PART : MATHEMATICS

1. $\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+.....+\frac{1}{2n}\right)=$

(1) 0 (2) $\ell n2$ (3) 1 (4) $\ell n3$

Ans. **(2)**

Sol. $\lim_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+.....+\frac{1}{2n}\right)$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{r+n}$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\left(\frac{r}{n}+1\right)}.\frac{1}{n}$

$\int_0^1 \frac{dx}{1+x} = \left[\ell n(1+x)\right]_0^1$

$\ell n2 - \ell n1 = \ell n2$

2. Sum of series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+......$upto 10 terms is

(1) $\frac{53}{111}$ (2) $\frac{54}{111}$ (3) $\frac{55}{111}$ (4) $\frac{56}{111}$

Ans. **(3)**

Sol. $t_n = \frac{n}{1+n^2+n^4} = \frac{n}{(n^2+n+1)(n^2-n+1)}$

$= \frac{1}{2}\left(\frac{1}{(n^2-n+1)}-\frac{1}{(n^2+n+1)}\right)$

$t_1 = \frac{1}{2}\left(1-\frac{1}{3}\right)$

$t_2 = \frac{1}{2}\left(\frac{1}{3}-\frac{1}{7}\right)$

$t_3 = \frac{1}{2}\left(\frac{1}{7}-\frac{1}{13}\right)$

|
|
|

$t_{10} = \frac{1}{2}\left(\frac{1}{91}-\frac{1}{111}\right)$

$\therefore S_{10} = t_1 + t_2 + t_3 + ... + t_{10}$

$= \frac{1}{2}\left(1-\frac{1}{111}\right) = \frac{55}{111}$

3. Number of three digit numbers which are divisible by 2 or 3 but not divisible by 7, is

Ans. (557)

Sol.

Divisible by 2 100 , 102 , 104, ... , 998

$t_n = a + (n-1)d \Rightarrow 998 = 100 + (n-1)2$

$\Rightarrow \frac{898}{2} = n - 1$

$\Rightarrow n = 449 + 1 = 450$

Divisible by 3 102, 105, 108, ... , 999

$t_n = a + (n-1)d \Rightarrow 999 - 102 = (n-1)3$

$\Rightarrow \frac{897}{3} = n - 1$

$\Rightarrow n = 299 + 1 = 300$

Divisible by 6 102, 108, 114, ... , 996

$996 - 102 = (n-1)6$

$n = 1 + \frac{894}{6} = 1 + 149 = 150$

$\therefore$ No. divisible by 2 or 3 = 450 + 300 – 150 = 600

No. divisible by 21 105, 126, 147, ... , 987

$987 - 105 = (n-1)21$

$\Rightarrow n = 1 + 42 = 43$

$\therefore$ Total required No. = 600 – 43 = 557

4. If $f'(x) + f(x) = \int_0^2 f(t)\,dt$ and $f(0) = e^{-2}$, then find value of $2f(0) - f(2)$

(1) 0 (2) – 1 (3) 1 (2) 2

Ans. (3)

Sol. Given $f'(x) + f(x) = \int_0^2 f(t)\,dt$

Let $\int_0^2 f(t)\,dt = a$

$\therefore f'(x) + f(x) = a$

$f'(x) = a - f(x)$

$\frac{f'(x)}{a - f(x)} = 1$

$\therefore -\ell n(a - f(x)) = x + c$

Put x = 0

$-\ell n(a - e^{-2}) = c$

$\therefore -\ell n(a - f(x)) = x - \ell n(a - e^{-2})$

$\ell n \frac{a - e^{-2}}{a - f(x)} = x$

$\Rightarrow \frac{a - e^{-2}}{a - f(x)} = e^x$

$a - f(x) = (a - e^{-2})e^{-x}$

$f(x) = a - (a - e^{-2})e^{-x}$

Now $\int_0^2 f(t)\,dt = a$

$\Rightarrow \int_0^2 (a-(a-e^{-2})e^{-t})dt = a$

$\Rightarrow \left[at + (a-e^{-2})e^{-t}\right]_0^2 = a$

$\Rightarrow 2a + (a - e^{-2})\, e^{-2} - (a - e^{-2}) = a$

$\Rightarrow a + ae^{-2} - e^{-4} - a + e^{-2} = 0$

$a = \dfrac{e^{-4} - e^{-2}}{e^{-2}} = e^{-2} - 1$

Now 2f(0) – f(2)

$= 2e^{-2} - a + (a - e^{-2})\, e^{-2}$

$= 2e^{-2} - e^{-2} + 1 - e^{-2}$

$= 1$

5. If $19^{200} + 23^{200}$ is divided by 49, then remainder is

Ans. **(29)**

Sol. $(21-2)^{200} + (21+2)^{200} = 49\lambda + 2^{201}$

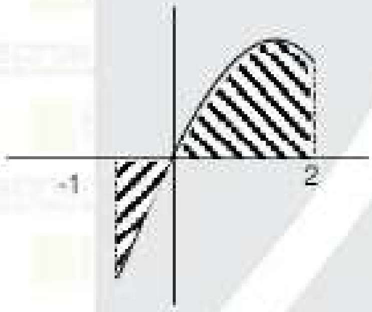
$2^{201} = 8^{67} = (7+1)^{67} = 49\lambda + 7\times 67 + 1$

$= 49\lambda + 470$

$= 49(\lambda + 9) + 29$

6. If area divided by curve $f(x) = x|x-3|$ and x –axis form $x = -1$ to $x = 2$ is A, then value of 12A is equal to

Ans. **(62)**



$A = -\int_{-1}^{0}(3x - x^2)dx + \int_0^2 (3x - x^2)dx$

$= -\left(\dfrac{3x^2}{2} - \dfrac{x^3}{3}\right)_{-1}^{0} + \left(\dfrac{3x^2}{2} - \dfrac{x^3}{3}\right)_0^2$

$= -\left(-\dfrac{3}{2} - \dfrac{1}{3}\right) + \left(6 - \dfrac{8}{3}\right)$

$= \dfrac{3}{2} + \dfrac{1}{3} + 6 - \dfrac{8}{3} = \dfrac{31}{6}$

$\Rightarrow 12A = 62$

7. Negation of statement $q \vee ((\sim q) \wedge p)$ is

(1) $\sim p \wedge q$ (2) $\sim p \vee q$ (3) $\sim p \vee \sim q$ (4) $\sim q \wedge \sim p$

Ans. (4)

Sol. Negation of given statement is

$\sim (q \vee (\sim q) \wedge p))$

$= \sim q \wedge \sim ((\sim q) \wedge p)$

$= \sim q \wedge (q \vee \sim p)$

$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$\sim q \wedge \sim p$

8. Sum of series $\frac{1}{1!50!} + \frac{1}{3!48!} + ... + \frac{1}{51!0!}$ is

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{51}}{51!}$ (3) $\frac{2^{50}}{50!}$ (4) $\frac{2^{51}}{50!}$

Ans. (1)

Sol. $\frac{1}{1!50} + \frac{1}{3!48!} + ... + \frac{1}{51!0!}$

$\frac{1}{51!}\left(\frac{51!}{1!50!} + \frac{51!}{3!48!} + ... \frac{51!}{51!0!}\right)$

$\frac{1}{51!}\left({}^{51}C_1 + {}^{51}C_3 + ... {}^{51}C_{51}\right)$

$\frac{1}{51!}\left(\frac{2^{51}}{2}\right) = \frac{2^{50}}{51!}$

9. If a curve $y = y(x)$ satisfy differential equation $\frac{dy}{dx} + y \tan x = x \sec x$ and $y(0) = 1$ then equation of curve is

(1) $y = \frac{x \tan x - \ell n \sec x + 1}{\sec x}$ (2) $y = \frac{x \cot x + 1}{\sec x}$

(3) $y = \frac{\ell n \sec x + 1}{\sec x}$ (4) $y = \frac{x \tan x - \ell n \cos x + 1}{\sec x}$

Ans. (1)

Sol. $\frac{dy}{dx} + y \tan x = x \sec x$ linear in y

$I.F. = e^{\int \tan x dx} = e^{\ell n \sec x} = \sec x$

$\therefore$ solution is

$y. \sec x = \int x \sec x. \sec x dx + c$

$y \sec x = \int x \sec^2 x dx + c$

$y \sec x = x \tan x - \int \tan x dx + c$

$y \sec x = x \tan x - \ell n \sec x + c$

$y(0) = 1 \quad \therefore 1 = 0 - 0 + c$

$c = 1$

$\therefore y \sec x = x \tan x - \ell n \sec x + 1$

$y = \frac{x \tan x - \ell n \sec x + 1}{\sec x}$

10. If $\left|\frac{z-2}{z-3}\right| = 2$ then locus of z is a circle whose centre is (α,β) and radius is γ then value of $3(\alpha+\beta+\gamma)$ is

Ans. **(12)**

Sol. $\left|\frac{z-2}{z-3}\right| = 2$

$(z-2)(\bar{z}-2) = 4(\bar{z}-3)(z-3)$

$\Rightarrow z\bar{z} - 2z - 2\bar{z} + 4 = 4(z\bar{z} - 3z - 3\bar{z} + 9)$

$3z\bar{z} - 10z - 10\bar{z} + 32 = 0$

$z\bar{z} - \frac{10}{3}z - \frac{10}{3}\bar{z} + \frac{32}{3} = 0$

$\alpha = -\frac{10}{3}$ $\qquad$ $c = \frac{32}{3}$

centre $\left(\frac{10}{3}, 0\right)$ $\qquad$ $\gamma = \sqrt{|\alpha|^2 - c}$

$= \sqrt{\frac{100}{9} - \frac{32}{3}} = \frac{2}{3}$

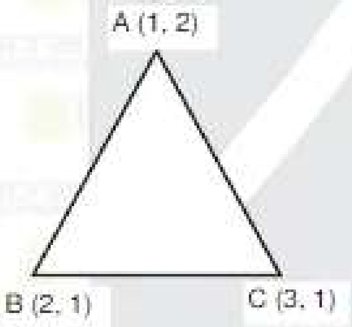
$\therefore \alpha+\beta+\gamma = \frac{10}{3} + \frac{2}{3} = \frac{12}{3}$ $\therefore 3(\alpha+\beta+\gamma) = 12$

11. Orthocentre of a triangle whose vertices are (1, 2), (2, 1) and (3, 1) is

(1) (1, 1) $\qquad$ (2) (–1, 1) $\qquad$ (3) (1, – 1) $\qquad$ (4) (0, 0)

Ans. **(3)**

Sol.



$M_{AB} = \frac{2-1}{1-2} = -1$

$\therefore$ Eq. of altitude through C is

$y - 1 = 1\,(x - 3)$

$x - y = 2$ $\qquad$ ____(1)

$M_{BC} = 0$

$\therefore$ Altitude through A is

$y - 2 = \frac{1}{0}(x - 1) \Rightarrow x = 1$ ____(2)

$\therefore$ Orthocentre is (1, – 1)

12. If mean and variance of 1, 3, 5, α, β are 5 and 8 respectively then the value of $\alpha^3 + \beta^3$, is

Ans. **(1072)**

Sol. Mean = $\frac{1+3+5+\alpha+\beta}{5} = 5$

$\alpha + \beta = 16$ ____(1)

$\sigma^2 = \frac{1+9+25+\alpha^2+\beta^2}{5} - 5^2$

$8 = \frac{35+\alpha^2+\beta^2}{5} - 25$

$33 \times 5 - 35 = \alpha^2 + \beta^2$

$\alpha^2 + \beta^2 = 130$ ____(2)

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$256 - 130 = 2\alpha\beta \Rightarrow \alpha\beta = 63$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= 4096 - 3024$

$= 1072$

13. Set S = $\left\{x : \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$ then n(S) equals.

(1) 2 (2) 3 (3) 4 (4) 6

Ans. **(3)**

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10$

$\left(\left(\sqrt{3}+\sqrt{2}\right)^2\right)^{\frac{x^2-4}{2}} + \left(\left(\sqrt{3}-\sqrt{2}\right)^2\right)^{\frac{x^2-4}{2}} = 10$

$\Rightarrow \left(5+2\sqrt{6}\right)^{\frac{x^2-4}{2}} + \left(5-2\sqrt{6}\right)^{\frac{x^2-4}{2}} = 10$

Now $\left(5+2\sqrt{6}\right)\left(5-2\sqrt{6}\right) = 25 - 24 = 1$

& $\left(5+2\sqrt{6}\right) + \left(5-2\sqrt{6}\right) = 10$

$\therefore \frac{x^2-4}{2} = \pm 1 \quad \Rightarrow x^2 = 4 \pm 2$ ⟨ 6, 2

$\Rightarrow x = \pm\sqrt{6}, \pm\sqrt{2}$

$\therefore$ n (S) = 4

14. If 8, a_2 , a_3......., are in A.P. such that sum of first 4 terms is 50 and sum of last 4 terms is 170, then find product of middle terms.

Ans. (754)

Sol. $S_4 = 50$

$2(16 + 3d) = 50$

$d = 3$ $\quad\quad 4a + d(4n - 10) = 170$

$32 + 3(4n - 10) = 170$

$4n - 10 = 46$

$n = 14$

Middle terms are T_7, T_8

$T_7\,T_8 = (8 + 6 \times 3)\,(8 + 7 \times 3) = 26 \times 29$

$= 754$

15. Number of all possible words by using all letters of the word "ASSASSINATION" with meaning or without meaning such that all vowels are together, is

(1) (10) 7! (2) (8) 7! (3) 9! (4) (12) 7!

Ans. (1)

Sol. **ASSASSINATION**

Vowels → AAIAIO

all vowels are together

$\therefore$ Now words $\frac{(13-6+1)!}{4!2!} \times \frac{6!}{3!2!}$

$= \frac{8!\,6 \times 5 \times 4!}{4!.2 \times 6 \times 2}$

$= (10)7!$

16. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$ then value of $f(4) - g(4)$ is equal to

Ans. (12)

Sol. $g(x) = 2x + f'(1)$

$g'(x) = 2$ and $g''(x) = 0$

$g'(1) = 2$ and $g''(2) = 0$

$f(x) = x^2 + 2x$ ....(1)

$f'(x) = 2x + 2$

$f'(1) = 4$

So, $g(x) = 2x + 4$

Now $f(4) - g(4)$

$= 16 + 8 - (8 + 4)$

$= 24 - 12 = 12$

17. Let region for $x \in [0, 1]$ given by

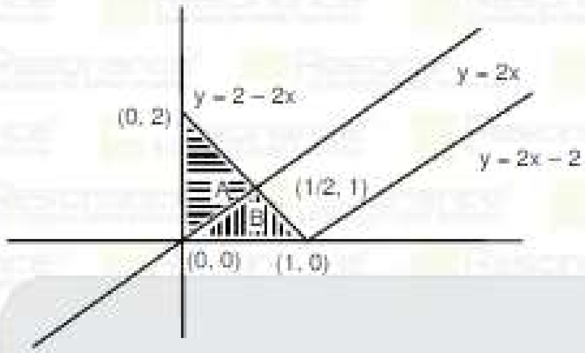
A : $2x \le y \le \sqrt{4(x-1)^2}$ with y-axis.

B : $y = \min\left\{2x, \sqrt{4(x-1)^2}\right\}$ with x-axis then $\frac{A}{B}$ equals

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (1)

Sol.



$A = \frac{1}{2} \times 2 \times \frac{1}{2} = \frac{1}{2}$

$B = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$, So $\frac{A}{B} = 1$

18 If system of linear equations $\lambda x + y + z = 1$, $x + \lambda y + z = 1$ and $x + y + \lambda z = 1$ is inconsistent for some values of λ then find value of $\Sigma\left(|\lambda|^2 + |\lambda|\right)$

(1) 2 (2) 6 (3) 4 (4) 1

Ans. (2)

Sol. $D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} \lambda-1 & 0 & 1-\lambda \\ 0 & \lambda-1 & 1-\lambda \\ 1 & 1 & \lambda \end{vmatrix}$

$= (\lambda-1)^2 \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda-1)^2.(1+1+\lambda)$

$D = (\lambda-1)^2(\lambda+2)$

$D = 0 \Rightarrow \lambda = 1, -2$

$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix} = (\lambda-1)^2$

$D_1 \neq 0$ for $\lambda = -2$

When $\lambda = 1$, all equations are identical, so number of solutions are infinite.

$\Sigma\left(|\lambda|^2 + |\lambda|\right) = |-2|^2 + |-2| = 6$

19. $\int_0^1 (x^{21}+x^{14}+x^7)(2x^{14}+3x^7+6)^{1/7}dx = \frac{1}{\ell}(11)^{m/n}$ then value of $\ell + m + n$ is equal to

Ans. (63)

Sol. $\int (x^{20}+x^{13}+x^6)(2x^{21}+3x^{14}+6x^7)^{1/7}dx$

$2x^{21}+3x^{14}+6x^7 = t$

$42(x^{20}+x^{13}+x^6)\,dx = dt$

$$\frac{1}{42}\int_0^{11} t^{\frac{1}{7}}dt = \left(\frac{t^{\frac{8}{7}}}{\frac{8}{7}} \times \frac{1}{42}\right)_0^{11}$$

$$= \frac{1}{48}\left(t^{\frac{8}{7}}\right)_0^{11} = \frac{1}{48}(11)^{8/7}$$

$\ell = 48,\ m = 8,\ n = 7$

$\ell + m + n = 63$

20. The shortest distance between lines.

$\frac{x-5}{3} = \frac{y-1}{7} = \frac{z-3}{5}$ & $\frac{x+7}{5} = \frac{y+5}{11} = \frac{z+1}{3}$ is

(1) $\frac{80}{\sqrt{354}}$ (2) $\frac{40}{\sqrt{354}}$ (3) $\frac{160}{\sqrt{354}}$ (4) $\frac{320}{\sqrt{354}}$

Ans. (3)

Sol. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 3 & 7 & 5 \\ 5 & 11 & 3 \end{vmatrix} = -34i + 16j - 2k$

$$\text{S.D.} = \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right| = \left|\frac{(12i+6j+4k)\cdot(34i-16j+2k)}{\sqrt{34^2+16^2+2^2}}\right|$$

$$\frac{408-96+8}{\sqrt{1156+256+4}} = \frac{320}{\sqrt{1416}} = \frac{160}{\sqrt{354}}$$

21. Let R = {(a, b) : $3a - 3b + \sqrt{7}$ is irrational} then

(1) R is an equivalence relation (2) R is symmetric but not reflexive.

(3) R is reflexive but not symmetric (4) R is reflexive and symmetric but not transitive

Ans. (3)

Sol. Clearly $\left(\frac{\sqrt{7}}{3}, 0\right) \in R$ but $\left(0, \frac{\sqrt{7}}{3}\right) \notin R$

So it is not symmetric

C is correct option

22. Let S be solution set for values of x satisfying $\cos^{-1}(2x) + \cos^{-1}\sqrt{1-x^2} = \pi$ then $\sum_{x\in S} 2\sin^{-1}(x^2-1)$ is equal to

(1) 0 (2) $-\sin^{-1}\left(\frac{24}{25}\right)$ (3) $\sin^{-1}(\sqrt{3}/4)$ (4) $\pi - \sin^{-1}(\sqrt{3}/4$

Ans. **(2)**

Sol. $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\sqrt{1-x^2} = -\cos(\cos^{-1} 2x)$ .... (1)

$\Rightarrow 1 - x^2 = 4x^2 \Rightarrow x = \pm\frac{1}{\sqrt{5}} \Rightarrow x = \frac{-1}{\sqrt{5}}$ from (1)

$\sum_{x\in S} 2\sin^{-1}(x^2-1) = 2\sin^{-1}\left(\frac{-4}{5}\right) = -\sin^{-1}\frac{24}{25}$

23. A triangle be such that cos2A + cos2B + cos2C is minimum. If in radius of the triangle is 3 then which of the following is correct.

(1) area of Δ is $\frac{6\sqrt{3}}{2}$ (2) perimeter of Δ is $18\sqrt{3}$

(3) area of Δ is $2\sqrt{3}$ (4) perimeter of Δ is $9\sqrt{3}$

Ans. **(2)**

Sol. min when A = B = C = 60°

$r = \frac{\Delta}{S} = 3 \Rightarrow \frac{\frac{\sqrt{3}}{4}a^2}{\frac{3a}{2}} = 3$

$\frac{a}{2\sqrt{3}} = 3 \Rightarrow a = 6\sqrt{3}$

Perimeter = 3a = $18\sqrt{3}$

24. If solution of $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$ is a circle and y(0) = 1, area of circle is 2π. P and Q are point of intersection of circle with y – axis. Normal at P and Q intersect. x –axis at R and S the length of RS is.

Ans. **(4)**

Sol. $(y-2)dy + (x+a)\,dx = 0$

$\frac{x^2}{2} + ax + \frac{y^2}{2} - 2y = c \Rightarrow y(0) = 1 \Rightarrow c = \frac{-3}{2}$

$x^2 + y^2 + 2ax - 4y + 3 = 0$

area = 2π

$\pi \times (a^2 + 4 - 3) = 2\pi \Rightarrow a = 1$

P (0 , 1) Q (0 ,3)

Normal at P (0, 1) $\Rightarrow x + y = 1$

$\Rightarrow$ R (1, 0)

Normal at Q (0, 3) $\Rightarrow y - x = 3$

$\Rightarrow$ S (-3, 0)

length of RS = 4