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JEE Main 2023 (Memory based)

1st February 2023 - Shift 1

Answer & Solutions

MATHEMATICS

1. $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$ equals :

A. $\ln 2$
B. $\ln\frac{3}{2}$
C. $\ln\frac{2}{3}$
D. 0

Answer (A)

Solution:

$$\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{n+r}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$$

$$0<\lim_{n\to\infty}\frac{r}{n}<1$$

$$=\int_0^1\frac{dx}{1+x}$$

$$=\ln(1+x)|_0^1=\ln\ 2$$

2. For solution of $\frac{dy}{dx}+y\tan x=\sec x$, $y(0)=1$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

A. $\frac{\sqrt{3}}{2}$
B. $\frac{1+\sqrt{3}}{2}$
C. $\frac{1}{2}$
D. $-\frac{\sqrt{3}}{2}$

Answer (B)

Solution:

$\frac{dy}{dx} + y \tan x = \sec x$

I. F $= e^{\int \tan x dx} = \sec x$

Solution of equation is

$y \cdot \sec x = \int \sec x \cdot \sec x$

$\Rightarrow y \cdot \sec x = \tan x + C$

At $x = 0,\ y = 1$ (given)

$\Rightarrow C = 1$

At $x = \frac{\pi}{6}$

$\Rightarrow y \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} + 1$

$\Rightarrow y = \frac{1+\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}$

$\Rightarrow y = \frac{1+\sqrt{3}}{2}$

3. The sum of $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \cdots \infty$ terms equals to:

A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$

Answer (A)

Solution:

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \cdots \infty$$
$$= \sum_{r=1}^{\infty} \frac{r}{1+r^2+r^4}$$
$$= \frac{1}{2} \sum_{r=1}^{\infty} \frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)(r^2-r+1)}$$
$$= \frac{1}{2} \sum_{r=1}^{\infty} \frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}$$
$$= \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{13} + \cdots\right)$$
$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

4. The number of ways by which letter of word $ASSASSINATION$ can be arranged such that all vowels come together is:

A. $\frac{8!3!}{6!}$
B. $\frac{8!}{4!3!}$
C. $\frac{8!6!}{4!(2!)^2 3!}$
D. $\frac{8!6!}{4!\,3!2!}$

Answer (C)

Solution:

$A \rightarrow 3$ times repeated

$S \rightarrow 4$ times repeated

$I \rightarrow 2$ times repeated

$N \rightarrow 2$ times repeated

$T \rightarrow 1$

$O \rightarrow 1$

A, I & O are vowels

$\therefore$ Number of ways $= \frac{8!}{4!2!} \cdot \frac{6!}{3!2!}$

5. $f(x) + f'(x) = \int_0^2 f(t)dt$ and $f(0) = e^{-2}$, then the value of $f(2) - 2f(0)$ is:

A. 0
B. -1
C. 1
D. 2

Answer (B)

Solution:

$f(x) + f'(x) = \int_0^2 f(t)dt$
Let $k = \int_0^2 f(t)dt$
$\Rightarrow \frac{dy}{dx} + y = k$
$\Rightarrow ye^x = ke^x + C$
$\because f(0) = e^{-2}$
$\Rightarrow e^{-2} = k + C$
$\Rightarrow C = e^{-2} - k$
$\Rightarrow ye^x = ke^x + e^{-2} - k$
$\Rightarrow y = k + (e^{-2} - k)e^{-x}$
Now, $\int_0^2 f(t)\, dt = k$
$\Rightarrow \int_0^2 (k + (e^{-2} - k)e^{-t})\, dt = k$
$\Rightarrow [kt]_0^2 - [e^{-t}(e^{-2} - k)]_0^2 = k$
$\Rightarrow 2k - (e^{-2} - k)(e^{-2} - 1) = k$
$\Rightarrow 2k - (e^{-4} - ke^{-2} - e^{-2} + k) = k$
$\Rightarrow 2k - e^{-4} + ke^{-2} + e^{-2} - k = k$
$\Rightarrow ke^{-2} = e^{-4} - e^{-2}$
$\Rightarrow k = e^{-2} - 1$
$\Rightarrow f(x) = e^{-2} - 1 + e^{-x}$
Now, $f(2) - 2f(0) = (e^{-2} - 1 + e^{-2}) - 2(e^{-2} - 1 + 1)$
$\Rightarrow f(2) - 2f(0) = 2e^{-2} - 1 - 2e^{-2}$
$\Rightarrow f(2) - 2f(0) = -1$

6. If set $S = \left\{(\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10\right\}$ then $n(S)$ equals:

A. 2
B. 3
C. 4
D. 6

Answer (C)

Solution:

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$

Let $(\sqrt{2}+\sqrt{3})^{x^2-4}=t$

$\therefore\ t+\frac{1}{t}=10$

$\Rightarrow\ t^2-10t+1=0$

$\Rightarrow\ (t-5)^2=24$

$\Rightarrow t=5\pm 2\sqrt{6}$

$\Rightarrow\ \left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5\pm 2\sqrt{6}$

If $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5+2\sqrt{6}$

$\Rightarrow \left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=\left(\sqrt{2}+\sqrt{3}\right)^2$

$\Rightarrow x^2-4=2 \Rightarrow x=\pm\sqrt{6}$

if $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5-2\sqrt{6}$

$\Rightarrow \left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=\left(\sqrt{2}+\sqrt{3}\right)^{-2}$

$\Rightarrow x^2-4=-2 \Rightarrow x^2=\pm\sqrt{2}$

$\therefore$ 4 solutions are possible in total.

7. $1,\ 3,\ 5,\ x,\ y$ are 5 observations. Mean of these observations is 5 and variance is 8. Sum of the cubes of the two missing number equals:

A. 1072
B. 513
C. 1079
D. 516

Answer (A)

Solution:

$\bar{x}=5$

$\Rightarrow 1+3+5+x+y=25$

$\Rightarrow x+y=16\cdots(i)$

$\sigma^2=8=\frac{\sum x_i^2}{5}-(\bar{x})^2$

$\Rightarrow 8=\frac{1^2+3^2+5^2+x^2+y^2}{5}-25$

$\Rightarrow 165=35+x^2+y^2$

$\Rightarrow x^2+y^2=130$

$\Rightarrow (x+y)^2-2xy=130$

$\Rightarrow xy=63\cdots(ii)$

From (*i*) & (ii),

$x=7, y=9$

Now, $x^3+y^3=7^3+9^3$

$x^3+y^3=343+729=1072$

8. Sum of the series $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\cdots+\frac{1}{5!0!}$ is:

A. $\frac{2^{51}}{50!}$

B. 2^{51}

C. $5!\cdot 2^{51}$

D. $\frac{2^{50}}{51!}$

Answer (D)

Solution:

$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\cdots+\frac{1}{5!0!}$

$=\frac{1}{51!}\left(\frac{51!}{1!50!}+\frac{51!}{3!48!}+\frac{51!}{5!46!}+\cdots+\frac{51!}{51!0!}\right)$

$=\frac{1}{51!}\left({}^{51}c_1 + {}^{51}c_3 + -+ {}^{51}c_{51}\right)$

$=\frac{1}{51!}\left(\frac{2^{51}}{2}\right)=\frac{2^{50}}{51!}$

9. If $R=\{(a,b):3a-3b+\sqrt{7}$ is irrational$\}$.Then which among the following options are correct

A. R is an equivalence relation

B. R is symmetric but not reflexive

C. R is reflexive but not symmetric

D. R is reflexive and symmetric but not transitive

Answer (C)

Solution:

For reflexive

$3a-3a+\sqrt{7}=\sqrt{7}$ is irrational

$\therefore (a,a)\in R, \therefore$ reflexive

For symmetric

$\left(\frac{\sqrt{7}}{3},0\right)\in R \ \ but \ \left(0,\frac{\sqrt{7}}{3}\right)\notin R$

$\Rightarrow$ Relation is not symmetric

For transitive

$\left(\frac{\sqrt{7}}{3},0\right)\in R$, $\left(0,\frac{2\sqrt{7}}{3}\right)\in R$

But $\left(\frac{\sqrt{7}}{3},\frac{2\sqrt{7}}{3}\right)\notin R$

$\Rightarrow$ Relation is not transitive

10. Negation of the statement $p\vee(p\wedge\sim q)$ is:

A. p

B. $\sim p$

C. q

D. $\sim q$

Answer (B)

Solution:

$\because \left(p \vee (p \wedge \sim q)\right) \equiv p$

$\Rightarrow \sim\left(p \vee (p \wedge \sim q)\right) \equiv \sim p$

11. Let S be solution set for values of x satisfying $\cos^{-1}(2x) + \cos^{-1}\sqrt{1-x^2} = \pi$, then $\sum_{x\in S} 2\sin^{-1}(x^2-1)$ is equal to:

A. 0

B. $-\sin^{-1}\left(\frac{24}{25}\right)$

C. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D. $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Answer (B)

Solution:

$\frac{\pi}{2} - \sin^{-1}(2x) + \frac{\pi}{2} - \sin^{-1}\sqrt{1-x^2} = \pi$

$\Rightarrow \sin^{-1}(2x) + \sin^{-1}\sqrt{1-x^2} = 0$

$\Rightarrow \sin^{-1}(-2x) = \sin^{-1}\sqrt{1-x^2}$

$\Rightarrow -2x = \sqrt{1-x^2}$

$4x^2 = 1 - x^2$

$\Rightarrow x = \pm\sqrt{\frac{1}{5}}$

$x = -\frac{1}{\sqrt{5}}$ is only possible solutions

$\sum_{x\in S} 2\sin^{-1}(x^2-1) = 2\sin^{-1}\left(-\frac{4}{5}\right)$

$= -2\sin^{-1}\frac{4}{5} \quad \cdots\left(2\sin^{-1}x = \sin^{-1}(2x\sqrt{(1-x^2)}\right)$

$= -\sin^{-1}\left(\frac{24}{25}\right)$

12. A triangle be such that $\cos 2A + \cos 2B + \cos 2C$ is minimum. If inradius of the triangle is 3, then which of the following is CORRECT?

A. Area of Δ $is \frac{6\sqrt{3}}{2}$ Sq. Units

B. Perimeter of Δ is $18\sqrt{3}$ Units

C. Area of Δ is $2\sqrt{3}$ Sq. Units

D. Perimeter of Δ is $9\sqrt{3}$ Units

Answer (B)

Solution:

If $K = \cos\ 2A + \cos\ 2\beta + \cos\ 2C$ is minimum then $k = \frac{-3}{2}$

$\&\ A = B = C = \pi/3$

$\therefore r = \frac{\Delta}{s} = 3 = \frac{\sqrt{3}a^2}{4\times 3a} \times 2$

$\Rightarrow\ a = 6\sqrt{3}$

$\therefore$ Area $= \frac{\sqrt{3}}{4} \times 36 \times 3 = 27\sqrt{3}$ Sq. Units

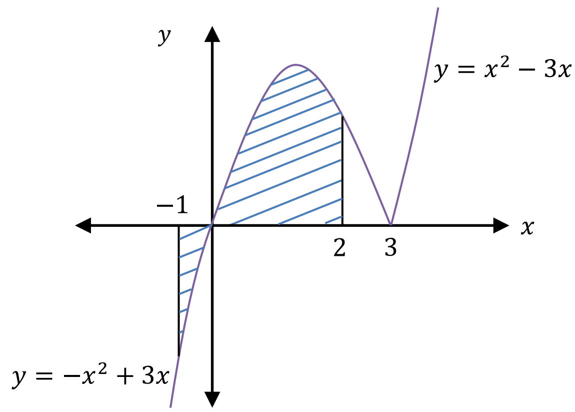
$s = 3a = 18\sqrt{3}$ units

$\therefore$ Perimeter is $18\sqrt{3}$ units

13. Area bounded by $y = x|x-3|$ & x –axis between $x = -1$ & $x = 2$ is A then $12A$ equals _______.

Answer : 62

Solution:



$$y = x|x-3| = \begin{cases} x(x-3)\ ; x \geq 3 \\ -x(x-3); x \leq 3 \end{cases}$$

Area= $\int_{-1}^{0}(x^2 - 3x)dx + \int_{0}^{2}(-x^2 + 3x)dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_{0}^{2}$

$= \left[0 - \left(-\frac{11}{6}\right)\right] - \left[\frac{-10}{3} - 0\right]$

$= \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$

$\Rightarrow 12A = 12 \times \frac{31}{6} = 62$

14. Remainder when $23^{200} + 19^{200}$ is divided by 49 equals ______.

Answer (2)

Solution:

$23^{200} + 19^{200} = (21+2)^{200} + (21-2)^{200}$
$= 2[\,{}^{200}C_0 21^{200} + {}^{200}C_2 21^{198} + {}^{200}C_4 21^{196} + \cdots + {}^{200}C_{198} 21^2 + {}^{200}C_{200}(21)^0]$
$= 2(49k + 1)$
Remainder $= 2$

15. $8, a_1, a_2, \ldots, a_n$ are terms in A.P. Sum of first 4 terms of series is 50 and sum of last 4 terms of series is 170. Then the product of middle terms of series is __________.

Answer (754)

Solution:

$\frac{4}{2}[16 + 3d] = 50$
$\Rightarrow d = 3$
$\frac{4}{2}[2a_n + 3(-d)] = 170$
$\Rightarrow 2a_n - 3d = 85$
$\Rightarrow 2a_n = 94$
$\Rightarrow a_n = 47$
$\Rightarrow 8 + (n-1)d = 47$
$\Rightarrow n = 14$
So 7^{th} & 8^{th} are middle Term

$T_7 = 8 + 6 \cdot 3 = 26$

$T_8 = 8 + 7 \cdot 3 = 29$

$\therefore T_7 \cdot T_8 = 754$

16. A circle is represented by $\frac{|z-2|}{|z-3|} = 2$. Its radius is γ units and centre is (α, β), then $3(\alpha + \beta + \gamma)$ is equal to ______.

Answer (12)

Solution:

Let $z = x + iy$
$\Rightarrow (x-2)^2 + y^2 = 4(x-3)^2 + 4y^2$
$\Rightarrow x^2 + y^2 - 4x + 4 = 4x^2 - 24x + 36 + 4y^2$
$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$
$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$
Centre$\equiv \left(\frac{10}{3}, 0\right)$
$\Rightarrow r = \sqrt{\left(\frac{10}{3}\right)^2 + 0^2 - \frac{32}{3}} = \frac{2}{3}$
$\Rightarrow 3(\alpha + \beta + \gamma) = 12$

17. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$ then $f(4) - g(4)$ equals _______.

Answer (12)

Solution:

$g(x) = 2x + f'(1)$
$\Rightarrow g'(x) = 2$
$\Rightarrow g'(1) = 2$ and $g''(x) = 0$
Now, $f(x) = x^2 + xg'(1) + g''(2)$
$f(x) = x^2 + 2x$
$\Rightarrow f'(x) = 2x + 2 \Rightarrow f'(1) = 4$
$\therefore g(x) = 2x + 4$
$f(4) - g(4) = (16 + 8) - (8 + 4)$
$= 12$

18. For some values of λ, system of equations
$\lambda x + y + z = 1,$
$x + \lambda y + z = 1,$
$x + y + \lambda z = 1$ has no solution, then $\sum(|\lambda|^2 + |\lambda|)$ equals __________.

Answer (6)

Solution:

$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$
$\Rightarrow \lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1 - \lambda) = 0$
$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda - 1 - 1) = 0$
$\Rightarrow \lambda = 1, -2$
For $\lambda = 1$ There are infinite solution
For $\lambda = -2$ system has no solution
$\sum(|\lambda|^2 + |\lambda|) = 4 + 2 = 6$

19. If solution of $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$ is a circle and $y(0) = 1$, area of circle is 2π. P and Q are point of intersection of circle with y-axis. Normal at P and Q intersect x –axis at R and S. The length of RS is:

Answer (4)

Solution:

$\frac{dy}{dx} + \frac{x+a}{y-2} = 0$

$\Rightarrow (y-2)dy = -(x+a)dx$

$\Rightarrow \frac{(y-2)^2}{2} = -\frac{(x+a)^2}{2} + C$

$\Rightarrow (x+a)^2 + (y-2)^2 = 2C$

$\because y(0) = 1$

$\Rightarrow a^2 + 1 = 2C$

Area $= 2\pi$

$\Rightarrow \pi(2C) = 2\pi \Rightarrow C = 1$

$\Rightarrow a^2 + 1 = 2 \Rightarrow a = \pm 1$

CASE I:

Equation of circle $(x+1)^2 + (y-2)^2 = 2$

$C \equiv (-1,\ 2)$

For P & Q, $x = 0$

$\Rightarrow y - 2 = \pm 1$

$\Rightarrow P\ \&\ Q \equiv (0,\ 3)\&(0,\ 1)$

Normal equation$\Rightarrow y - 3 = \frac{3-2}{(0+1)}(x-0)$

$\Rightarrow x - y + 3 = 0$

$y - 1 = \frac{1-2}{0+1}(x-0)$

$\Rightarrow y + x - 1 = 0$

$R\ \&\ S \equiv (-3,\ 0)\ \&\ (1,\ 0)$

$\Rightarrow RS = 4$

CASE II:

Equation of circle $(x-1)^2 + (y-2)^2 = 2$

$C \equiv (1,\ 2)$

For P & Q, $x = 0$

$\Rightarrow y - 2 = \pm 1$

$\Rightarrow P\ \&\ Q \equiv (0,\ 3)\&(0,\ 1)$

Normal equations at P & Q are

$y - 3 = \frac{3-2}{(0-1)}(x-0)$

$\Rightarrow x + y - 3 = 0$ and

$y - 1 = \frac{1-2}{(0-1)}(x-0)$

$\Rightarrow x - y + 1 = 0$

$R\ \&\ S \equiv (3,\ 0)\ \&\ (-1,\ 0)$

$\Rightarrow RS = 4$

20. Number of 3-digit numbers which are divisible by 2 or 3 but not divisible by 7 is ____________.

Answer (514)

Solution:

We know that

$T_n = a + (n-1)d$

So, numbers divisible by 2 is:

$998 = 100 + (n_2 - 1)2$

$\Rightarrow n_2 = 450$

Numbers divisible by 3 is:

$999 = 102 + (n_3 - 1)3$

$\Rightarrow n_3 = 300$

Numbers divisible by 2 & 3 is:

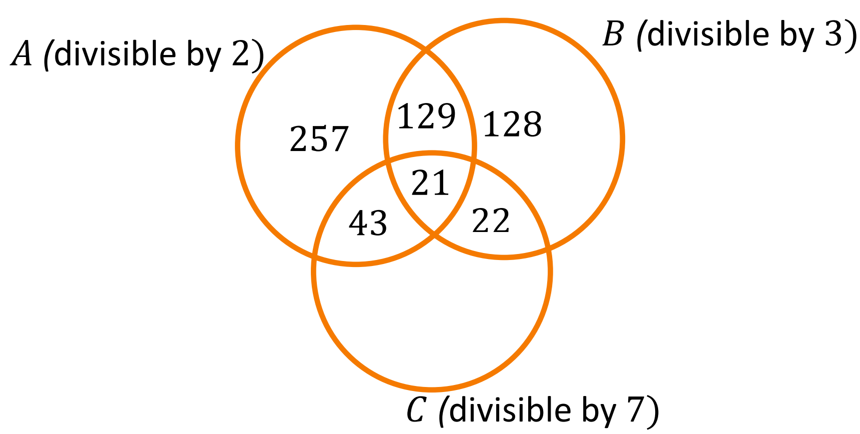
$996 = 102 + (n_{2\,\&\,3} - 1)6$

$\Rightarrow n_{2\,\&\,3} = 150$

Numbers divisible by 2 & 7 is:

$994 = 112 + (n_{2\,\&\,7} - 1)14$

$\Rightarrow n_{2\,\&\,7} = 64$



Numbers divisible by 3 & 7 is:
$987 = 105 + (n_{3\ \&\ 7} - 1)21$
$\Rightarrow n_{3\ \&\ 7} = 43$
Numbers divisible by 2, 3 & 7 is:
$966 = 126 + \left(n_{2,3\ \&\ 7} - 1\right)42$
$\Rightarrow n_{2,3\ \&\ 7} = 21$
Only $A = 450 - (43 + 150) = 257$
Only $B = 300 - (22 + 150) = 128$
Total numbers which are divisible by 2 or 3 but not divisible by $7 = 257 + 129 + 128 = 514$