JEE-Main-31-01-2023 (Memory Based)
[EVENING SHIFT]

Physics

Question: In a circular coil carrying I current, the turns changed form n to N. Then the ratio of initial and final magnetic field at the center of the coil is -

Options:

(a) $\frac{n}{N}$

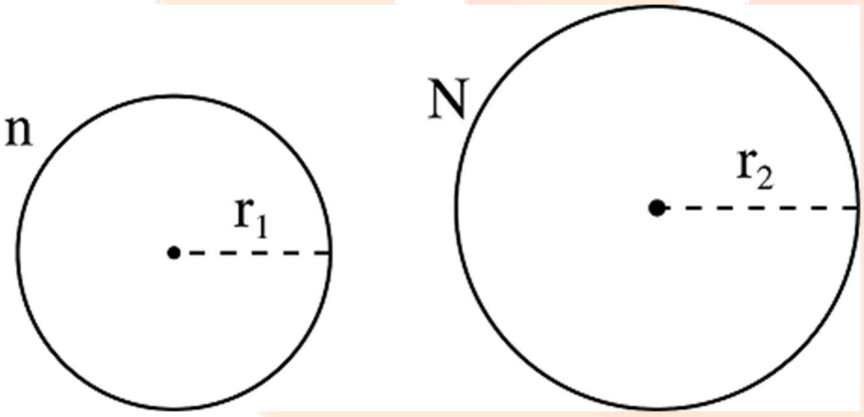
(b) $\frac{N}{n}$

(c) $\frac{n^2}{N^2}$

(d) $\frac{N^2}{n^2}$

Answer: (c)

Solution:



$L = 2\pi r_1 n = 2\pi r_2 N$

$\left(\frac{r_2}{r_1} = \frac{n}{N}\right)$

$$\frac{B_1}{B_2} = \frac{n\mu_0 i}{2r_1} \times \frac{2r_2}{N\mu_0 i} = \frac{n}{N} \times \frac{r_2}{r_1} = \frac{n}{N} \times \frac{n}{N} = \frac{n^2}{N^2}$$

Question: Match the column

Column I	Column II
a. X-ray	p. Eye surgery
b. Infrared rays	q. Crystallography
c. Microwaves	r. Physiotherapy
d. Ultra violet rays	s. Airplane navigation

Options:

(a) $a \to r; b \to q; c \to p; d \to s$

(b) $a \to r; b \to s; c \to p; d \to q$

(c) $a \to q; b \to r; c \to s; d \to p$

(d) $a \to p; b \to s; c \to r; d \to s$

Answer: (c)

Solution:

Question: If a body moving with 20 m/s stops in 5s. Find μ.

Options:

(a) 0.2

(b) 0.4

(c) 0.5

(d) 0.3

Answer: (b)

Solution: $a = -\mu g$

$V = u + at$

$0 = 20 - 4 \times 10 \times 5$

$U = \frac{20}{50} = 0.4$

Question: In a series L-C-R circuit, the value of voltage is $V = 5\sin(\omega t)$ Value of resistance is 30Ω. $X_L = 10\Omega$ and $X_C = 50\Omega$. Then the current amplitude is

Options:

(a) $\frac{1}{\sqrt{2}}$ ampere

(b) 1 ampere

(c) $\frac{1}{20\sqrt{2}}$ ampere

(d) $\frac{1}{10\sqrt{2}}$ ampere

Answer: (b)

Solution: $z = \sqrt{R^2 + \left(X_C - X_L\right)^2}$

$z = \sqrt{30^2 + 40^2}$

$z = 50$

$i_{\max} = \frac{V_{\max}}{z}$

$i_{\max} = 0.1\,\text{Ampere}$

Question: In an LCR circuit connected to an AC source of frequency 100 Hz. Find the inductive reactance for inductor of 5mH.

Options:

(a) 1.57Ω

(b) 3.14Ω

(c) 6.28Ω

(d) 9.42Ω

Answer: (b)

Solution:

Question: In an adiabatic process, pressure of an ideal gas becomes $\frac{16}{81}$ times of the initial pressure where as its volume becomes $\frac{27}{8}$ times. Then its $\frac{C_P}{C_V}$ is

Options:

(a) $\frac{7}{5}$

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) 2

Answer: (b)

Solution: $P_0V_0^r = \frac{16}{81}P_0\left(\frac{27}{8}V_0\right)^r$

$\Rightarrow 1 = \frac{16}{81}\times\left(\frac{27}{8}\right)^r$

$\Rightarrow \left(\frac{3}{2}\right)^4 = \left(\frac{3}{2}\right)^{3r}$

$\Rightarrow 3r = 4$

$\Rightarrow r = \frac{4}{3}$

Question: Two projectiles are thrown with same speed at angles $60°$ and $30°$ and maximum height attained by them is H_1 and H_2 respectively. Find $H_1 + H_2$.

Options:

(a) $\frac{u^2}{2g}$

(b) $\frac{u^2}{g}$

(c) $\frac{u^2}{4g}$

(d) $\frac{2u^2}{g}$

Answer: (a)

Solution: $H\max_1 \rightarrow \frac{\mu^2 \sin^2\theta_1}{2g}$

$H\max_2 \rightarrow \frac{\mu^2 \sin^2\theta_2}{2g}$

$H\max_1 + H\max_2 \rightarrow \frac{\mu^2}{2g}\left[\sin^2 60 + \sin^2 30\right]$

$\frac{\mu^2}{2g}\left[\frac{3}{4}+\frac{1}{4}\right]$

$H\max = \left[\frac{\mu^2}{2g}\right]$

Question: If weight of a ball at the surface of earth is W, then find its weight at height 9R from the surface.

Options:

(a) $\frac{W}{200}$

(b) $\frac{W}{50}$

(c) $\frac{W}{20}$

(d) $\frac{W}{100}$

Answer: (d)

Solution: At surface $W = mg$

$\left(m = \frac{w}{g}\right)$

Wave surface $= \frac{Gme\frac{w}{g}}{R^2}$

$\left(\frac{GMr}{gR^2} = 1\right)$

$\Rightarrow$ At length '9R' weight becomes $\left(W_{9R}\right)$

$$W_{9R} = \frac{Gme\left(\frac{w}{g}\right)}{(R+9R)^2}$$

$$W_{9R} = \frac{Gmew}{100R^2 g}$$

$$W_{9R} = \frac{Gme}{2R^2} \times \frac{w}{100}$$

$$\left[W_{9R} = \frac{W}{100}\right]$$

Question: A source of P = 1000 W with $\eta = 70\%$ is used to increase temperature of 5 kg water from $60°C$ to $70°C$. Find time.

Options:

(a) 200 s

(b) 300 s

(c) 400 s

(d) 500 s

Answer: (b)

Solution: $P \times t \times \eta = ms\Delta T$

$1000 \times t \times 0.7 = 5 \times 1000 \times 4.2 \times 10$

$t = 300s$

Question: H amount of heat is produced when 4 Ampere current is passed through a resistor for 10 seconds. Find the heat produced when 16 Ampere current is passed through the same resistor for 10 seconds

Options:

(a) 4H

(b) 16H

(c) 12H

(d) H

Answer: (b)

Solution: $H = i^2 Rt$

$H = 4^2 \times R \times 10 = 160R$

$H_2 = 16 \times 16 \times R \times 10$

$= 16 \times 160R$

$H_2 = 16H$

Question: During an adiabatic process performed on a diatomic gas 725 J of work is done on the gas. The change in internal energy of the gas is equal to

Options:

(a) 495 J

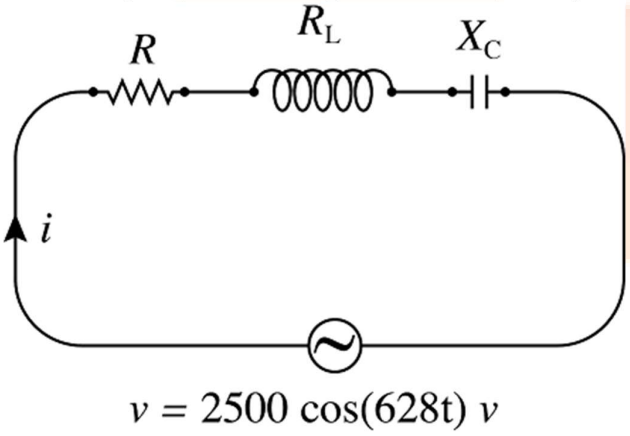
(b) 725 J

(c) 225 J
(d) zero
Answer: (b)
Solution:

Question: In a transistor, the doping level of bass, emitter and collector respectively are
Options:
(a) High, Moderate, Low
(b) Low, High, Moderate
(c) Low, Moderate, Moderate
(d) High, Low, Low
Answer: (b)
Solution: Factual

Question: Find ionization energy of 2^{nd} excited state of Li^{2+}. It is given that ionization energy of ground state of hydrogen atom is 13.6 eV.
Options:
(a) 20.4 eV
(b) 27.2 eV
(c) 6.8 eV
(d) 13.6 eV
Answer: (d)
Solution:

Question: In a series RLC circuit, $R = 80\Omega, X_L = 100\Omega, X_C = 40\Omega$. If the source voltage is 2500 cos (628t) volts, find peak current (in Amperes)



Options:
(a) 25 A
(b) 50 A
(c) 40 A
(d) 30 A
Answer: (a)
Solution:

Question: Match the physical quantities given in Column – I with the physical dimensions in column – II

Column I	Column II
(A) Torque	(P) $ML^{-1}T^{-2}$
(B) Stress	(Q) $ML^{2}T^{-2}$
(C) Pressure Gradient	(R) $ML^{-2}T^{-2}$
(D) Angular momentum	(S) $ML^{2}T^{-1}$

Options:

(a) $A \to S, B \to P, C \to R, D \to Q$

(b) $A \to Q, B \to P, C \to R, D \to S$

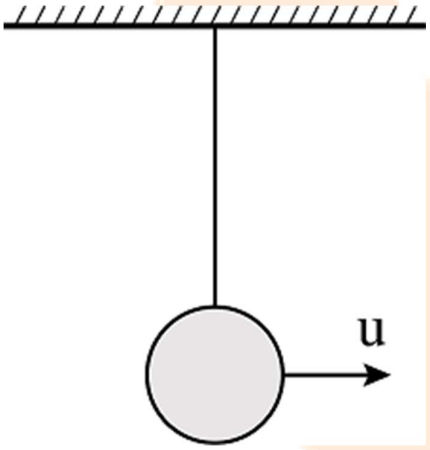
(c) $A \to P, B \to S, C \to R, D \to Q$

(d) $A \to Q, B \to P, C \to S, D \to R$

Answer: (b)

Solution:

Question: A ball of mass 1 kg is hanging from 1 m long inextensible string which can withstand maximum tension of 400 N. Find the maximum speed u that should be given to the ball



Options:

(a) $\sqrt{390} m/s$

(b) $\sqrt{410} m/s$

(c) $20 m/s$

(d) $22 m/s$

Answer: (a)

Solution:

Question: Two discs of same mass, radii r_1, r_2 thickness 1 mm and 0.5 mm, have densities in the ratio 3 : 1. The ratio of their moment of inertia about diameter is 1 : x. Find x.

Answer: 6

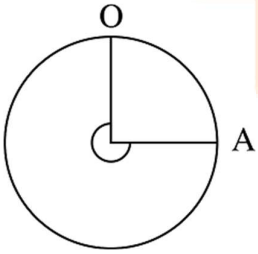
Solution:

Question: A ball was dropped from 20 m height from ground. Find the height (in m) up to which it rises after the collision. (Use $e=\frac{1}{2}, g=10m/s^2$)

Options:
(a) 10 m
(b) 15 m
(c) 5 m
(d) 20 m
Answer: (c)
Solution:

Question: For a group of positive charges, which is correct:
Options:
(a) Potential can be zero but electric field can't
(b) Potential can't be zero but electric field can
(c) Both zero not possible
(d) Bot non zero is possible
Answer: (b)
Solution:

Question: A particle coves circumference of circle of R = 10m in 4s. Find displacement in 3s
Options:
(a) 10 m
(b) $10\sqrt{2}m$
(c) 12 m
(d) $5\sqrt{2}m$
Answer: (b)
Solution:



In 3s it goes $O \rightarrow A$
$S=\sqrt{2}R=10\sqrt{2}m$

JEE-Main-31-01-2023 (Memory Based) [Evening Shift]

Chemistry

Question: Correct order of Lewis acid strength for boron trihalides
Options:
(a) $BF_3 > BCl_3 > BBr_3 > BI_3$
(b) $BF_3 < BCl_3 < BBr_3 < BI_3$
(c) $BF_3 < BI_3 < BBr_3 < BCl_3$
(d) $BI_3 > BCl_3 < BF_3 < BBr_3$
Answer: (b)
Solution: Due to back bonding in BX_3 Lewis acid nature reduces as size of halogen decreases.

Question: Methyl orange will not used in
Options:
(a) Strong acid and strong base
(b) Weak acid and Strong base
(c) Weak acid and weak base
(d) Strong acid and weak base
Answer: (b)
Solution: Choice of Indicator: In the titration of strong acid and a weak base, methyl orange is chosen as indicator. When titration between strong base and weak acid is to be performed then phenolphthalein is a good indicator. In the titration of strong acid versus strong base any indicators can be used. For the titration of weak acid vs weak base no indicator is available.

Question: Complete combustion hydrocarbon A uses 11 equivalents of oxygen and gives 4 equivalents of water. Formula of A.
Options:
(a) C_6H_8
(b) C_9H_8
(c) C_7H_{16}
(d) C_8H_8
Answer: (c)
Solution: 11 equivalents of O_2 means = 2.75 moles and 4 equivalent H_2O = 2 moles
Since O_2 is used for both CO_2 and H_2O

$\therefore$ 2.75 O_2 – 1 O_2 for 2 mole H_2O

Thus, for CO_2 = 1.75 mole = mole of carbon

Thus, $C_{1.75}H_4$ or C_7H_{16}

Question: Normal rainwater is slightly acidic and has pH 5.6, it is due to which reaction
Options:

(a) $H_2O + CO_2 \rightleftharpoons H^+ + HCO_3^-$

(b) $H_2O + NO_2 \rightleftharpoons HNO_3 + H^+$

(c) $H_2O + SO_3 \rightleftharpoons H^+ + HSO_4^-$

(d) All of these

Answer: (a)

Solution: Acid rain: We are aware that normally rain water has a pH of 5.6 due to the presence of H^+ ions formed by the reaction of rain water with carbon dioxide present in the atmosphere.

$H_2O(l) + CO_2(g) \rightleftharpoons H_2CO_3(aq)$

$H_2CO_3(aq) \rightleftharpoons H^+(aq) + HCO_3^-(aq)$

When the pH of the rain water drops below 5.6, it is called acid rain.

Question: How many of the following are linear shape molecule

XeF_2, I_3^-, I_3^+, CO_2, C_3O_2, $BeCl_2$, BCl_2^-, SO_2

Options:

(a) 5

(b) 4

(c) 6

(d) 7

Answer: (a)

Solution:

XeF_2, and I_3^- = sp^3d

CO_2 and C_3O_2 = sp

$BeCl_2$ = sp

Question: Which of the following element has 4f Half shell electronic configuration

(A) Sm (B) Eu (C) Gd

Options:

(a) Sm, Eu, Gd

(b) Only Eu

(c) Only Sm

(d) Both Eu and Gd

Answer: (d)

Solution:

Sm = $4f^6 4s^2$

Eu = $4f^7 4s^2$

Gd = $4f^7 5d^1 6s^2$

Question: $2C + O_2 \Rightarrow 2CO$

Given C = 12g, O = 48g, then find the volume of CO at STP =?

Options:

(a) 22.4 lt

(b) 67.2 lt

(c) 44.8 lt

(d) 11.2 lt
Answer: (a)
Solution: Here carbon is limiting agent thus 1 mole of CO is formed

Question: Statement-1: H_2O_2 used in manufacture of cephalosporin.
Statement-2: It is used to reduce anaerobic respiration.
Options:
(a) Both Statement 1 and 2 are correct
(b) Statement 1 is correct
(c) Statement 2 is correct
(d) Statement 1 is correct and 2 is incorrect
Answer: (a)
Solution: It is used in the synthesis of hydroquinone, tartaric acid and certain food products and pharmaceuticals (cephalosporin) etc.
Restoration of aerobic conditions to sewage wastes, etc.

Question: Which ion is used in manufacture of neuromuscular function and interneuronal transmission
Options:
(a) Ca
(b) Be
(c) Mg
(d) Li
Answer: (a)
Solution: About 99 % of body calcium is present in bones and teeth. It also plays important roles in neuromuscular function. Inter neuronal transmission, cell membrane integrity and blood coagulation.

Question: Molarity = 0.8 M, resistivity = 2×10^{-4} ohm cm. Calculate molar conductance?
Options:
(a) 625 S m^2 mol^{-1}
(b) 6.25×10^5 S cm^2 mol^{-1}
(c) 6.25×10^4 S cm^2 mol^{-1}
(d) 6250 S m^2 mol^{-1}
Answer: (a)
Solution:

$$\Lambda_m = \frac{K \times 1000}{M}$$

$$\Lambda_m = \frac{1 \times 1000}{2 \times 10^{-4} \times 0.8} = \frac{10^7}{1.6}$$

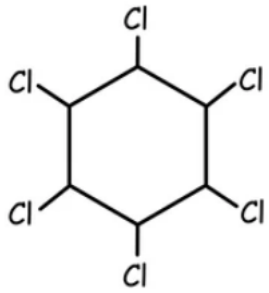
$= 6.25 \times 10^6$ S cm^2 mol^{-1}

Question: Which of the following contain maximum number of chlorine atom?
Options:
(a) Gammaxene
(b) Chloropicrin
(c) Freon-12

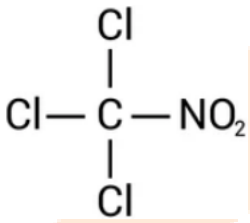
(d) Chloral Hydrate

Answer: (a)

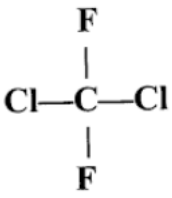
Solution:



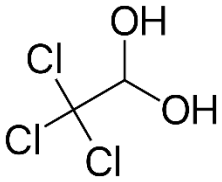
Benzene hexachloride (Gammaxene)



Chloropicrin



Freon-12



Chloral Hydrate

Question: How many of the following are disinfectant?

Cl_2, SO_2, H_2O_2, Phenol, Alcohol, Bithional, Terpineol

Options:

(a) 5

(b) 6

(c) 7

(d) 4

Answer: (a)

Solution: Disinfectants are applied to inanimate objects such as floors, drainage system, instruments, etc. Same substances can act as an antiseptic as well as disinfectant by varying the concentration. For example, 0.2 per cent solution of phenol is an antiseptic while its one percent solution is disinfectant.

Chlorine in the concentration of 0.2 to 0.4 ppm in aqueous solution and sulphur dioxide in very low concentrations, are disinfectants.

Question: In dumas method the nitrogen containing organic compound, when heated with copper oxide in an atmosphere of carbon dioxide, yields free nitrogen in addition to carbon dioxide and water. Traces of nitrogen oxides formed, are reduced to nitrogen by passing the gaseous mixture of which catalyst:
Options:
(a) Ni
(b) Pd
(c) CuO
(d) Copper gauze
Answer: (d)
Solution: Traces of nitrogen oxides formed, if any, are reduced to nitrogen by passing the gaseous mixture over a heated copper gauze.

Question: Statement-1: Cu reacts with borax to give green colour
Statement-2: Cu forms copper(l) metaborate
Options:
(a) Statement 1 and 2 both are correct
(b) Statement 1 is correct
(c) Statement 2 is correct
(d) Statement 1 and 2 both are incorrect
Answer: (b)
Solution: On heating, borax loses its water of crystallisation and decomposes to give sodium metaborate and boric anhydride.

$$\underset{\text{Borax}}{Na_2B_4O_7.10H_2O} \rightarrow Na_2B_4O_7 + 10H_2O$$

$$Na_2B_4O_7 \rightarrow \underset{\text{Sodium metaborate}}{2NaBO_2} + \underset{\text{Boric anhydride}}{B_2O_3}$$

On treatment with metal salt, boric anhydride forms metaborate of the metal which gives different colours in oxidising and reducing flame. For example, in the case of copper sulphate, following reactions occur.

$$CuSO_4 + B_2O_3 \xrightarrow{\text{Non-luminous flame}} \underset{\text{Cupric metaborate Blue-green}}{CU(BO_2)_2} + SO_3$$

Two reactions may take place in the reducing flame:

(i) The blue $Cu(BO_2)_2$ is reduced to colourless cuprous metaborate as follows:

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow{\text{luminous flame}} 2CuBO_2 + Na_2B_4O_7 + CO$$

or (ii) Cupric metaborate may be reduced to metallic copper and the bead appears red and opaque.

$$2CU(BO_2)_2 + 4NaBO_2 + 2C \xrightarrow{\text{luminous flame}} 2Cu + 2Na_2B_4O_7 + 2CO$$

Question:

	CCl_4	+	$2H_2O$	→	CO_2	+	$4HCl$
$\Delta_f H^o$ (in KJ/mol) ⇒	(–394)		(–191)		(–236)		(–92)

$\Delta_r H^o$ = ?

Options:

(a) 172 KJ

(b) –172 KJ

(c) –257 KJ

(d) 19 KJ

Answer: (a)

Solution: $\Delta_r H = \sum \Delta H^o_P - \sum \Delta H^o_R$

$\therefore \Delta_r H = (-236 - 4 \times 92) - (394 - 2 \times 191) = +172$ KJ

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[Evening Shift]

Mathematics

Question: Minimum value of $\left|x^2-x+1\right|+\left[x^2-x+1\right]$ for $x\in[-1,2]$ is ____

where $[.]$ is GIF.

Answer: $\frac{3}{4}$

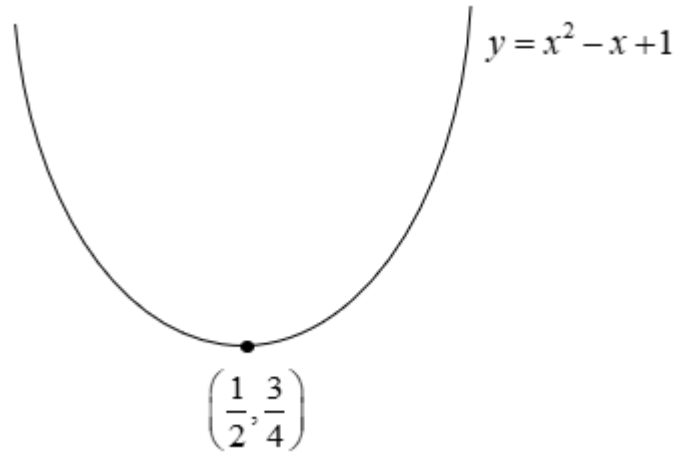
Solution:

We have $\left|x^2-x+1\right|+\left[x^2-x+1\right]$

Let $y=x^2-x+1$

In $[-1,2]$

Minimum $y=\frac{3}{4}$



$\therefore$ Minimum value of $\left|x^2-x+1\right|+\left[x^2-x+1\right]$ is

$\left|\frac{3}{4}\right|+\left[\frac{3}{4}\right]=\frac{3}{4}+0=\frac{3}{4}$

Question: If the foci of a hyperbola are $\left(1+\sqrt{2},0\right)$ and $\left(1-\sqrt{2},0\right)$ and its eccentricity is $\sqrt{2}$, then its latus rectum is

Answer: 2.00

Solution:

Given: foci of hyperbola are $\left(1+\sqrt{2},0\right)$ and $\left(1-\sqrt{2},0\right)$

Eccentricity, $e=\sqrt{2}$

Now, $2ae=2\sqrt{2}$

$\Rightarrow ae = \sqrt{2}$

$\Rightarrow a = 1$

Since, $e = \sqrt{2}$, thus the hyperbola is rectangular hyperbola,

Hence $a = b = 1$

$\therefore$ Latus rectum $= \dfrac{2b^2}{a} = 2$

Question: Coefficient of x^{-6} in the expansion of $\left(\dfrac{4x}{5} - \dfrac{5}{2x^2}\right)^9$ is

Answer: -5040.00

Solution:

General term of $\left(\dfrac{4x}{5} - \dfrac{5}{2x^2}\right)^9$ is

$T_{k+1} = {}^9C_k \left(\dfrac{4x}{5}\right)^{9-k} \left(\dfrac{-5}{2x^2}\right)^k$

$= (-1)^k \, {}^9C_k \, (2)^{18-3k} \cdot 5^{2k-9} \cdot x^{9-3k}$

For coefficient of x^{-6}, we must have

$9 - 3k = -6$

$3k = 15$

$k = 5$

$\therefore$ Coefficient of x^{-6} is $= (-1)^5 \, {}^9C_5 \cdot (2^3) \cdot 5$

$= -\dfrac{9 \times 8 \times 7 \times 6 \times 8 \times 5}{4 \times 3 \times 2} = -5040$

Question: If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$, then $n^2 + n + 15 = ?$

Answer: 45.00

Solution:

Given that: ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$

$\Rightarrow \dfrac{(2n+1)!}{(n+2)!} \times \dfrac{(n-1)!}{(2n-1)!} = \dfrac{11}{21}$

$\Rightarrow \dfrac{(2n+1)(2n)}{(n+2)(n+1)(n)} = \dfrac{11}{21}$

$\Rightarrow 21n(4n+1) = 11n(n^2 + 3n + 2)$

$\Rightarrow 84n + 42 = 11n^2 + 33n + 22$

$\Rightarrow 11n^2 - 51n - 20 = 0$

$\Rightarrow n = 5, \dfrac{-4}{11}$

$\therefore n^2+n+15=25+5+15=45$

Question: Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$, and $\vec{c}=5\hat{i}-3\hat{j}+3\hat{k}$ be three vectors. It is given that $\vec{r}\times\vec{b}=\vec{b}\times\vec{c}$ and $\vec{r}.\vec{a}=0$, then $25|\vec{r}|^2=?$

Answer: 339.00

Solution:

Given, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=5\hat{i}-3\hat{j}+3\hat{k}$

$\vec{r}\times\vec{b}=\vec{b}\times\vec{c}$

$\vec{r}\times\vec{b}-\vec{b}\times\vec{c}=0$

$(\vec{r}-\vec{c})\times\vec{b}=0$

$\vec{r}-\vec{c}\parallel\vec{b}$

$\vec{r}-\vec{c}=\lambda\vec{b}$

$\vec{r}=\vec{c}+\lambda\vec{b}$ ....(1)

Now, $\vec{r}.\vec{a}=0$

$\vec{r}.\vec{a}=\vec{c}.\vec{a}+\lambda(\vec{b}.\vec{a})$

$\lambda=\dfrac{-(\vec{c}.\vec{a})}{(\vec{b}.\vec{a})}$

$\lambda=\dfrac{-(8)}{5}$

Put λ in (1)

$\vec{r}=\vec{c}-\dfrac{8}{5}\vec{b}$

$\vec{r}=(5,-3,3)-\dfrac{8}{5}(1,-1,2)$

$5\vec{r}=(25,-15,15)-(8,-8,16)$

$5\vec{r}=(17,-7,-1)$

$5\vec{r}.5\vec{r}=289+49+1$

$25|\vec{r}|^2=339$

Question: $\lim\limits_{x\to\infty}\dfrac{\left[\left(\sqrt{3x+1}\right)^6+\left(\sqrt{3x-1}\right)^6\right]+\left[\left(\sqrt{3x+1}\right)^6-\left(\sqrt{3x-1}\right)^6\right]\times x^3}{\left(x+\sqrt{x^2+1}\right)^6+\left(x-\sqrt{x^2+1}\right)^6}=?$

Answer: $\dfrac{27}{32}$

Solution:

Given, $\lim_{x\to\infty}\dfrac{\left[\left(\sqrt{3x+1}\right)^6+\left(\sqrt{3x-1}\right)^6\right]+\left[\left(\sqrt{3x+1}\right)^6-\left(\sqrt{3x-1}\right)^6\right]\times x^3}{\left(x+\sqrt{x^2+1}\right)^6+\left(x-\sqrt{x^2+1}\right)^6}$

$$\frac{\left[\left(3^3+3^3\right)+\left(3^3-3^3\right)\right]}{(1+1)^6+(1-1)^6}=\frac{54}{64}=\frac{27}{32}$$

Question: Range of $\dfrac{x^2+2x+1}{x^2-8x+12}$ is

Answer: $\left(-\infty,-\dfrac{21}{4}\right]\cup[0,\infty)$

Solution:

Given, $\dfrac{x^2+2x+1}{x^2-8x+12}$

Let $y=\dfrac{x^2+2x+1}{x^2-8x+12}$

$y\left(x^2-8x+12\right)=x^2+2x+1$

$x^2y-8xy+12y-\left(x^2+2x+1\right)=0$

$x^2(y-1)+x(-8y-2)+12y-1=0$

Taking $D\geq 0$

$(-8y-2)^2-4(y-1)(12y-1)\geq 0$

$(4y+1)^2-(y-1)(12y-1)\geq 0$

$16y^2+1+8y-12y^2+13y-1\geq 0$

$4y^2+21y\geq 0$

$y(4y+21)\geq 0$

By solving the range will be in $\left(-\infty,-\dfrac{21}{4}\right]\cup[0,\infty)$

Question: If $\int_0^{\alpha}\dfrac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx=\dfrac{16+20\sqrt{2}}{15}$, then α is equal to

Answer: 2.00

Solution:

Given, $\int_0^{\alpha}\dfrac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx=\dfrac{16+20\sqrt{2}}{15}$

By rationalizing

$\int_0^{\alpha}\dfrac{x\left(\sqrt{x+\alpha}+\sqrt{x}\right)}{\alpha}$

$$\frac{1}{\alpha}\int_0^{\alpha} x\sqrt{x+\alpha}+x^{\frac{3}{2}}$$

$$\frac{1}{\alpha}\int_0^{\alpha}(x+\alpha)^{\frac{3}{2}}-\alpha\sqrt{x+\alpha}+x^{\frac{3}{2}}dx$$

$$\frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{\frac{5}{2}}-\alpha(x+\alpha)^{\frac{3}{2}}\times\frac{2}{3}+\frac{2}{5}x^{\frac{5}{2}}\right]$$

$$\frac{1}{\alpha}\left[\frac{2}{5}\times 2^{\frac{5}{2}}\alpha^{\frac{5}{2}}-\alpha\frac{2^{\frac{5}{2}}}{3}\alpha^{\frac{3}{2}}+\frac{2}{5}\alpha^{\frac{5}{2}}-\frac{2}{5}\alpha^{\frac{5}{2}}+\alpha^{\frac{5}{2}}\times\frac{2}{3}-0\right]$$

$$=\frac{2^{\frac{7}{2}}}{5}\alpha^{\frac{3}{2}}-\alpha^{\frac{5}{2}}\times\frac{2^{\frac{5}{2}}}{3}+\alpha^{\frac{5}{2}}\times\frac{2}{3}$$

By equating

$$\frac{2^{\frac{7}{2}}}{5}\alpha^{\frac{3}{2}}-\alpha^{\frac{3}{2}}\times\frac{2^{\frac{5}{2}}}{3}+\alpha^{\frac{3}{2}}\times\frac{2}{3}=\frac{16+20\sqrt{2}}{15}$$

$$\frac{\alpha^{\frac{3}{2}}\left(3\times 2^{\frac{7}{2}}-5\times 2^{\frac{5}{2}}+10\right)}{15}=\frac{16+20\sqrt{2}}{15}$$

$$\frac{\alpha^{\frac{3}{2}}\left(24\sqrt{2}-20\sqrt{2}+10\right)}{15}=\frac{16+20\sqrt{2}}{15}$$

$$\alpha^{\frac{3}{2}}\left(\frac{4\sqrt{2}+10}{15}\right)=\frac{16+20\sqrt{2}}{15}$$

$$\alpha^{\frac{3}{2}}=2\sqrt{2}$$

$$\Rightarrow \alpha=2$$

Question: $\dfrac{(i-1)}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}=?$

Answer: $\sqrt{2}(\cos 75^\circ+i\sin 75^\circ)$

Solution:

$$\frac{(i-1)}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}=\frac{\sqrt{2}(\cos 135^\circ+i\sin 135^\circ)}{\cos 60^\circ+i\sin 60^\circ}$$

$$=\sqrt{2}(\cos 75^\circ+i\sin 75^\circ)$$

Question: If $|A_n|=2$ and $\left|adj\left(2\,adj\,2A^{-1}\right)\right|=2^{84}$, then $n=?$

Answer: 5.00

Solution:

$$\left|adj\left(2\,adj\,2A^{-1}\right)\right| = 2^{84}$$

$$\Rightarrow \left|2^{n-1}adj\left(adj\,2A^{-1}\right)\right| = 2^{84}$$

$$\Rightarrow 2^{(n-1)n}\left|adj\left(adj\,2A^{-1}\right)\right| = 2^{84}$$

$$\Rightarrow 2^{(n-1)n}\left|2A^{-1}\right|^{(n-1)^2} = 2^{84}$$

$$\Rightarrow \frac{2^{(n-1)n}\times 2^{n(n-1)^2}}{|a|^{(n-1)^2}} = 2^{84}$$

$$\Rightarrow \frac{2^{(n-1)n}\times 2^{n(n-1)^2}}{2(n-1)^2} = 2^{84}$$

$$\Rightarrow (n-1)n + n(n-1)^2 - (n-1)^2 = 84$$

$$\Rightarrow (n-1)n + (n-1)(n-1)^2 = 84$$

$$\Rightarrow (n-1)n + (n-1)^3 = 84$$

$$\Rightarrow (n-1)\left(n + (n-1)^2\right) = 84$$

$$\Rightarrow (n-1)\left(n + (n-1)^2\right) = 4\times 21$$

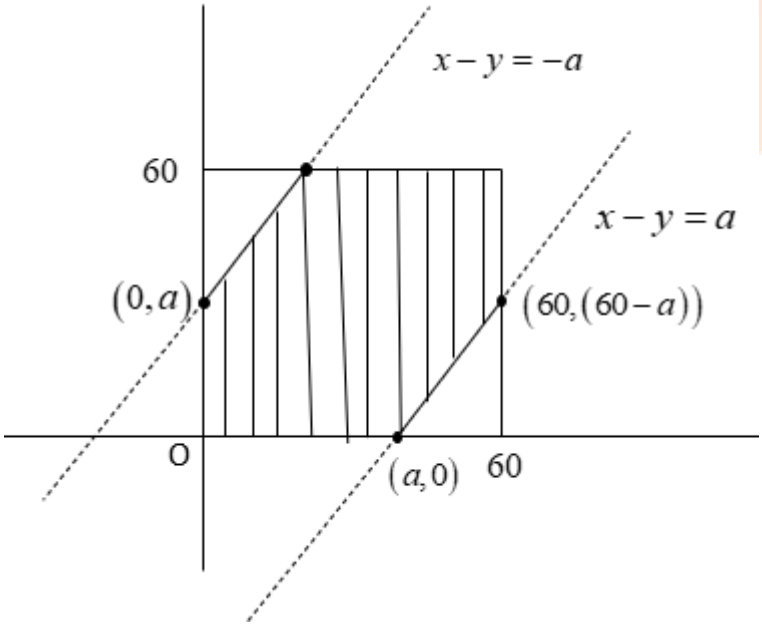
Comparing both sides, we get

$n = 5$

Question: Probability that magnitude of difference of two real numbers belonging to [0, 60] is less than 'a' is $\frac{11}{36}$. Find 'a'.

Answer: 10.00

Solution:



Let x, y be two numbers, then

$(x, y) \in [0, 60]$

$P(|x-y| < a) = \frac{11}{36}$

$-a < x - y < a$

Now, A.T.Q

$$\frac{11}{36} = \frac{3600 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$$

$\Rightarrow 1100 = 3600 - (60-a)^2$

$\Rightarrow (60)^2 + a^2 - 120a = 2500$

$\Rightarrow a^2 - 120a + 3600 - 2500 = 0$

$\Rightarrow a^2 - 120a + 1100 = 0$

$\Rightarrow a^2 - 10a - 110a + 1100 = 0$

$\Rightarrow a(a-10) - 110(a-10) = 0$

$\Rightarrow a = 10, 110$

$\Rightarrow a = 10$

Question: Number of roots $e^{4x} - 8e^{3x} + e^{2x} + 8e^{x} + 13$ is

Answer: 2 real roots

Solution:

Given, $e^{4x} - 8e^{3x} + e^{2x} + 8e^{x} + 13$

Put $e^x = t$

$t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

$$\left(t^2 + \frac{1}{t^2}\right) + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

Put $t - \frac{1}{t} = u$

$\Rightarrow u^2 + 2 + 8u + 13 = 0$

$\Rightarrow u^2 + 8u + 15 = 0$

$\Rightarrow u = -5$ or $u = -3$

$\Rightarrow t - \frac{1}{t} = -5$ or $t - \frac{1}{t} = -3$

$\Rightarrow t^2 + 5t - 1 = 0$ or $t^2 + 3t - 1 = 0$

$\Rightarrow e^x = \frac{-5 \pm \sqrt{29}}{2}$ or $e^x = \frac{-3 \pm \sqrt{13}}{2}$

$\therefore$ 2 real roots and 2 non-real roots are there.

Question: $S:\left\{(a,b):a,b\in I \ \& \ 2+\frac{a}{b}>0\right\}$

$T:\left\{(a,b):a,b\in I \ \& \ a^2-b^2\in I\right\}$, then

Options:

(a) T & S both are symmetric

(b) T & S both are transitive

(c) T is symmetric but S is not

(d) S is transitive but T is not

Answer: (c)

Solution:

$S:\left\{(a,b):a,b\in I \ \& \ 2+\frac{a}{b}>0\right\}$

$T:\left\{(a,b):a,b\in I \ \& \ a^2-b^2\in I\right\}$

Checking T for symmetric

$a^2-b^2\in I$

$\Rightarrow b^2-a^2\in I$

$\therefore T$ is symmetric

Checking T for transitivity:

$a^2-b^2\in I$ and $b^2-c^2\in I$

Adding both we get

$a^2-b^2+b^2-c^2=a^2-c^2\in I$ [$\because$ sum of two integers is also an integer]

$\therefore T$ is transitive.

Checking S for Symmetric

$(a,b)\in S\Rightarrow 2+\frac{a}{b}>0$

$\Rightarrow \frac{a}{b}>-2$

Now consider $(-1,1,0)$, then

$\frac{-1}{10}>-2\Rightarrow (a,b)\in S$

Now, $\frac{b}{a}=-10<-2\Rightarrow (b,a)\notin S$

Thus, S is not symmetric

Now, if $2+\frac{a}{b}>0$ & $2+\frac{b}{c}>0$, then

$2+\frac{a}{c}$ can be positive or negative

Thus, S is not transitive

Question: Foot of perpendicular from origin to the plane intersecting axes at A, B and C is $(2,a,4)$. Volume of tetrahedron OABC is 144 square units. Find value of 'a'.

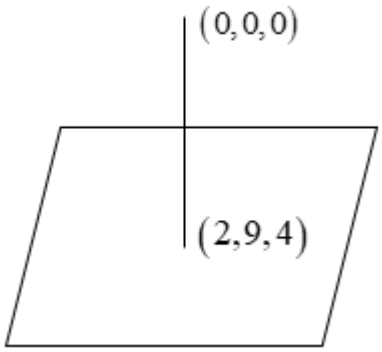
Answer: 2.00

Solution:

We observe that

Drs of plane: $\langle 2,9,4\rangle$

Any point on plane: $(2,9,4)$

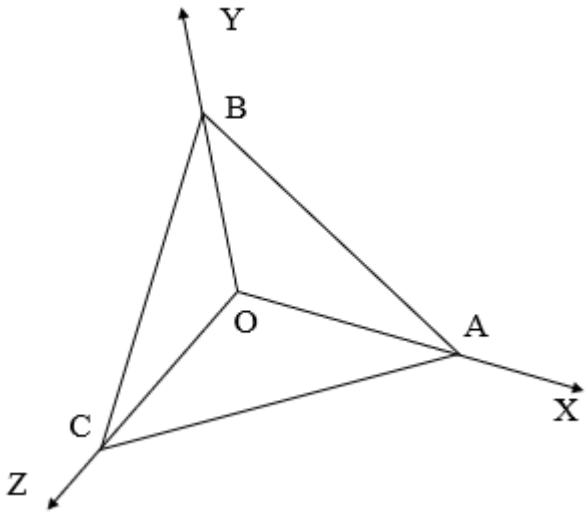


$\therefore$ Equation of the plane is

$2(x-2)+a(y-a)+4(z-4)=0$

$\Rightarrow 2x+ay+uz=20+a^2$

Now, the plane is intersecting the axes at A, B & C



Thus, the coordinates of points are

$A\equiv\left(10+\frac{a^2}{2},0,0\right)$

$B\equiv\left(0,\frac{20+a^2}{a},0\right)$

$C\equiv\left(0,0,\frac{20+a^2}{4}\right)$

Volume of tetrahedron $=\frac{1}{6}\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]=144$

$$\Rightarrow \frac{1}{6}\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]=144$$

$$\Rightarrow \frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right)=144$$

$$\Rightarrow \left(20+a^2\right)^3=144\times 48a$$

$$\Rightarrow a=2$$

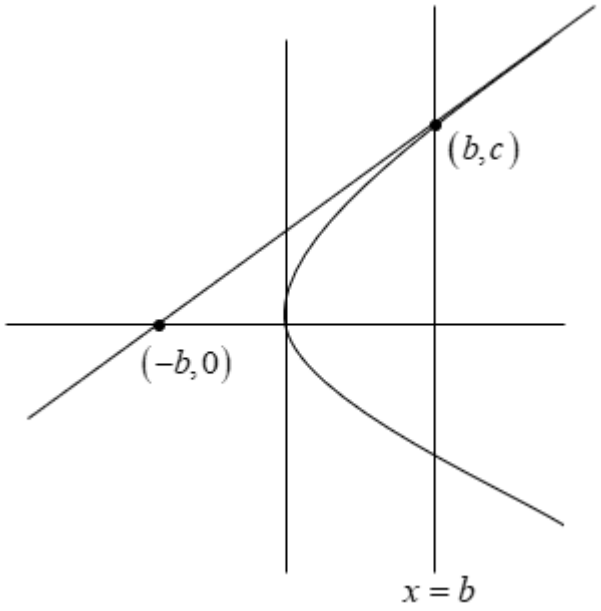
Question: (b,c) lies on $y^2=2ax$. Tangent at (b,c) makes a triangle of area 16 with $x=b$ and $y=0$. Find $\sum a$, where $a,b,c\in z$.

Answer: 146.00

Solution:

Given $y^2=2ax$

Equation of tangent $y(c)=a(x+b)$



Area $=\frac{1}{2}\times 2b\times c=16$

$bc=16$

$c^2=2ab$

$a=\frac{c^2}{2b}$

These are the possibilities

$b=16,\ c=1\rightarrow$ Rejected

$b=1,\ c=16\rightarrow a=128$

$b=4,\ c=4\rightarrow a=2$

$b=8,\ c=2\rightarrow$ rejected

$b=2,\ c=8\rightarrow a=16$

Now, $\sum a = 128 + 2 + 16 = 146$

Question: Find θ, if $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0;\ \theta \in [0, 2\pi]$

Answer: $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

Solution:

Given that:

$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0$

$\Rightarrow \sin^{-1}(\sin\theta) - \frac{\pi}{2} + \sin^{-1}(\sin\theta) > 0$

$\Rightarrow 2\sin^{-1}(\sin\theta) > \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(\sin\theta) > \frac{\pi}{4}$

$\therefore \frac{\pi}{2} > \sin^{-1}(\sin\theta) > \frac{\pi}{4}$

$1 > \sin\theta > \frac{1}{\sqrt{2}}$

$\theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$