



QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

📅 1 FEBRUARY, 2023

🕒 9:00 AM to 12:00 Noon

SHIFT - 1

Duration : 3 Hours

Maximum Marks : 300

SUBJECT - MATHEMATICS

RESULT JEE ADVANCED 2022

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MATHEMATICS

1. Sum of series $\frac{1}{1!50!} + \frac{1}{3!48!} + \dots + \frac{1}{51!0!}$ is

- (1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{49}}{51!}$ (3) $\frac{2^{51}}{51!}$ (4) $\frac{2^{52}}{51!}$

Ans. (1)

Sol.
$$\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)} \frac{1}{51!}$$

$$= \frac{1}{51!} \{ {}^{51}C_1 + {}^{51}C_3 + \dots + {}^{51}C_{51} \} = \frac{1}{51!} (2^{50})$$

2. $\sum_{r=1}^{10} \frac{r}{1+r^2+r^4}$ is equal to

- (1) $\frac{56}{111}$ (2) $\frac{57}{111}$ (3) $\frac{55}{111}$ (4) $\frac{58}{111}$

Ans. (3)

Sol.
$$T_r = \frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$$

$$\Rightarrow T_r = \frac{1}{2} \left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1} \right]$$

$$T_1 = \frac{1}{2} \left[\frac{1}{1} - \frac{1}{3} \right]$$

$$T_2 = \frac{1}{2} \left[\frac{1}{3} - \frac{1}{7} \right]$$

$$T_3 = \frac{1}{2} \left[\frac{1}{7} - \frac{1}{13} \right]$$

⋮

$$T_{10} = \frac{1}{2} \left[\frac{1}{91} - \frac{1}{111} \right]$$

$$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2} \left[1 - \frac{1}{111} \right] = \frac{55}{111}$$

3. The value of $\lim_{n \rightarrow \infty} \left(\frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + \dots + \frac{1}{2n} \right)$ is

- (1) $\ln 4$ (2) $\ln 2$ (3) $\ln 3$ (4) $\ln 5$

Ans. (2)

Sol.
$$\lim_{n \rightarrow \infty} \left(\frac{1}{1+n} + \dots + \frac{1}{n+n} \right) = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n+r} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \left(\frac{1}{1 + \frac{r}{n}} \right)$$

$$= \int_0^1 \frac{1}{1+x} dx = [\ln(1+x)]_0^1 = \ln 2$$

4. If $y = f(x)$ satisfies $\frac{dy}{dx} + y \tan x = x \sec x$ and $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{6} - \frac{\sqrt{3}}{2} \ln \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

(2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \ln \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \ln \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

(4) $\frac{\pi}{6} + \frac{\sqrt{3}}{2} \ln \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

Ans. (3)

Sol. Here I.F. = $\sec x$

Then solution of D.E :

$$y(\sec x) = x \tan x - \ln(\sec x) + c$$

Given $y(0) = 1 \Rightarrow c = 1$

$$\therefore y(\sec x) = x \tan x - \ln(\sec x) + 1$$

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12} + \frac{\sqrt{3}}{2} \ln \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

5. Find the number of ways in which the word ASSASSINATION can be arrange such that all vowels come together.

(1) 50400

(2) 50200

(3) 51400

(4) 50000

Ans. (1)

Sol. Vowels : A,A,A,I,I,O

Consonants : S,S,S,S,N,N,T

\therefore Total number of ways in which vowels come together

$$= \frac{|8}{|4|2} \times \frac{|6}{|3|2}$$

$$= 50400$$

6. If system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1 \text{ is inconsistent then find } \sum(|\lambda^2| + |\lambda|)$$

Ans. (6)

Sol.
$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2)[1(\lambda^2 - 1) - 1(\lambda - 1) + (1 - \lambda)] = 0$$

$$(\lambda + 2)[\lambda^2 - 2\lambda + 1] = 0$$

$$(\lambda + 2)(\lambda - 1)^2 = 0 \Rightarrow \lambda = -2, \lambda = 1$$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$

$$\text{so } \sum(|-2^2| + |-2|) = 6$$

7. Negation of $p \vee \sim(p \vee \sim q)$ is equivalent to

$$(1) \sim p \wedge q \quad (2) p \wedge \sim q \quad (3) \sim(p \wedge q) \quad (4) \sim p \wedge \sim q$$

Ans. (4)

Sol. negation of given statement

$$\sim(p \vee \sim(p \vee \sim q)) \equiv \sim p \wedge (p \vee \sim q)$$

$$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$$

$$\equiv f \vee (\sim p \wedge \sim q)$$

$$\equiv \sim p \wedge \sim q$$

8. For a binomial distribution $B(n, p)$, sum and product of mean & variance is 5 & 6 respectively, then find $6(n + p - q)$

Ans. (52)

Sol. $np + npq = 5, np \cdot npq = 6$

$$np(1 + q) = 5, n^2 p^2 q = 6$$

$$n^2 p^2 (1 + q)^2 = 25, n^2 p^2 q = 6$$

$$\frac{6}{q}(1 + q)^2 = 25$$

$$6q^2 + 12q + 6 = 25q$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$(3q - 2)(2q - 3) = 0$$

$$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3} \text{ is accepted}$$

$$p = \frac{1}{3} \Rightarrow n \cdot \frac{1}{3} + n \cdot \frac{1}{3} \cdot \frac{2}{3} = 5$$

$$\frac{3n + 2n}{9} = 5$$

$$n = 9$$

$$\text{so } 6(n + p - q) = 6 \left(9 + \frac{1}{3} - \frac{2}{3} \right) = 52$$

9. If n -numbers a_1, a_2, \dots, a_n are in A.P. such that its first term is 8, sum of first four terms is 50 and sum of last four terms is 170, then the value of $(a_7 \cdot a_8)$ is equal to

Ans. (754)

Sol. $a_1 + a_2 + a_3 + a_4 = 50$

$$\Rightarrow 32 + 6d = 50$$

$$\Rightarrow d = 3$$

and, $a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$

$$\Rightarrow 32 + (4n - 10) \cdot 3 = 170$$

$$\Rightarrow n = 14$$

$$a_7 = 26, a_8 = 29$$

$$\Rightarrow a_7 \cdot a_8 = 754$$

10. A relation on real numbers defined by $R = \{3a - 3b + \sqrt{7} \text{ is Irrational}, a, b \in \mathbb{R}\}$ then relation R is

- (1) Reflexive, Symmetric and Transitive
- (2) Reflexive but not Symmetric and Transitive
- (3) Reflexive and Transitive but not Symmetric
- (4) Equivalence

Ans. (2)

Sol. Reflexive (a, a)

$$3a - 3a + \sqrt{7} = \sqrt{7} \text{ is irrational}$$

so reflexive true

Symmetric ($a, b \leftrightarrow b, a$)

$$3a - 3b + \sqrt{7} \text{ is irrational}$$

$$3b - 3a + \sqrt{7} \text{ Irrational, not always true}$$

so not symmetric

Transitive ($(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ so not transitive

11. Remainder when $23^{200} + 19^{200}$ is divided by 49

- (1) 21 (2) 29 (3) 31 (4) 39

Ans. (2)

Sol. $(21 + 2)^{200} + (21 - 2)^{200}$

$$\Rightarrow 2[{}^{200}C_0 21^{200} + {}^{200}C_2 21^{198} \cdot 2^2 + \dots + {}^{200}C_{198} 21^2 \cdot 2^{198} + 2^{200}]$$

$$\Rightarrow 2[49I_1 + 2^{200}] = 49I_1 + 2^{201}$$

$$\text{Now, } 2^{201} = (8)^{67} = (1 + 7)^{67} = 49I_2 + {}^{67}C_0 + {}^{67}C_1 \cdot 7 = 49I_2 + 470 = 49I_2 + 49 \times 9 + 29$$

\therefore Remainder is 29

12. If 1, 3, 5, a, b have mean 5 and variance 8 then the value of $a^3 + b^3$ is

- (1) 1072 (2) 1702 (3) 1027 (4) 1207

Ans. (1)

Sol. $\frac{1+3+5+a+b}{5} = 5$

$a + b = 16$ (1)

$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{\sum x}{5}\right)^2$

$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$

$a^2 + b^2 = 130$ (2)

by (1), (2)

$a = 7, b = 9$

or $a = 9, b = 7$

13. Let A(1,2), B(2, 3) and C(3, 1) are vertices of ΔABC and orthocentre of ΔABC is (α, β) then quadratic equation whose roots are $(\alpha + 4\beta)$ and $(\beta + 4\alpha)$ is

(1) $x^2 - 9x + 11 = 0$

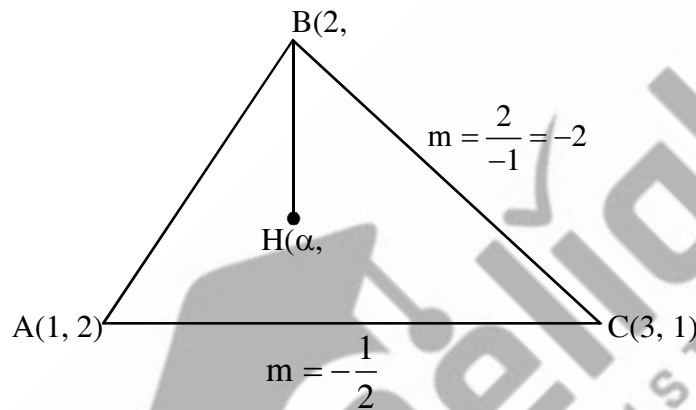
(2) $x^2 - 11x + 9 = 0$

(3) $x^2 - 20x + 99 = 0$

(4) $x^2 - 9x + 20 = 0$

Ans. (3)

Sol.



Here $m_{BH} m_{AC} = -1$

$\left(\frac{\beta-3}{\alpha-2}\right)\left(\frac{1}{-2}\right) = -1$

$\beta - 3 = 2\alpha - 4$

$\beta = 2\alpha - 1$

...(i)

$m_{AH} m_{BC} = -1$

$\Rightarrow \left(\frac{\beta-2}{\alpha-1}\right)(-2) = -1$

$\Rightarrow 2\beta - 4 = \alpha - 1$

$\Rightarrow 2(2\alpha - 1) = \alpha + 3$

$\Rightarrow 3\alpha = 5$

$\alpha = \frac{5}{3}, \beta = \frac{7}{3} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$

$x^2 - 20x + 99 = 0$

14. If $y = f(x) = x|x - 3|$; $x \in [-1, 2]$ and area bounded by $y = f(x)$ in $x \in [-1, 2]$ is A , then $12A$ is

Ans. (62)

Sol. $A = \int_{-1}^0 (x^2 - 3x) dx + \int_0^2 (3x - x^2) dx$

$$\Rightarrow A = \left. \frac{x^3}{3} - \frac{3x^2}{2} \right|_{-1}^0 + \left. \frac{3x^2}{2} - \frac{x^3}{3} \right|_0^2$$

$$\Rightarrow A = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$$

$$\therefore 12A = 62$$

15. For the circle $\left| \frac{z-1}{z-3} \right| = 2$ if center of circle is (α, β) & radius is γ find $\alpha + \beta + \gamma$.

- (1) 4 (2) 5 (3) $\frac{5}{2}$ (4) $\frac{5}{3}$

Ans. (2)

Sol. $\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$$x^2 + y^2 - 2x + 1 = 4x^2 + 4y^2 - 24x + 36$$

$$x^2 + y^2 - \frac{22}{3}x + \frac{35}{3} = 0$$

$$\alpha + \beta + \gamma = \frac{11}{3} + 0 + \frac{4}{3} = 5$$

16. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$, then $f(4) - g(4)$ is equal to

Ans. (12)

Sol. $g'(x) = 2, g''(x) = 0$

$$\Rightarrow f(x) = x^2 + 2x$$

$$\Rightarrow f'(x) = 2x + 2$$

$$\Rightarrow f'(1) = 4$$

$$\therefore g(x) = 2x + 4$$

$$f(4) - g(4) = 12$$

17. Let $S = \left\{ x : (\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10 \right\}$, then $n(S)$ is

- (1) 2 (2) 3 (3) 4 (4) 5

Ans. (3)

Sol. Let $(\sqrt{3} + \sqrt{2})^{x^2-4} = t$

$$t + \frac{1}{t} = 10$$

$$\Rightarrow t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$$

$$\Rightarrow (\sqrt{3} + \sqrt{2})^{x^2-4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$$

$$\Rightarrow x^2 - 4 = 2, -2 \quad \text{or} \quad x^2 = 6, 2$$

$$\Rightarrow x = \pm\sqrt{2}, \pm\sqrt{6}$$

18. If $f(x) + f'(x) = \int_0^2 f(t) dt$ and $f(0) = e^{-2}$, then the value of $f(2) - 2f(0)$ is -

(1) 0

(2) 1

(3) -1

(4) 2

Ans. (3)

Sol. $\frac{dy}{dx} + y = k$

$$y \cdot e^x = k \cdot e^x + c$$

$$f(0) = e^{-2}$$

$$\Rightarrow c = e^{-2} - k$$

$$\therefore y = k + (e^{-2} - k)e^{-x}$$

$$\text{now } k = \int_0^2 (k + (e^{-2} - k)e^{-x}) dx$$

$$\Rightarrow k = e^{-2} - 1$$

$$\therefore y = (e^{-2} - 1) + e^{-x}$$

$$f(2) = 2e^{-2} - 1, f(0) = e^{-2}$$

$$f(2) - 2f(0) = -1$$

19. The number of 3-digit numbers divisible by 2 or 3 but not by 7 is

Ans. (514)

Sol. Divisible by 2 \rightarrow 450

Divisible by 3 \rightarrow 300

Divisible by 7 \rightarrow 128

Divisible by 2 & 7 \rightarrow 64

Divisible by 3 & 7 \rightarrow 43

Divisible by 2 & 3 \rightarrow 150

Divisible by 2, 3 & 7 \rightarrow 21

$$\therefore \text{Total numbers} = 450 + 300 - 150 - 64 - 43 + 21 = 514$$

20. A triangle is given such that $\cos 2A + \cos 2B + \cos 2C$ is minimum. If radius of incenter is 3 cm then which of the following is incorrect ? (where O is incenter)

(1) $\vec{OB} \cdot \vec{OA} = -18$

(2) Area of $\Delta ABC = \frac{27\sqrt{3}}{2}$

(3) $\sin A + \sin B + \sin C = \sin 2A + \sin 2B + \sin 2C$

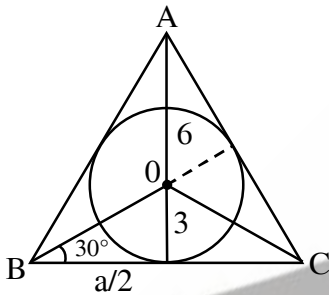
(4) Length of altitude from vertex A is 9

Ans. (B)

Sol. If $\cos 2A + \cos 2B + \cos 2C$ is minimum then

$A = B = C = 60^\circ$

so ΔABC is equilateral now



(A) $\vec{OA} \cdot \vec{OB} = |\vec{OA}| |\vec{OB}| \cos 120$

$6 \cdot 6 \cdot \left(\frac{1}{2}\right) = -18$ correct option

(B) $\Delta ABC = \frac{\sqrt{3}}{4} \cdot a^2 = \frac{\sqrt{3}}{4} \cdot (6\sqrt{3})^2 = \frac{\sqrt{3} \cdot 36 \cdot 3}{4} = 27\sqrt{3}$ incorrect option

(C) $\sin A + \sin B + \sin C = \sin 2A + \sin 2B + \sin 2C$ correct option

(D) length of altitude = 9 correct option

#IITkipooritaiyyari



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