridge so

$$R_{\text{net}} = \frac{2R \times 2R}{2R + 2R} = R$$

2. An object of height *h* is placed in front of a convex mirror (radius of curvature = 20 cm).

Find the height of the image.



(1)  $\frac{n}{2}$ 

(2)  $\frac{h}{3}$ 

(3)  $\frac{h}{6}$ 

 $(4) \frac{h}{4}$ 

Answer (2)

**Sol.** 
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{-20} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{v} = \frac{3}{20}$$

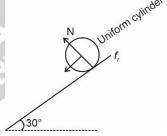
$$\Rightarrow v = \frac{20}{3}$$

$$\Rightarrow m = -\frac{v}{u} = \frac{1}{3}$$

- A uniform solid cylinder of radius R, is released from a 600 m long ramp, inclined at 30° from the horizontal. Find the time taken to reach the bottom of ramp. (Consider sufficient friction for pure rolling)
  - (1) 60 sec
  - (2)  $6\sqrt{10} \sec$
  - (3)  $3\sqrt{10}$  sec
  - (4) 20 sec

#### Answer (2)

Sol.



$$mg\sin\theta - f_r = ma$$

Also, 
$$\frac{3}{2}mR^2\alpha = mg\sin\theta \times R$$

$$\Rightarrow \frac{3}{2}mR = mg\sin\theta$$

$$a = \frac{2}{3}g\sin\theta = \frac{2}{3} \times g \times \sin 30^\circ = \left(\frac{g}{3}\right) = \frac{10}{3} \text{ m/s}^2$$

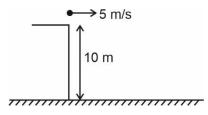
S = 600 m

time = 
$$\sqrt{\frac{2s}{a}}$$
 =  $\sqrt{\frac{1200 \times 3}{10}}$   
=  $\sqrt{360}$ 

 $=6\sqrt{10}$  seconds



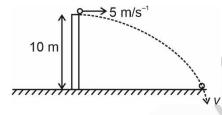
4. A ball is thrown horizontally from a height of 10 m with a speed of 5 ms<sup>-1</sup> as shown. Find the speed with which it strikes the ground.



- (1) 15 m/s
- (2) 5 m/s
- (3) 10 m/s
- (4) 20 m/s

#### Answer (1)

Sol.



$$v^2 = u^2 + 2gh$$

$$v^2 = 25 + 2 \times 10 \times 10$$

$$v^2 = 225$$

$$v = \sqrt{225}$$

$$= 15 \text{ m/s}$$

- 5. An ideal gas (adiabatic constant = 3/2) undergoes an adiabatic expansion process where change in temperature is -T. If there are 2 moles of the gas, find the work done by the gas.
  - (1) 3RT
  - (2) 2RT
  - (3) 4RT
  - (4) -RT

#### Answer (3)

Sol. 
$$W = \frac{nR\Delta T}{1 - \gamma}$$
$$= \frac{2 \cdot R \cdot (-T)}{1 - \frac{3}{2}}$$
$$= 4RT$$

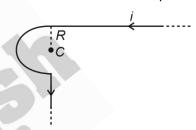
6. **Statement-1**: Value of acceleration due to gravity is same at all the point inside earth assuming it to be made up of uniform density.

**Statement-2**: Value of gravitational field increases as we go towards centre in a uniform spherical shell.

- (1) Both statement-1 and statement-2 are true
- (2) Statement-1 is true but statement-2 is false
- (3) Statement-1 is false but statement-1 is true
- (4) Both statement-1 and statement-2 are false

#### Answer (4)

- **Sol.** Value of acceleration due to gravity decreases as we go inside the earth and it does not change inside the spherical shell.
- 7. An infinite wire is bent in the shape as shown:



Find the magnetic field at point C.

(1) 
$$\frac{\mu_0 i(1+\pi)}{4\pi R}$$

(2) 
$$\frac{\mu_0 i(2+\pi)}{4\pi R}$$

(3) 
$$\frac{\mu_0 i(1+\pi)}{2\pi R}$$

(4) 
$$\frac{\mu_0 i}{2R}$$

#### Answer (1)

**Sol.** 
$$B = \frac{\mu_0 i}{4\pi R} [\sin 90^\circ + \sin 0^\circ] + \frac{\mu_0 i}{4R} + 0$$

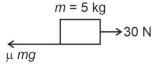
$$=\frac{\mu_0 i}{4\pi R}[1+\pi]$$

A force of 30 N is applied on a block of mass
 kg. The block travels a distance of 50 m in
 sec starting from rest. Find coefficient of friction

- (1) 0.5
- (2) 0.7
- (3) 0.3
- (4) 0.8

#### Answer (1)

Sol.



$$30 - \mu \ \textit{mg} = \textit{ma} \Rightarrow \textit{a} = \left(\frac{30 - 50\mu}{5}\right)$$

$$S = \frac{1}{2}at^2$$

$$\Rightarrow 50 = \frac{1}{2}a \times t^2$$

$$\Rightarrow \frac{100}{100} = a$$

$$a = 1 \text{ m/s}^2$$

$$\Rightarrow \quad \frac{30-50\mu}{5}=1$$

$$\Rightarrow \quad \mu = \frac{25}{50} = \frac{1}{2} = 0.5$$

- Which of the following is not the frequency of frequency modulated (FM) signal?
  - (1) 90 MHz
  - (2) 89 MHz
  - (3) 106 MHz
  - (4) 100 KHz

#### Answer (4)

Sol. Band of FM is in MHz

10. For a real gas, the equation of gas is given by  $\left(P + \frac{an^2}{V^2}\right)(V - bn) = nRT$ . If symbols have their

usual meaning then the dimensions of  $\frac{V^2}{ar^2}$  is same

as that of

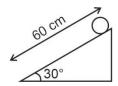
- (1) Compressibility
- (2) Bulk modulus
- (3) Viscosity
- (4) Energy density

#### Answer (1)

**Sol.**  $[P] = \left| \frac{an^2}{V^2} \right| = \text{dimension of bulk modulus.}$ 

So 
$$\left[\frac{V^2}{an^2}\right]$$
 has dimension of compressibility.

11. A solid cylinder starts pure rolling from top of a fixed inclined plane of slant length 60 cm as shown in the figure. Velocity of centre of mass when the cylinder reaches the bottom is equal to  $(g = 10 \text{ m/s}^2)$ 



- (1) 1 m/sec
- (2) 2 m/sec
- (3) 3 m/sec
- (4) 4 m/sec

#### Answer (2)

Sol. Using energy conservation

$$mg(0.3) = \frac{1}{2}mv_{\rm cm}^2 + \frac{1}{2} \times \frac{mR^2}{2} \frac{v_{\rm cm}^2}{R^2}$$

$$mg \times 0.3 = \frac{3}{4} m v_{\rm cm}^2$$

 $v_{cm} = 2 \text{ m/sec}$ 

A stone is thrown vertically up with speed  $v_0$  from a cliff of height H. Find the average speed of the ball till the moment it reaches ground. Given that  $H = 100 \text{ m}, v_0 = 10 \text{ m/s}, g = 10 \text{ m/s}^2.$ 

(1) 
$$\frac{64}{1+\sqrt{21}}$$
 m/s

(2) 55 m/s

(3) 
$$110(1+\sqrt{21})$$
 m/s (4)  $\frac{110}{1+\sqrt{21}}$  m/s

(4) 
$$\frac{110}{1+\sqrt{21}}$$
 m/s

Answer (4)

**Sol.** Total distance = 
$$\frac{v_0^2}{2g} \times 2 + 100$$

= 110 m

Total time =  $t_0$ 

$$\Rightarrow$$
 - 100 = 10 $t_0$  -  $\frac{1}{2}$  × 10 ×  $t_0^2$ 

$$\Rightarrow t_0 = 1 + \sqrt{21} s$$

$$\Rightarrow$$
 Avg. speed =  $\frac{110}{1+\sqrt{21}}$  m/s



- 13. A drop of mercury is divided into 125 drops of equal radius of 10<sup>-3</sup> m each. If surface tension of mercury is equal to 0.45 N m<sup>-1</sup>. Magnitude of change in surface energy is equal to nearly
  - (1)  $1.41 \times 10^{-4} \text{ J}$
  - (2)  $7.06 \times 10^{-4} \text{ J}$
  - (3)  $8.47 \times 10^{-4} \text{ J}$
  - (4)  $5.65 \times 10^{-4} \text{ J}$

## Answer (4)

Sol. Let radius of bigger drop was R so

$$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi \times (10^{-3})^3$$

$$\Rightarrow R = 5 \times 10^{-3} \text{ m}$$

So  $U_i = 4\pi R^2 \sigma = 4\pi \times (5 \times 10^{-3})^2 \times 0.45 = 1.41 \times 10^{-4} \text{ J}$  $U_i = 125 \times 4\pi I^2 \sigma = 500\pi (10^{-3})^2 \times 0.45 = 7.06 \times 10^{-4} \text{ J}$ 

So 
$$\Delta U = U_f - U_i$$

$$= 5.65 \times 10^{-4} \text{ J}$$

- 14. A charged particle with charge 2 x 10<sup>-6</sup> C, at rest, is first accelerated through a potential difference of 100 V and then it is subjected to a transverse magnetic field of 4 mT. In region of magnetic field, it undergoes a circular path of radius 3 cm. Mass of the particle is equal to
  - (1)  $1.44 \times 10^{-16} \text{ kg}$
  - (2)  $7.2 \times 10^{-16} \text{ kg}$
  - (3)  $1.44 \times 10^{-10} \text{ kg}$
  - (4)  $7.2 \times 10^{-10} \text{ kg}$

#### Answer (1)

**Sol.** 
$$R = \frac{\sqrt{2mqV}}{qB}$$

$$3 \times 10^{-2} = \frac{\sqrt{2m \times 100}}{\sqrt{2 \times 10^{-6} \times 4 \times 10^{-3}}}$$

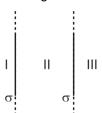
$$\Rightarrow m = 1.44 \times 10^{-16} \text{ kg}$$

- 15. A string of mass per unit length equal to  $7 \times 10^{-3}$  kg/m is subjected to a tension equal to 70 N. The speed of transverse wave on this string is equal to
  - (1) 10 m/sec
  - (2) 50 m/sec
  - (3) 100 m/sec
  - (4) 200 m/sec

#### Answer (3)

Sol. 
$$v = \sqrt{\frac{T}{\mu}}$$
  
=  $\sqrt{\frac{70}{7 \times 10^{-3}}}$ 

16. Two thin insulating sheets (each having charge density  $+\sigma$ ) are arranged as shown:



Then find the net electric field (magnitude) in the 3 regions:

(1) 
$$E_1 = \frac{\sigma}{\varepsilon_0}$$

$$E_2 = 0$$

$$E_3 = \frac{\sigma}{\varepsilon_0}$$

(2) 
$$E_1 = E_2 = E_3 = 0$$

(3) 
$$E_1 = 0$$

$$E_2 = \frac{\sigma}{2\varepsilon_0}$$

$$E_3 = \frac{\sigma}{\epsilon_0}$$

(4) 
$$E_1 = \frac{\sigma}{\varepsilon_0}$$

$$E_2 = 0$$

$$E_3 = \frac{\sigma}{2\epsilon_0}$$

#### Answer (1)

**Sol.** 
$$E_1 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$E_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

$$E_3 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$



17. Match the two columns:

	Column I		Column II
a.	Intrinsic semiconductor	1.	Fermi level is closer to conduction band
b.	p-type semiconductor	2.	Fermi level is closer to valence band
C.	n-type semiconductor	3.	Fermi level is inside conduction band
d.	Metal	4.	Fermi level is in between the two bands

(1) 
$$a-4$$
,  $b-2$ ,  $c-1$ ,  $d-3$ 

(2) 
$$a-1$$
,  $b-3$ ,  $c-2$ ,  $d-4$ 

(3) 
$$a-3$$
,  $b-2$ ,  $c-1$ ,  $d-4$ 

(4) 
$$a-2$$
,  $b-1$ ,  $c-3$ ,  $d-4$ 

#### Answer (1)

**Sol.** In intrinsic, fermi level is between the two bands. In p-type, fermi level is closer to valence band. In n-type, fermi level is closer to conduction band. In metal, fermi level is inside conduction band.

18.

19.

20.

#### **SECTION - B**

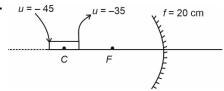
**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A cylindrical rod of length 10 cm is placed along the principal axis of concave mirror (f = 20 cm) in such a way that the COM of the rod is at 40 cm from the pole of mirror. The length of image of rod is <sup>x</sup>/<sub>2</sub> cm.

Find x.

#### **Answer (32.00)**

Sol.



For u = -45 cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{45} = \frac{-1}{20}$$

$$\frac{1}{v} = \frac{-1}{20} + \frac{1}{45} = \frac{-9+4}{180} = \frac{-5}{180}$$

$$v = -36 \text{ cm}$$

For u = -35 cm

$$\frac{1}{v} = -\frac{1}{20} + \frac{1}{35}$$

$$=\frac{-7+4}{140}=\left(\frac{-3}{140}\right)$$

$$u_2 = -\left(\frac{140}{3}\right)$$

$$|v_1 - v_2| = \left| \frac{140}{3} - 36 \right|$$

Length of image 
$$=$$
  $\left(\frac{32}{3}\right)$  cm

22. In an series LCR circuit connected across 220 V, 50 Hz A.C supply. If the inductive reactance of the circuit is 79.6  $\Omega$ . If the power delivered in the circuit is maximum, the capacitance of the circuit is x  $\mu$ F. Find x.

#### Answer (40.00)

**Sol.** For maximum power, LCR should be resonance condition,

$$X_{l} = X_{C}$$

$$\Rightarrow$$
 79.6 =  $\frac{1}{\omega_0}$ 

as 
$$\omega = 2\pi f$$

$$= 2\pi \times 50 = 100\pi \text{ rad/sec}$$



$$C = \frac{1}{(79.6 \times 100\pi)}$$

$$= 4 \times 10^{-5}$$

$$= 40 \times 10^{-6} F$$

$$= 40 \mu F$$

23. An alpha particle and a proton having same de-broglie wavelengths will have kinetic energies in the ratio

## Answer (00.25)

**Sol.** Charge on  $\alpha$  particle = 2e

Mass of proton = m

Mass of  $\alpha$  particle = 4m

$$\frac{\lambda_{p}}{\lambda_{\alpha}} = \frac{\left(P_{\alpha}\right)}{\left(P_{p}\right)} = \frac{\sqrt{2K_{\alpha}m_{\alpha}}}{\sqrt{2K_{p}m_{p}}} = 1$$

$$\frac{K_{\alpha}}{K_{p}} \times \left(\frac{m_{\alpha}}{m_{p}}\right) = 1$$

$$\frac{K_{\alpha}}{K_{\alpha}} \times 4 = 1$$

$$\frac{K_{\alpha}}{K_{D}} = \frac{1}{4}$$

24. If mass of a planet is 9 times that of earth and radius is 2 times that of earth, then escape speed from this planet is  $\frac{xv_e}{\sqrt{2}}$ . Find  $x. v_e$  is escape speed from earth.

#### Answer (03)

**Sol.** 
$$v'_{e} = \sqrt{\frac{2GM'}{R'}}$$

$$V_{\theta} = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \frac{v_e'}{v_e} = \sqrt{\frac{9}{2}}$$

$$\Rightarrow v'_{e} = \frac{3v_{e}}{\sqrt{2}}$$

25. There are n number of polarizers arranged one after the other. Each polarizer pass axis is inclined at 45° with respect to the previous polarizer. Unpolarized light of intensity  $l_0$  is incident on this setup. Final transmitted light has intensity  $\frac{l_0}{64}$ . Find n.

**Sol.** 
$$I = \frac{I_0}{2} \times \frac{\cos^2 45^\circ \times \cos^2 45^\circ \times ...}{(n-1) \text{ times}}$$

$$\Rightarrow \frac{I_0}{64} = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^{n-1}$$

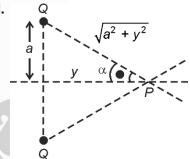
$$\Rightarrow n-1=5$$

$$\Rightarrow n = 6$$

26. Two point charges each of magnitude Q is kept at a separation of 2a. The distance from mid point on perpendicular bisector where a point charge will experience maximum force is  $\frac{a}{\sqrt{x}}$ . Find the value of x.

## Answer (02.00)

Sol.



E due to one charge = 
$$\frac{KQ}{(a^2 + y^2)}$$

 $E_{\text{net}}$  at point  $P = 2E\cos\alpha$ 

$$=\frac{2KQy}{\left(y^2+a^2\right)^{\frac{3}{2}}}$$

Force =  $q E_{net}$ 

$$\frac{dF}{dy} = 0$$
, for maximum force, on solving  $\frac{dF}{dy} = 0$ ,

we get, 
$$y = \left(\frac{a}{\sqrt{2}}\right)$$

$$\therefore x = 2$$

- 27.
- 28.
- 29.
- 30.



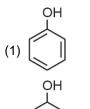
## **CHEMISTRY**

#### SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

#### Choose the correct answer:

1. Which of the following compound shows fastest rate of dehydration?





#### Answer (3)

(3)

**Sol.** The alcohol which produces stable carbocation shows faster rate of dehydration.

- Which one of the following complexes has maximum splitting?
  - (1) [Fe(CN)<sub>6</sub>]<sup>4-</sup>
  - (2) [Fe(NH<sub>3</sub>)<sub>6</sub>]<sup>2+</sup>
  - (3) [FeCl<sub>6</sub>]<sup>4-</sup>
  - (4) [Fe(OX)<sub>3</sub>]<sup>4-</sup>

#### Answer (1)

**Sol.** CN<sup>-</sup> is strongest field ligand out of given ligands.

- 3. Average kinetic energy of an ideal gas depends on
  - (1) Nature of the gas
  - (2) Pressure of the gas
  - (3) Temperature of the gas
  - (4) Volume of the gas

#### Answer (3)

**Sol.** Average kinetic energy of an ideal gas per mole is given by

$$\overline{KE} = \frac{3}{2} RT$$

.. KE depends on temperature

4. **Assertion:** Hydrogen is an environment/ecofriendly fuel

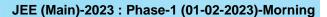
Reason: Hydrogen is the lightest element

- Assertion is true and reason is true and reason is the correct explanation of assertion
- (2) Assertion is true and reason is true but reason is not the correct explanation of assertion
- (3) Assertion is true but reason is false
- (4) Assertion is false but reason is true

#### Answer (2)

Sol. The correct option is (2)

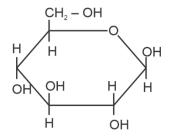
5. Pyranose form of the given compound is?





#### Answer (2)

**Sol.** The correct pyranose form of the given compound is



$$6. \hspace{0.5cm} X_{(g)} \ensuremath{ \Longleftrightarrow } 2Y_{(g)} \hspace{0.5cm} ; \hspace{0.1cm} k_{p_1} \ldots (i)$$

$$\mathsf{A}_{(g)} \ \Longleftrightarrow \ \mathsf{B}_{(g)} + \mathsf{C}_{(g)} \quad \ \ ; \ \mathsf{k}_{\mathsf{p}_2} \ldots (ii)$$

If degree of dissociation is same for both the reactions. Find out ratio of total pressure  $p_1$  and  $p_2$  respectively.

(1) 
$$\frac{k_{p_1}}{k_{p_2}}$$

(2) 
$$\frac{4k_{p_1}}{k_{p_2}}$$

(3) 
$$\frac{k_{p_1}}{4k_{p_2}}$$

(4) 
$$\frac{k_{p_1}}{2k_{p_2}}$$

#### Answer (3)

Sol.

$$\begin{array}{ccc}
X & \Longleftrightarrow & 2Y \\
1 & & - \\
(1-\alpha) & & 2\alpha
\end{array}$$

$$k_{p_1} = \frac{2\alpha \cdot 2\alpha \left(1 + \alpha\right)}{\left(1 + \alpha\right)\left(1 + \alpha\right)\left(1 - \alpha\right)} \cdot p_1$$

$$A_{(g)} \iff B_{(g)} + C_{(g)}$$
  
 $(1-\alpha) \qquad \alpha \qquad \alpha$ 

$$k_{p_2} = \frac{\alpha \cdot \alpha}{(1+\alpha)(1-\alpha)} \cdot p_2$$

$$\frac{k_{p_1}}{k_{p_2}} = \frac{4p_1}{p_2}$$

$$\Rightarrow \frac{p_1}{p_2} = \frac{k_{p_1}}{4k_{p_2}}$$

7. Which of the following is not correctly matched?

List-I

List-II

(A) Antibiotic

Penicillin

(B) Antiseptic

Chloroxylenol

(C) Tranquilizer

Erythromycin

(D) Analgesic

Aspirin

(1) A

(2) B

(3) C

(4) D

#### Answer (3)

**Sol.** Erythromycin is an antibiotic

Hence correct answer is option (3)

8. 
$$CH_3 - C \equiv C - CH_3$$

$$+H_2$$
Lindlar's catalyst (A)

Identify the correct statement on physical properties of (A) and (B)

(1) Melting point: A > B

Boiling point: A > B

(2) Melting point: A < B

Boiling point: A > B

(3) Melting point: A > B

Boiling point: A < B

(4) Melting point: A < B

Boiling point: A < B

#### Answer (2)

Sol. 
$$CH_3 - C \equiv C - CH_3 + H_2 \xrightarrow{\text{Lindlar's}} CH_3 + C = CH_3$$
(A)

$$CH_3 - C \equiv C - CH_3 \xrightarrow{\text{Na/Liq. NH}_3} \xrightarrow{\text{CH}_3} + C = C \xrightarrow{\text{CH}_3}$$
(Birch reduction)



Compound (A) is polar and has a permanent dipole moment whereas compound (B) is non-polar. Compound (A) will have higher boiling point than (B) due to dipole-dipole interaction. However, compound (B) is more symmetrical than compound (A). Therefore, (B) will have higher melting point that (A).

9. Which of the following option contains the correct match?

#### List-I

#### List-II

- (A) Caustic soda
- (P) CaSO<sub>4</sub>
- (B) Washing soda
- (Q) Na<sub>2</sub>CO<sub>3</sub>·10H<sub>2</sub>O
- (C) Dead burnt plaster
- (R) Ca(OH)<sub>2</sub>
- (D) Slaked lime
- (S) NaOH
- (1)  $A \rightarrow S$ ,  $B \rightarrow Q$ ,  $C \rightarrow P$ ,  $D \rightarrow R$
- (2)  $A \rightarrow P$ ,  $B \rightarrow Q$ ,  $C \rightarrow R$ ,  $D \rightarrow S$
- (3)  $A \rightarrow S$ ,  $B \rightarrow P$ ,  $C \rightarrow Q$ ,  $D \rightarrow R$
- (4)  $A \rightarrow R$ ,  $B \rightarrow S$ ,  $C \rightarrow Q$ ,  $D \rightarrow P$

#### Answer (1)

- Sol. Caustic soda
- → NaOH
- Washing soda
- → Na<sub>2</sub>CO<sub>3</sub>·10H<sub>2</sub>O
- Dead burnt plaster
- → CaSO<sub>4</sub>
- Slaked lime
- → Ca(OH)<sub>2</sub>
- 10. How photochemical smog can be controlled in automobiles?
  - (1) Using catalytic convertors which will increase release of nitrogen oxide
  - (2) Using catalytic convertors which will decrease the release of nitrogen oxide
  - (3) By increasing sulphur content in fuel
  - (4) By decreasing sulphur content in fuel

#### Answer (2)

**Sol.** Catalytic convertors will prevent the release of nitrogen oxide and hydrocarbons to the atmosphere.

- 11. When K<sub>4</sub>[Fe(CN)<sub>6</sub>] is added to FeCl<sub>3</sub>, the Prussian blue complex compound formed is
  - (1)  $Fe_3[Fe(CN)_6]_4$
- (2) Fe<sub>4</sub>[Fe(CN)<sub>6</sub>]<sub>3</sub>
- (3)  $K_2Fe[Fe(CN)_6]$
- (4)  $K_2Fe_3[Fe(CN)_6]_2$

#### Answer (2)

- Sol. Prussian blue is Fe<sub>4</sub>[Fe(CN)<sub>6</sub>]<sub>3</sub>
- 12. Match the tests given in column-I with the compounds given in column-II

Column-I

Column-II

- (A) Schiff's test
- 1. Carbohydrate
- (B) Carbylamine test
- 2. Peptide
- (C) Molisch test
- 3. Aldehyde
- (D) Biuret Test
- 4. 1º Amine
- (1) A-1; B-2; C-4; D-3
- (2) A-2; B-4; C-3; D-1
- (3) A-4; B-3; C-2; D-1
- (4) A-3; B-4; C-1; D-2

#### Answer (4)

- Sol. Schiff's test is given by aldehydes.
  - Carbylamine test is given by 1º amines only
  - Molisch test is given by carbohydrates
  - Biuret test is given by peptide.
- Electrons are emitted in cathode ray tube with a velocity of 1000 m/s. Select the correct statement among the following.
  - (1) The de-Broglie wavelength of e<sup>-</sup> is 666.67 nm
  - (2) The cathode rays travel from cathode to anode
  - (3) The characteristics of e<sup>-</sup> depends on the metal used in cathode
  - (4) The characteristics of the e<sup>-</sup> depends on the gas field inside the cathode tube

#### Answer (2)

**Sol.** 
$$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34} \text{ Js}}{9.1 \times 10^{-31} \text{kg} \times 10^3}$$

$$= 725 \times 10^{-9} \text{ m}$$

Cathode rays travel from cathode to anode.



- 14. Choose correct statement from following statements
  - A: Beryllium Oxide is an Acidic Oxide.
  - B : Beryllium Sulphate is soluble in aqueous medium
  - C: Beryllium Carbonate is thermally stable
  - D: Beryllium shows anomalous behaviour in comparison to other Group-2 elements.
  - (1) A & B
- (2) B & C
- (3) B & D
- (4) C & D

#### Answer (3)

Sol. BeO: Amphoteric Oxide

BeSO<sub>4</sub>: Soluble in water

15. In which of the following options the reaction does not match with its correct product?

(1) 
$$CI$$

$$CH_3 - C - CI$$

$$AICI_3$$
 $CH_3$ 

#### Answer (4)

Sol. Br 
$$\xrightarrow{\text{alc. KOH}}$$

With alc. KOH alkyl halides elimination reaction.

16. For given reaction in Acidic medium,

$$5e^{-} + 8H^{+} + \frac{MnO_{4}^{-}}{.001 M} \rightarrow \frac{Mn^{2+}}{.1 M} + 4H_{2}O$$

$$E_{cell}^{\circ} = 1.54 \, V$$

$$E_{cell} = 1.2832 \, V$$

Find out pH of solution

- (1) 2.46
- (2) 3.52
- (3) 6.38
- (4) 1.02

## Answer (1)

**Sol.** 
$$1.2832 = 1.54 - \frac{.0591}{5} \log \frac{10^{-1}}{\left(10^{-3}\right)\left(H^{+}\right)^{8}}$$

$$-.2568 = \frac{-.0591}{5} \left( log 10^2 - 8 log H^+ \right)$$

$$21.72 = 2 + 8pH$$

$$19.72 = 8pH$$

$$pH = \frac{19.72}{8} = 2.46$$

 Statement I : Chlorine easily forms oxides and the compounds are explosive

Statement II: The higher oxidation states of chlorine, bromine and iodine are realised when the halogens form oxides and fluorides

- (1) Statement I is correct
  - Statement II is incorrect
- (2) Statement I is incorrect Statement II is correct
- (3) Both the statements are correct
- (4) Both the statements are incorrect

#### Answer (3)

**Sol.** The oxides of chlorine are easily formed and the compounds are explosive. The higher oxidation states of chlorine, bromine and iodine are stable in their oxides and fluorides.

Therefore, both the statements are correct.

- 18.
- 19.
- 20.

#### **SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.



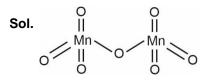
21. Consider structure of Mn<sub>2</sub>O<sub>7</sub>

X: No. of Mn-Mn bonds

Y: No. of Mn-O-Mn bonds

Find out (X + Y)

#### Answer (1)



$$\frac{X=0}{Y=1} \Rightarrow (X+Y)=1$$

22. X : oxidation No. of Br in Bromic AcidY : oxidation No. of Br in perbromic acid

Find out (X + Y)

#### Answer (12)

**Sol.** Bromic Acid (HBrO<sub>3</sub>) Br = +5 = X Perbromic acid (HBrO<sub>4</sub>) Br = +7 = Y (X + Y) = 12

23. Consider the following first order reactions

A 
$$\longrightarrow$$
 C and B  $\longrightarrow$  D  
 $t_{\frac{1}{2}} = 15 \text{ min}$   $t_{\frac{1}{2}} = 5 \text{ min}$ 

The initial concentration of A and B are 1 molar and 8 molar respectively. The time when concentration of A and B becomes equal is 'X' min. Then find 2X (to the nearest integer)

#### Answer (45)

Sol. Let final conc. becomes k

$$\therefore \quad \frac{1}{k_1} ln \frac{1}{[A]} = \frac{1}{k_2} ln \frac{8}{[A]}$$

$$A = \frac{1}{\sqrt{8}}$$

$$\therefore \quad t = \frac{15}{0.693} ln \left( \sqrt{8} \right)$$

= 22.5 min

24. H-atom in ground state absorbs 12.75 eV of energy. The orbital angular momentum of the electron becomes  $\frac{nh}{2\pi}$ , the value of n is

#### Answer (4)

**Sol.** 
$$\Delta E = 13.6 \left( 1 - \frac{1}{n^2} \right) = 12.75$$
  
 $\Rightarrow -\frac{1}{n^2} = \frac{12.75}{13.60} - 1$   
 $\Rightarrow n^2 = 16 \Rightarrow n = 4$ 

25. The density of a 3 M NaCl solution is 1 g/ml. The molality of the solution is 'x'. Then find 2x. (Round off to nearest integer)

#### Answer (7)

**Sol.** Molality = 
$$\frac{3}{824.5} \times 1000$$
 = 3.63 molal

26. Find out  $(\Delta T_F)$  (Depression in Freezing point) of a KCl solution having (i = 2). If 25 ml of this KCl solution requires 20 mL of 1 m AgNO<sub>3</sub> solution for complete precipitation of KCl solution.  $K_F = 1.86$  K.kg mole<sup>-1</sup> (closest Integer assuming molarity = molality)

#### Answer (03.00)

$$m = \frac{20}{25} = \frac{4}{5} = 0.8$$

**Sol.**  $20 \times 1 = 25 \times m$ 

$$\Delta T_F$$
 = (i) (K<sub>F</sub>) (m)  
= (2) (1.86)(0.8)  
⇒ 2.976  
≈ 3 K

27.

28.

29.

30.



## **MATHEMATICS**

#### **SECTION - A**

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

#### Choose the correct answer:

- 1.  $\lim_{n\to\infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n} \right)$  equals
  - (1) ln 2
- (2)  $\ln \frac{3}{2}$
- (3)  $\ln \frac{2}{3}$
- (4) 0

## Answer (1)

**Sol.**  $\lim_{n\to\infty}\sum_{r=1}^n \left(\frac{1}{n+r}\right)$ 

$$\Rightarrow \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \left( \frac{1}{1+\frac{r}{n}} \right)$$

$$\Rightarrow \int_{0}^{1} \frac{dx}{1+x}$$

$$\Rightarrow |\ln(1+x)|_0^1 = \ln 2$$

2. For solution of  $\frac{dy}{dx} + y \tan x = \sec x$ , y(0) = 1, then

$$y\left(\frac{\pi}{6}\right) =$$

- (1)  $\frac{\sqrt{3}}{2}$
- (2)  $\frac{1+\sqrt{3}}{2}$
- (3)  $\frac{1}{2}$
- (4)  $-\frac{\sqrt{3}}{2}$

#### Answer (2)

**Sol.** IF =  $e^{\int \tan x \, dx} = |\sec x|$ 

Solution of equation is

 $y \cdot |\sec x| = \int \sec x |\sec x| dx$ 

$$\Rightarrow$$
  $y \sec x = \tan x + C$ 

at x = 0, y = 1

C = 1

at  $x = \frac{\pi}{6}$ 

$$y \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} + 1$$

$$y = \left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)\frac{\sqrt{3}}{2}$$

$$=\frac{1+\sqrt{3}}{2}$$

3. The sum  $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \dots \infty$  terms equals

 $(1) \frac{1}{1}$ 

(2)  $\frac{1}{3}$ 

(3)  $\frac{1}{4}$ 

(4)  $\frac{1}{5}$ 

#### Answer (1)

Sol. 
$$\sum_{r=1}^{\infty} \frac{r}{1+r^2+r^4}$$

$$\Rightarrow \frac{1}{2} \sum_{r=1}^{\infty} \frac{(r^2 + r + 1) - (r^2 - r + 1)}{(r^2 + r + 1) \times (r^2 - r + 1)}$$

$$\Rightarrow \frac{1}{2} \sum_{r=1}^{\infty} \left( \frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1} \right)$$

$$\Rightarrow \frac{1}{2} \begin{pmatrix} 1 - \frac{1}{3} \\ + \frac{1}{3} - \frac{1}{7} \\ + \frac{1}{7} - \frac{1}{13} \\ \vdots \end{pmatrix}$$

$$\Rightarrow \frac{1}{2} \times 1 = \frac{1}{2}$$



- The number of ways by which letter of word ASSASSINATION can be arranged such that all vowels come together is
  - (1)  $\frac{8!3!}{6!}$
- (2)  $\frac{8!}{4!3!}$
- (3)  $\frac{8! \cdot 6!}{4!(2!)^2 \cdot 3!}$
- (4)  $\frac{8! \cdot 6!}{4!3!2!}$

## Answer (3)

Sol.  $A \rightarrow 3$ 

 $S \rightarrow 4$ 

 $I \rightarrow 2$ 

 $N \rightarrow 2$ 

 $T \rightarrow 1$ 

 $0 \rightarrow 1$ 

- $\therefore$  A  $\rightarrow$  3, I  $\rightarrow$  2 & O  $\rightarrow$  1 are vowels
- $\therefore \text{ Number of ways} = \frac{8!}{4!2!} \cdot \frac{6!}{3!2!}$
- 5.  $f(x) + f'(x) = \int_{0}^{2} f(t)dt$  and  $f(0) = e^{-2}$ , then the value

of f(2) - 2f(0) is

(1) 0

(2) -1

(3) 1

(4) 2

#### Answer (2)

**Sol.** 
$$f(x) + f'(x) = \int_{0}^{2} f(t)dt = k$$
 (let)

$$\Rightarrow e^{x}f(x) + e^{x} \cdot f'(x) = k \cdot e^{x}$$

$$\Rightarrow \int d(f(x) \cdot e^x) = \int k \cdot e^x dx$$

$$\Rightarrow f(x) \cdot e^x = ke^x + c$$

$$f(0) = e^{-2} \implies x = 0, y = f(x) = e^{-2}$$

$$e^{-2} = k + c \Rightarrow c = e^{-2} - k$$

$$\mathbf{v} \cdot \mathbf{e}^{\mathbf{x}} = \mathbf{k} \cdot \mathbf{e}^{\mathbf{x}} + (\mathbf{e}^{-2} - \mathbf{k})$$

$$\Rightarrow$$
  $y = k + (e^{-2} - k)e^{-x}$ 

Now, 
$$\int_{0}^{2} f(x) = k$$

$$\Rightarrow \int_{0}^{2} \left( k + \left( e^{-2} - k \right) e^{-x} \right) dx = k$$

$$\Rightarrow kx|_0^2 - e^{-x} \left(e^{-2} - k\right)|_0^2 = k$$

$$\Rightarrow 2k - (e^{-2} - k)(e^{-2} - 1) = k$$

$$\Rightarrow$$
 2k - (e<sup>-4</sup> - ke<sup>-2</sup> - e<sup>-2</sup> + k) = k

$$\Rightarrow$$
 2k - e<sup>-4</sup> + ke<sup>-2</sup> + e<sup>-2</sup> - k = k

$$\Rightarrow k \cdot e^{-2} = e^{-4} - e^{-2}$$

$$\Rightarrow k = e^{-2} - 1$$

$$f(x) = e^{-2} - 1 + e^{-x}$$

Now, 
$$f(2) - 2f(0)$$

$$= (e^{-2} - 1 + e^{-2}) - 2(e^{-2} - 1 + 1)$$

$$\Rightarrow 2e^{-2} - 1 - 2e^{-2}$$

= -1

6. If set 
$$S = \left\{ \left( \sqrt{3} + \sqrt{2} \right)^{x^2 - 4} + \left( \sqrt{3} - \sqrt{2} \right)^{x^2 - 4} = 10 \right\}$$

then n(S) equals

(1) 2

(2) 3

(3) 4

(4) 6

#### Answer (1)

**Sol.** Let 
$$(\sqrt{2} + \sqrt{3})^{x^2 - 4} = t$$

$$\therefore t + \frac{1}{t} = 10$$

$$\Rightarrow t^2 - 10t + 1 = 0$$

$$\Rightarrow (t-5)^2 = 24$$

$$\Rightarrow \left(\sqrt{2} + \sqrt{3}\right)^{x^2 - 4} = 5 \pm 2\sqrt{6}$$

:. if 
$$(\sqrt{2} + \sqrt{3})^{x^2 - 4} = 5 + 2\sqrt{6}$$

then 
$$x^2 - 4 = 2 \implies x = \pm \sqrt{6}$$

if 
$$\left(\sqrt{2} + \sqrt{3}\right)^{x^2 - 4} = 5 - 2\sqrt{6}$$

then 
$$x^2 - 4 = -2 \implies x^2 = -2$$
 not possible

∴ 2 solutions

- 7. 1, 3, 5, x, y are 5 observations. Mean of the observations is 5 and variance is 8. The sum of the cubes of the two missing number equals
  - (1) 1072
- (2) 513
- (3) 1079
- (4) 516

## Answer (1)

Sol. 
$$\overline{x} = 5$$

$$\Rightarrow$$
 1 + 3 + 5 + x + y = 25

$$\Rightarrow x + y = 16$$

$$\sigma^2 = 8 = \frac{\sum x_i^2}{5} - (\bar{x})^2$$

$$\Rightarrow 8 = \frac{1^2 + 3^2 + 5^2 + x^2 + y^2}{5} - 25$$

$$\Rightarrow$$
 165 = 35 +  $x^2$  +  $y^2$ 

$$\Rightarrow$$
  $x^2 + y^2 = 130$ 

$$\Rightarrow$$
  $(x + y)^2 - 2xy = 130$ 

$$\Rightarrow xy = 63$$

$$\Rightarrow x = 7, y = 9$$

Now, 
$$x^3 + y^3$$

$$\Rightarrow$$
 7<sup>3</sup> + 9<sup>3</sup>

$$\Rightarrow$$
 343 + 729

- ⇒ 1072
- 8. Sum of series

$$\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \dots + \frac{1}{51!0!}$$
 equals

- (1)  $\frac{2^{51}}{50!}$
- (2) 2<sup>5</sup>
- (3) 51·2<sup>50</sup>
- (4)  $\frac{2^{50}}{51!}$

## Answer (4)

**Sol.** 
$$\frac{1}{51!} \left( \frac{51!}{1!50!} + \frac{51!}{3!48!} + \frac{51!}{5!46!} + \dots + \frac{51!}{51!0!} \right)$$

$$= \frac{1}{51!} \left( {}^{51}C_1 + {}^{51}C_3 + ... + {}^{51}C_{51} \right)$$

$$=\frac{1}{51!} \left(\frac{2^{51}}{2}\right) = \frac{2^{50}}{51!}$$

- 9. Let  $R = \{(a, b) : 3a 3b + \sqrt{7} \text{ is irrational}\}$ 
  - (1) R is an equivalence relation
  - (2) R is symmetric but not reflexive
  - (3) R is reflexive but not symmetric
  - (4) R is reflexive and symmetric but not transitive

#### Answer (3)

Sol. For reflexive

$$3a-3b+\sqrt{7}=\sqrt{7}$$
 is irrational

$$\therefore$$
  $(a, a) \in R$ 

For symmetric

$$(a, b) \in R$$

$$\Rightarrow$$
 3a-3b+ $\sqrt{7}$  is irrational

$$\Rightarrow$$
 3b-3a+ $\sqrt{7}$  is irrational

$$\Rightarrow$$
  $(b, a) \in R$ 

.. Not symmetric

For transitive

$$(a, b) \in R$$
 and  $(b, c) \in R$ 

$$\Rightarrow$$
 3a-3b+ $\sqrt{7}$  is irrational

$$3b-3c+\sqrt{7}$$
 is irrational

$$3a-3c+\sqrt{7}$$
 is irrational

- ∴ R is not transitive
- 10. Negation of the statement  $p \lor (p \land \neg q)$  is
  - (1) p
  - $(2) \sim p$
  - (3) q
  - $(4) \sim q$

Answer (2)

**Sol.** 
$$p \lor (p \land \sim q)$$

$$\equiv p$$

$$\sim (p \lor (p \land \sim q)) \equiv \sim p$$



11. Let S be solution set for values of x satisfying  $\cos^{-1}(2x) + \cos^{-1}\sqrt{1-x^2} = \pi$  then  $\sum_{n=1}^{\infty} 2\sin^{-1}(x^2-1)$ 

is equal to

(2) 
$$-\sin^{-1}\left(\frac{24}{25}\right)$$

$$(3) \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

(3) 
$$\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$
 (4)  $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ 

## Answer (2)

**Sol.** 
$$\frac{\pi}{2} - \sin^{-1}(2x) + \frac{\pi}{2} - \sin^{-1}\sqrt{1 - x^2} = \pi$$

$$\Rightarrow \sin^{-1}(2x) + \sin^{-1}\sqrt{1-x^2} = 0$$

$$\Rightarrow \sin^{-1}(-2x) = \sin^{-1}\sqrt{1-x^2}$$

$$\Rightarrow -2x = \sqrt{1-x^2}$$

$$4x^2 = 1 - x^2$$

$$\Rightarrow x = \pm \sqrt{\frac{1}{5}}$$

$$x = -\frac{1}{\sqrt{5}}$$
 is only possible solution

$$\sum_{x \in S} 2 \sin^{-1}(x^2 - 1) = 2 \sin^{-1}\left(-\frac{4}{5}\right)$$
$$= -2 \sin^{-1}\frac{4}{5}$$

$$=-\sin^{-1}\left(\frac{24}{25}\right)$$

- 12. A triangle be such that  $\cos 2A + \cos 2B + \cos 2C$  is minimum. If inradius of the triangle is 3 then which of the following is CORRECT?
  - (1) Area of  $\triangle$  is  $\frac{6\sqrt{3}}{2}$
  - (2) Perimeter of  $\Delta$  is  $18\sqrt{3}$
  - (3)  $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B \sin C$
  - (4) Perimeter of triangle is  $9\sqrt{3}$

#### Answer (2)

**Sol.** If 
$$k = \cos 2A + \cos 2B + \cos 2C$$
 is minimum then

$$k = \frac{-3}{2}$$
 and  $A = B = C = \frac{\pi}{3}$ 

$$\therefore r = \frac{\Delta}{s} = 3 = \frac{\sqrt{3}a^2 \cdot 2}{4 \cdot 3a}$$

$$\Rightarrow a = 6\sqrt{3}$$

$$\therefore \quad \text{Area} = \frac{\sqrt{3}}{4} \cdot 36.3 = 27\sqrt{3}$$

Perimeter is  $18\sqrt{3}$ 

#### **SECTION - B**

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation. truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Area bounded by y = x|x - 3| and x-axis between x = -1 and x = 2 is A then 12A equals

#### Answer (18.00)

**Sol.** 
$$y = \begin{cases} x(x-3) & x > 3 \\ -x(x-3) & x \le 3 \end{cases}$$

$$A = \int_{-1}^{2} -x(x-3) dx = \int_{-1}^{2} (-x^{2} + 3x) dx$$

$$=\frac{-x^3}{3}+\frac{3x^2}{2}\bigg|_{-1}^2$$

$$=\left(-\frac{8}{3}+\frac{3}{2}\cdot 4\right)-\left(\frac{1}{3}+\frac{3}{2}\right)$$

$$=\frac{-8}{3}+6-\frac{11}{6}$$

$$=6-\frac{27}{6}=\frac{9}{6}$$

$$12A = 18$$

22. Remainder when  $23^{200} + 19^{200}$  divided by 49 equals

## Answer (02.00)

**Sol.** 
$$23^{200} + 19^{200} = (21 + 2)^{200} + (21 - 2)^{200}$$
  
=  $2 \left[ {}^{200}C_0 21^{200} + {}^{200}C_2 21^{198} + {}^{200}C_4 21^{196} + ... + {}^{200}C_{198} 21^2 + {}^{200}C_{200} 21^0 \right]$ 

$$= 2(49K + 1)$$

Remainder = 2

23. 8,  $a_1$ ,  $a_2$  ...  $a_n$  are terms m A.P. Sum of first 4 terms of series is 50 and sum of last 4 terms of series is 170. Find product of middle terms of series

#### **Answer (754)**

**Sol.** 
$$\frac{4}{2}[16+3d] = 50$$
  
 $\Rightarrow d = 3$   
 $\frac{4}{2}[2a_n + 3(-d)] = 170$   
 $2a_n - 3d = 85$   
 $2a_n = 94$   
 $a_n = 47$   
 $8 + (n-1)d = 47$ 

So 7th and 8th are middle terms

$$T_7 = 8 + 6.3 = 26$$

$$T_8 = 8 + 7.3 = 29$$

$$T_7.T_8 = 754$$

24. A circle is represented by  $\frac{|z-2|}{|z-3|} = 2$ . Its radius is  $\gamma$ units and centre is  $(\alpha, \beta)$  then find  $3(\alpha + \beta + \gamma)$ 

#### **Answer (12.00)**

**Sol.** Let 
$$z = x + iy$$
  

$$(x-2)^2 + y^2 = 4(x-3)^2 + 4y^2$$

$$x^2 + y^2 - 4x + 4 = 4x^2 - 24x + 36 + 4y^2$$

$$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$$

# or $x^2 + y^2 - \frac{20x}{3} + \frac{32}{3} = 0$

centre = 
$$\left(\frac{10}{3}, 0\right)$$

$$\gamma = \sqrt{\left(\frac{10}{3}\right)^2 + 0^2 - \frac{32}{3}} = \frac{2}{3}$$

$$3(\alpha + \beta + \gamma) = 12$$

25. If 
$$f(x) = x^2 + g'(1)x + g''(2)$$
 and  $g(x) = 2x + f'(1)$ , then  $f(4) - g(4)$  equals

#### **Answer (12.00)**

**Sol.** 
$$g(x) = 2x + f'(1)$$
  
 $\Rightarrow g'(x) = 2 \Rightarrow g'(1) = 2 \text{ and } g''(x) = 0$ 

Now,

$$f(x) = x^2 + g'(1)x + g''(1)$$

$$f(x) = x^2 + 2x$$

$$f'(x) = 2x + 2 \implies f'(1) = 4$$

$$g(x) = 2x + 4$$

$$f(4) - g(4) = (16 + 8) - (8 + 4)$$

$$= 24 - 12 = 12$$

26. For some values of  $\lambda$ , system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

has no solution, then  $\sum (|\lambda|^2 + |\lambda|)$  equal

#### Answer (06.00)

Sol. 
$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$
$$\Rightarrow \lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1 - \lambda) = 0$$
$$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda - 1 - 1) = 0$$
$$\Rightarrow \lambda = 1, -2$$

For  $\lambda = 1$  there are infinite solution

for  $\lambda = -2$ , system has no solution



27. If solution of  $\frac{dy}{dx} + \frac{(x+a)}{y-2} = 0$  is a circle and

y(0) = 1, area of circle is =  $2\pi$ . P and Q are point of intersection of circle with y-axis. Normals at P and Q intersect x-axis at R and S. The length of RS is

## Answer (0, 4)

**Sol.** 
$$(y-2)dy + (x+a)dx = 0$$

$$\Rightarrow \frac{y^2}{2} - 2y + \frac{x^2}{2} + ax = c$$

$$\Rightarrow x^2 + y^2 + 2ax - 4y + 3 = 0$$

$$\Rightarrow$$
  $(x + a)^2 + (y - 2)^2 = a^2 + 1$ 

Also 
$$\sqrt{a^2 + 1} = \sqrt{2}$$
 (as area is  $2\pi$  sq. units)

$$\therefore C \equiv (x \pm 1)^2 + (y-2)^2 = 2$$

For P and Q put x = 0

$$\Rightarrow$$
 y-2 = 1 or -1  $\Rightarrow$  P = (0, 3) and Q = (0, 1)

If 
$$C = (x + 1)^2 + (y - 2)^2 = 2$$
 then normal at P

$$y = x + 3 \Rightarrow R \equiv (-3, 0)$$

and normal at Q

$$x + y = 1 \Rightarrow S \equiv (1, 0)$$

$$\therefore$$
 RS = 4

If 
$$C = (x-1)^2 + (y-2)^2 = 2$$

Normal at 
$$Py = x + 1 \Rightarrow R \equiv (-1, 0)$$

Normal at  $Qx + y = 3 \Rightarrow S \equiv (3, 0)$ 

$$\therefore$$
 RS = 4

28. Find number of 3-digit number which are divisible by 2 or 3 but not divisible by 7.

## Answer (536.00)

**Sol.** Numbers divisible by 2 = 450 = n(A)

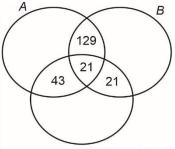
Numbers divisible by 
$$3 = 300 = n(B)$$

Numbers divisible by 6 = 150

Numbers divisible by 2 and 7 = 64

Numbers divisible by 3 and 7 = 42

Numbers divisible by 2, 3 and 7 = 21



C (divisible by 7)

Total numbers = 
$$450 + 300 - 150 - 43 - 21$$
  
=  $600 - 64 = 536$ 

- 29. ?
- 30. ?