UNIT AND MEASUREMENT

- **Fundamental Units :**

Sr. No.	Physical Quantity	SI Unit	Symbol
1	Length	Metre	m
2	Mass	Kilogram	Kg
3	Time	Second	S
4	Electric Current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous Intensity	Candela	Cd
7	Amount of Substance	Mole	mol

- **Supplementary Units :**

Sr. No.	Physical Quantity	SI Unit	Symbol
1.	Plane Angle	Radian	r
2	Solid Angle	Steradian	Sr

Relative error in the measurement of a quantity $= \frac{\Delta a_{mean}}{a_{mean}}$

Percentage error $= \frac{\Delta a_{mean}}{a_{mean}} \times 100$

When $z = \frac{a^p \cdot b^q}{c^r}$, then maximum relative in z is $\frac{\Delta z}{z} = p\frac{\Delta a}{a} + q\frac{\Delta b}{b} + r\frac{\Delta C}{C}$

Motion in a Straight Line

(1). For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity V_0, final velocity v and acceleration a are related by a set of kinematic equations of motions. These are

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax$$

The above equations are the equations of motion for particle. If the position of the particle at t = 0 is 0. If the particle starts at $x = x_0$ i.e. if it is at x_0 at t = 0, then in the above equation x is replaced by $(x - x_0)$.

MOTION IN A PLANE

(1). Law of cosines, if $\vec{R} = \vec{P} + \vec{Q}$ then $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Here, $\theta =$ angle between $\vec{P}$ and $\vec{Q}$

(2). Direction of $\vec{R}$ $\Rightarrow \tan\alpha = \dfrac{Q\cos\theta}{P + Q\sin\theta}$: $\alpha =$ angle between $\vec{R}$ and $\vec{P}$

(3). Position of an object at time t, if it is initially at $\vec{r}_0$, having initial velocity v_0 and moving with constant acceleration $\vec{a}$, is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}^2$$

LAWS OF MOTION

(1). Force: $\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$, when m is constant $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

(2). Conservation of linear momentum: $\sum \vec{p}_i = \sum \vec{p}_j$

WORK ENERGY AND POWER

(1). The **work-energy theorem** states that for conservative forces acting on the body, the change in kinetic energy of a body equal to the net work done by the net force on the body.

$$K_f - K_i = W_{net}$$

Where K_i and K_f are initial and final kinetic energies and W_{net} is the net work done.

(2). For a conservative force in one dimension, Potential energy function V(x) is defined such that

$$F(x) = -\frac{dV(x)}{dx}$$

(3). Average power of a force is defined as the ratio of the work, W, to the total time ***t*** taken.

$$\Rightarrow P_{av} = \frac{W}{t}$$

(4). The instantaneous power is defined as the limiting value of the average power as time interval approaches zero. $P = \frac{dW}{dt}$

Power can also be expressed as

$P = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$ here, $d\vec{r}$ is displacement vector.

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Work-Energy theorem

$$W_C + W_{NC} + W_{PS} = \Delta K$$

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

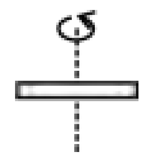
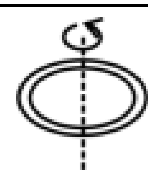
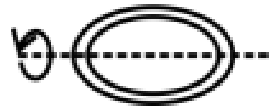
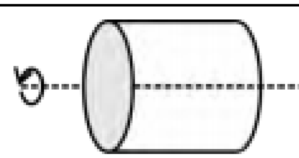
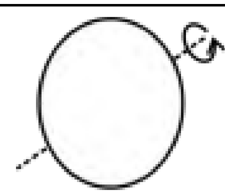
(1). According to the theorem of perpendicular axes moment of inertia of a body about perpendicular axis is $I_z = I_x + I_y$.

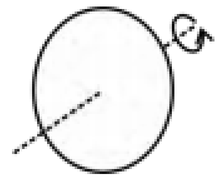
Where I_x, I_y, I_z, are the moment of inertia of the rigid body about x, y and z axes respectively x and y axes lie in the plane of the body and z-axis lies perpendicular to the plane of the body and passes through the point of intersection of x and y.

(2). According to the theorem of parallel axes $I = I_C + Md^2$

Where I_C is the moment of inertia of the body about an axis passing through its centre of mass and d is the perpendicular distance between the two axes.

Table 1: Moment of inertia of some symmetrical bodies

Body	Axis	Figure	M.I.
Rod (Length L)	Perpendicular to rod, at the midpoint centre of mass		$\frac{ML^2}{12}$
Circular ring (radius R)	Passing through centre and perpendicular the plane		MR^2
Circular ring (Radius R)	Diameter		$\frac{MR^2}{2}$
Solid cylinder (radius R)	Axis of cylinder		$\frac{MR^2}{2}$
Solid sphere (radius R)	Diameter		$\frac{2}{5}MR^2$

Hollow sphere (radius R)	Diameter		$\frac{2}{3}MR^2$

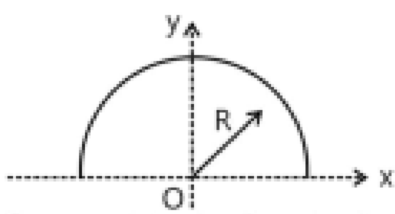
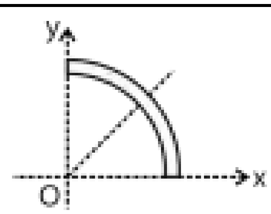
(3). Relation between moment of inertia (I) and angular momentum $\vec{L}$ is given by $\vec{L} = I\,\vec{\omega}$

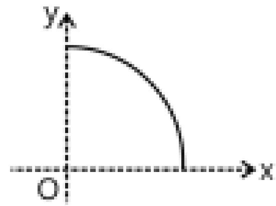
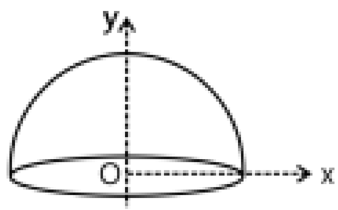
(4). Relation between moment of inertia (I) and kinetic energy of rotation is given by

$$K.E._{rotation} = \frac{1}{2}I\omega^2$$

(5). Relation between of inertia (I) and torque $(\vec{\tau}) \Rightarrow \vec{\tau} = I\vec{\alpha}$

Given below are the positions of centre of mass of some commonly used objects.

Object	Location of centre of mass
 Uniform semicircular disc of radius R	$x_{CM} = 0,\ y_{CM} = \frac{4R}{3\pi},\ z_{CM} = 0$
 Uniform quarter of a ring of radius R	$x_{CM} = \frac{2R}{\pi},\ y_{CM} = \frac{2R}{\pi},\ z_{CM} = 0$

	 Uniform quarter of a disc of radius R	$x_{CM} = \frac{4R}{3\pi},\ y_{CM} = \frac{4R}{3\pi},\ z_{CM} = 0$
	 Uniform spherical shell of radius R	$x_{CM} = 0,\ y_{CM} = \frac{R}{2},\ z_{CM} = 0$

GRAVITATION

(1). Newton's universal law of gravitation $F = \frac{Gm_1m_2}{r^2}$

In vector form, $\vec{F} = \frac{Gm_1m_2}{r^2} \cdot (\vec{r})$

. Acceleration due to gravity 'g' is $g = \frac{GM_e}{R_e^2}$

. Variation of g at altitude 'h' is $g_h = g\left[\frac{R_e}{R_e + h}\right]^2$

If $h << R$ then, $g_h = g\left[1 - \frac{2h}{R_e}\right]$

MECHANICAL PROPERTIES OF SOLIDS

Relation between Y,B,S $\Rightarrow Y = \frac{9BS}{3B+S}$

Elongation produced in rod of length 'L' due to its own weight is $\Delta L = \frac{\rho g L^2}{2Y} = \frac{MgL}{2AY}$

Relation between Y,B,η, $Y = 3B(1-2\eta)$

i. Poisson's Ratio $\eta = \frac{3B-2S}{6B+2S}$

. Relation between Y,S $Y = 2S(1+\sigma)$

. Depression at the middle of a beam $y = \frac{Wt^3}{4Ybd^3}$

Sheer Modulus $S = \frac{Fh}{Ax}$

MECHANICAL PROPERTIES OF FLUIDS

Relative density of a substance $\rho_{rel} = \frac{\rho_{substance}}{\rho_{water} \text{ at } 4^\circ C}$

Gauge pressure $P_g = \rho g h$

Apparent weight of a body of density σ in a fluid of density ρ

$W' = W\left(1 - \frac{\rho}{\sigma}\right)$, W = weight of the body in air

Excess pressure inside an air bubble is $P_i - P_0 = \frac{4S}{R}$

Height of a liquid in a capillary tube is $h = \frac{2S\cos\theta}{r\rho g}$

THERMAL PROPERTIES OF MATTER

$\beta = 2\alpha,\ \gamma = 3\alpha$ (Relation between α, β, γ)

(a) $Q = \frac{kA(T_1 - T_2)t}{x}$

$H = \frac{dQ}{dt} = -kA\left(\frac{dT}{dx}\right)$ H is called the heat current.

. (a) Coefficient of reflectivity is $r = \frac{Q_1}{Q}$

(b) Coefficient of absorptivity $a = \frac{Q_2}{Q}$

(c)Coefficient of transitivity $t = \frac{Q_3}{Q}$

Where Q_1 is the radiant energy reflected, Q_2 is the radiant energy absorbed and Q_3 is the radiant energy transmitted through a surface on which Q is the incident radiant energy

THERMODYNAMICS

(1). First law of thermodynamics $\Delta Q = \Delta U + \Delta W$

(2). Work done, $\Delta W = P\Delta V$

$\therefore \quad \Delta Q = \Delta U + P\Delta V$

(3). Relation between specific heats for a gas $C_p - C_r = R$

(4). For isothermal process, (i) according to Boyle's law PV = constant

According to Charles law (For volume) $V \propto T$ constant and Charles law (for pressure) $P \propto T$

And (ii) Work done is $W = \mu RT \ln \frac{V_2}{V_1} = 2.303\, \mu RT \log \frac{V_2}{V_1}$

(5). For adiabatic process, (i) According to Boyle's law PV^{γ} = constant Where, $\gamma = \frac{C_p}{C_v}$

And (ii) Work done is $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{\mu R[T_1 - T_2]}{\gamma - 1}$

(6). Slope of adiabatic = γ (slope of isotherm)

KINETIC THEORY OF GASSES

R.M.S. velocity $v_{rms} = \sqrt{\frac{3k_BT}{m}}$ (3). Average velocity $v_{av} = \sqrt{\frac{8K_BT}{\pi m}}$

Most probable velocity $v_{mp} = \sqrt{\frac{2K_BT}{m}}$

Mean free path $(\vec{\lambda}) = \frac{1}{\sqrt{2}n\pi d^2}$

OSCILLATIONS

(1) Displacement equation for SHM $x = A\sin(\omega t + \phi)$ Or

$x = A\cos[\omega t + \phi]$ where A is amplitude and $(\omega t + \phi)$ is phase of the wave

(2). Velocity in SHM $v = A\omega\cos\omega t$ and $v = \omega\sqrt{A^2 - x^2}$

(3). Acceleration in SHM is $a = -\omega^2 A\sin\omega t$ and $a = -\omega^2 x$

Force action on oscillation body is $m \cdot \frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$

Equation of motion is $x = Ae^{-u/2m}\cos(\omega' t + \phi)$ Where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

WAVES

(1). Equation of a plane progressive harmonic wave travelling along positive direction of X-axis is

$y(x, t) = a \sin(kx - \omega t + \phi)$

And along negative direction of X-axis is $y(x, t) = a \sin(kx + \omega t + \phi)$

Where, $y(x,t) \rightarrow$ Displacement as a function of position x and time t,

$a \rightarrow$ Amplitude of the wave, $\omega \rightarrow$ Angular frequency of the wave,

$k \rightarrow$ Angular wave number, $(kx - \omega t + \phi) \rightarrow$ Phase,

And $\phi \rightarrow$ Phase constant or initial phase angle

The effect of density on velocity of sound $\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}$

The effect of temperature on velocity of sound $\frac{v_1}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$

CURRENT ELECTRICITY

(1). Resistance of a uniform conductor of length L, area of cross-section A and resistivity ρ along its length, $R = \rho \frac{\ell}{A}$

(2). Current density $j = \frac{di}{ds}$

(3). Conductance $G = \frac{1}{R}$.

i4) Drift velocity $v_d = \frac{eE}{m} t. = \frac{i}{neA}$

(5). Current $i = neAv_d$

(6) Resistivity is $\rho = \frac{m}{ne^2 t} = \frac{1}{\sigma}$ where σ is resistivity.

For n identical cells in parallel across load R, current through load $i = \frac{n\varepsilon}{nR + r}$

Comparison of emf $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$ (ii) Internal resistance of cell $r = \left(\frac{\ell_1}{\ell_2} - 1\right)R$

MOVING CHARGES AND MAGNETIC FIELD

Situation	Formula
Lorentz force	$q[\vec{E} + \vec{v} \times \vec{B}]$
A charge particle thrown at some angle to a uniform magnetic field (i) Path (ii) Radius (iii) Time period (iv) Pitch	(i) Helix (ii) $r = \frac{mv\sin\theta}{qB}$ (iii) $t = \frac{2\pi m}{qB}$ (iv) $T \cdot v\cos\theta$
Cyclotron frequency	$f = \frac{qB}{2\pi m}$
Maximum kinetic energy of a charged particle in a cyclotron (With R as radius of dee)	$K = \frac{q^2B^2R^2}{2m}$
Force on a straight current carrying conductor in a uniform magnetic field	$\vec{F} = i(\vec{l} \times \vec{B})$
Force on a arbitrary shaped current carrying conductor in a uniform magnetic field	$\vec{F} = i\int d\vec{\ell} \times \vec{B} = i\ \vec{\ell} \times \vec{B}$
Magnetic moment of a current carrying loop	$\vec{M} = i\ \vec{A}$
Torque on a current carrying loop placed in a uniform magnetic field	$\vec{\tau} = \vec{M} \times \vec{B}$
Biot-Savart Law	$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$
Magnetic field at a point distance x from the centre of a current carrying circular loop	$\frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

MAGNETISM AND MATTER

Bar magnet : The electrostatic Analog

Electrostatics	Magnetism
Permittivity = $\frac{1}{\varepsilon_0}$	Permittivity = μ_0
Charge q	Magnetic pole strength (q_m)
Dipole Moment $\vec{p} = q \cdot l$	Magnetic Dipole Moment $\vec{M} = q_m l$

$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$	$\vec{F} = \frac{\mu_0}{4\pi} \frac{q_{m(1)} q_{m(2)}}{r^2}$
$\vec{F} = q\vec{E}$	$\vec{F} = q_m \vec{B}$
Axial Field $\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^2}$	$\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{M}}{r^3}$
Equatorial Field $\vec{E} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^2}$	$\vec{B} = -\frac{\mu_0}{4\pi} \frac{\vec{M}}{r^2}$
Torque $\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{M} \times \vec{B}$

ELECTROMAGNETIC INDUCTION

$\phi_B = Li$ $\Rightarrow$ $L \rightarrow$ self inductance of the coil

Induced emf $e = -L\frac{di}{dt}$ (8). $L = \mu_0 \mu_r n^2 \times A \times i$ Where L coefficient of self-inductance

$\phi_2 = Mi_1$ and $\theta_2 = -M\frac{di_1}{dt}$ Where M is co efficient of mutual inductance

. The emf induced (in dynamo) $e_{(t)} = BA\omega(\sin \omega t)$

. Mutual inductance $M = \mu_0 \mu_1 n_1 n_2 Ai$

RAY OPTICS AND OPTICAL INSTRUMENTS

(1). The distance between the pole and centre of curvature of the mirror called radius of curvature

$$f = \frac{R}{2}$$

Relationship between u, v and focal length f is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ in case lens.

Longitudinal magnification = (Lateral magnification)^2

Refractive index of material of prism $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$ (δ_m) Minimum deviation angle

WAVE OPTICS

$\frac{\sin i}{\sin t} = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ (Snell's law)

Ratio of maximum to minimum intensity $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

(a) Fringe width $\beta = \frac{\lambda D}{d}$ (b) Condition of maxima $\Delta\phi = 2n\pi$ where $n = 0, 1, 2..$

Radius of central bright spot in diffraction pattern $r_0 = \frac{1.22\,\lambda f}{2a}$

Fresnel distance $Z_f = \frac{a^2}{\lambda}$ (8) Malus law $I = I_0 \cos^2\theta$

Brewster's law $\tan i_B = \mu$

DUAL NATURE OF RADIATION AND MATTER

(1). Einstein's photoelectric cell equation, $\frac{1}{2}mv_{max}^2 = hf - hf_0$

Where f, f_0 are frequencies of incident radiation.

(2). Work function and threshold frequency or threshold wavelength, $\phi_0 = hf_0 = \frac{hc}{\lambda_0}$

(3). Energy of photon, $E = hf = \frac{hc}{\lambda}$

(4). Momentum of photon, $P = \frac{E}{c} = \frac{h}{\lambda}$

(5). De Broglie wavelength of a material particle, $\lambda = \frac{h}{mv}$

COMMUNICATION SYSTEM

. In amplitude modulation $P_1 = P_2\left[1 + \frac{\mu^2}{2}\right]$

. Maximum frequency can be reflected from ionosphere $f_{max} = 9(N_{max})^{1/2}$

. Maximum modulated frequency can be detected by diode detector $f_m = \frac{1}{2\pi R\mu}$

NUCLEI

Nuclear radius (R) is given by $R = R_0 A^{1/3}$ Here $R_0 = 1.2 \times 10^{-15}\,m$.

Density of all nuclei is constant.

Total binding energy $= [Z\,m_p + (A - Z)m_n - M]c^2$ J

Av. BE/nucleon = Total B.E/A

Radioactivity decay law $\frac{dN}{dt} = -\lambda N \Rightarrow N = N_0 e^{-\lambda t}$