MADE EASY

# GATE 2023

## ELECTRONICS ENGINEERING

## Memory based Questions & Solutions

Exam held on 05th Feb, 2023 Afternoon Session

**Corporate Office:** 44-A/1, Kalu Sarai, New Delhi-110016
MADE EASY Centres : Delhi , Hyderabad , Jaipur, Bhopal, Lucknow, Bhubaneswar, Pune, Kolkata

9021300500

www.madeeasy.in

## SECTION - A GENERAL APTITUDE

Q.1 What is the smallest number with distinct digits whose digit will add upto 45?

(a) 99999 (b) 123456789

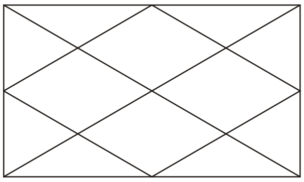
(c) 123457869 (d) 123555789

Ans. (b)

The digits should be distinct and smallest number is 123456789.

End of Solution

Q.2 How many rectangles in the figure?



(a) 12 (b) 10

(c) 9 (d) 8

Ans. (b)

Number of rectangles = 10 as square is also called as rectangle.

End of Solution

Q.3 In the class of 100 students

(i) there are 30 students who neither like romantic movies nor comedy movies.

(ii) the number of students who like romantic movies is twice the number of student who like comedy movies, and

(iii) the number of students who like both romantic and comedy movies is 20.

How many students in the class with like romantic movies?

(a) 20 (b) 30

(c) 60 (d) 40

Ans. (c)

Let students who like Romantic Movies = $R$.

Students who like Comedy Movies = $C$.

Given $R = 2C$

Also, 30 students do not like Romantic and Comedy Movies both.

$\therefore$ $100 - 30 = 70 = n(R \cap C)$

and $n(C \cap R) = 20$

$$n(R \cap C) = n(R) + n(C) - n(C \cap R)$$

$$70 = 2C + C - 20$$

$$3C = 90$$

$$C = 30$$

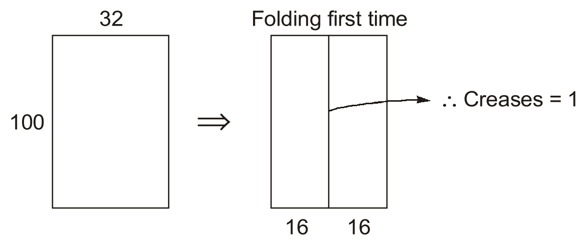
$$R = 2C = 60$$

End of Solution

**Q.4** A 100 cm × 32 cm rectangular sheet is folded 5 times. Each time the sheet is folded, the long Edge aligns with its opposite side. Eventually, the folded sheet is a rectangular of dimension 100 cm × 1 cm. The total number of creases visible when the sheet is unfolded is ____.

(a) 32 (b) 63
(c) 31 (d) 5

**Ans.** (c)




$\therefore$ After folding 5 times $= 1 + 2^1 + 2^2 + 2^3 + 2^4$

$= 1 + 2 + 4 + 8 + 16 = 31$

*End of Solution*

**Q.5** Courts : ______ : : Parliament : Legislature.

(a) Governmental (b) Legal
(c) Judiciary (d) Executive

**Ans.** (c)

*End of Solution*

**Q.6** "I cannot support this proposal. My ______ will not permit it".

(a) Consent (b) Conscience
(c) Consicious (d) Consensus

**Ans.** (b)

*End of Solution*

**Q.7** When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet. [Excerpt from the truth about stories by T.King]

Which option can be inferred from the given passage above?

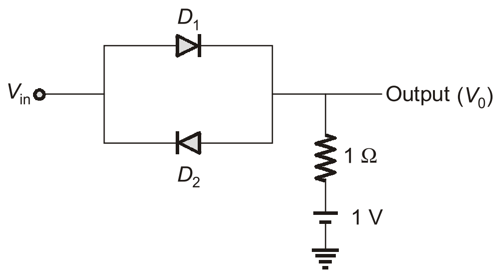
(a) It is an adult's memory of what he or she liked as a child.
(b) The child in the passage read stories about interplanetary travels only in parts.
(c) It is child's description of what he or she likes.
(d) It teaches us that stories are good for children.

**Ans.** (a)

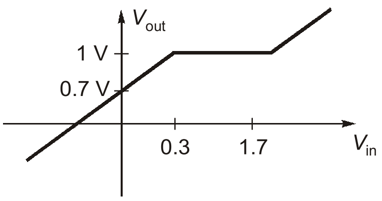
*End of Solution*

## SECTION - B TECHNICAL

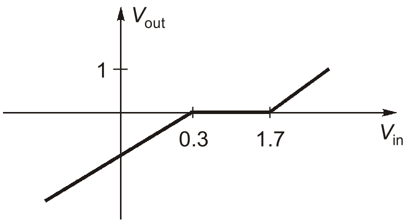
**Q.8** If the cut-in voltage of the diode is 0.7 V, then the transfer characteristics of the below circuit is



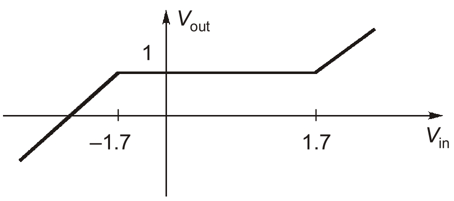

(a)



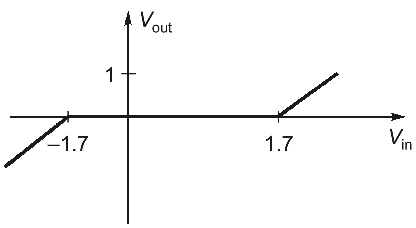

(b)




(c)




(d)




**Ans.** (a)

Case I:

$$V_\gamma = 0.7\text{V}$$

For the +ve half cycle if input $V_{in}$,

$$D_1 \rightarrow \text{ON and } D_2 \rightarrow \text{OFF}$$

For diode $D_1$: $V_{in} - 1V > 0.7$

$$V_{in} > 1.7\text{V}$$

$$V_0 = V_{in} - 0.7$$

**Case II:**

For the +ve half cycle if input $V_{in}$,

$$D_1 \rightarrow \text{OFF and } D_2 \rightarrow \text{ON}$$

For diode $D_2$: $1 - V_{in} > 0.7$

$$V_{in} < 0.3\text{V}$$

$$V_0 = V_{in} + 0.7$$

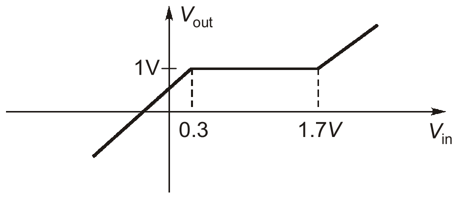
**Case III:**

$$0.3V < V_{in} < 1.7V$$

$$D_1 \rightarrow \text{OFF and } D_2 \rightarrow \text{OFF}$$

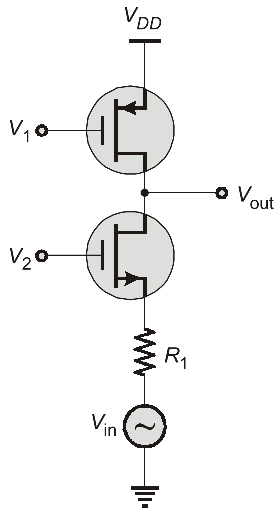
$$V_0 = 1\text{V}$$

Transfer characteristics,



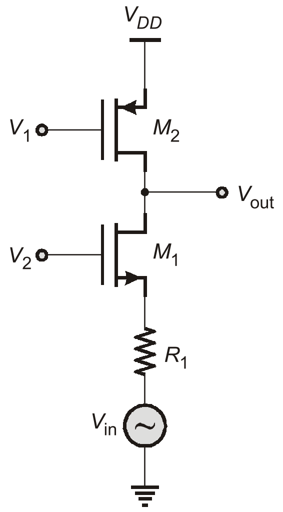

End of Solution

**Q.9** In the circuit shown below, $V_1$ and $V_2$ are bias voltages. Based on input and output impedances the circuit behaves as a




(a) current controlled current source
(b) current controlled voltage source
(c) voltage controlled current source
(d) voltage controlled voltage source

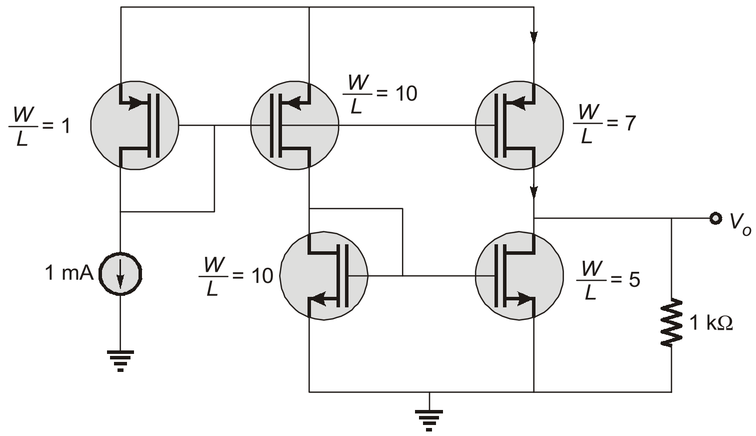
**Ans.** (a)




Here from circuit,
$M_1$ is common-gate amplifier and $M_2$ behaves as an active load.
By using properties of common gate ($CG$) amplifier,
Input impedance ($R_i$) is low
Output impedance ($R_0$) is high
So, it is a current amplifier.
Current amplifier is a current controlled current source.

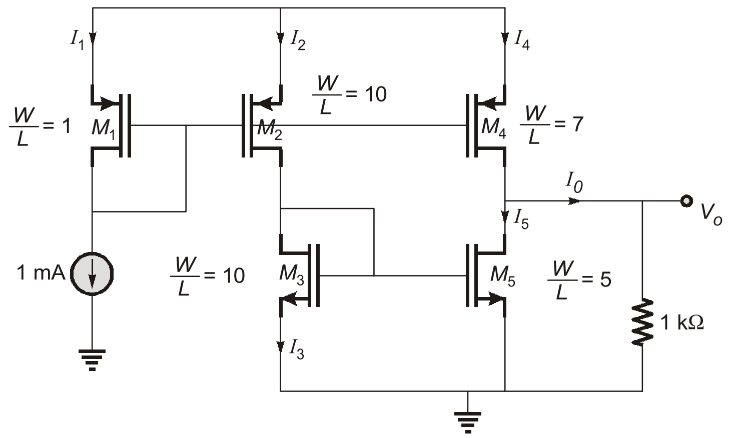
*End of Solution*

**Q.10** Find output voltage ($V_o$).




(a) 1 V
(b) 2 V
(c) 3 V
(d) 4 V

Ans. (b)




We know,

$$I_D \propto \left(\frac{W}{L}\right)$$

$$I_1 = 1 \text{ mA}$$

$$I_2 = \frac{10}{1} \times 1 = 10 \text{ mA}$$

$$I_3 = 10 \text{ mA}$$

$$I_4 = 7 \text{ mA}$$

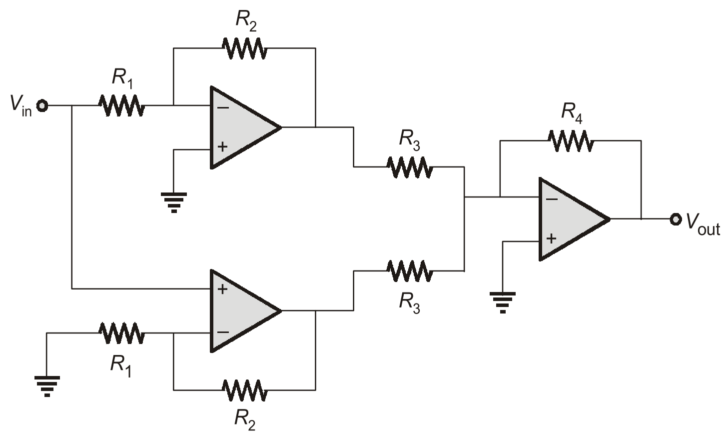
$$I_5 = 5 \text{ mA}$$

$$I_0 = I_4 - I_5 = 7 - 5 = 2 \text{ mA}$$

$$V_0 = 2 \times 1 = 2\text{V}$$

End of Solution

Q.11 For the op-amp circuit shown below, the gain $V_{out}/V_{in}$ will be



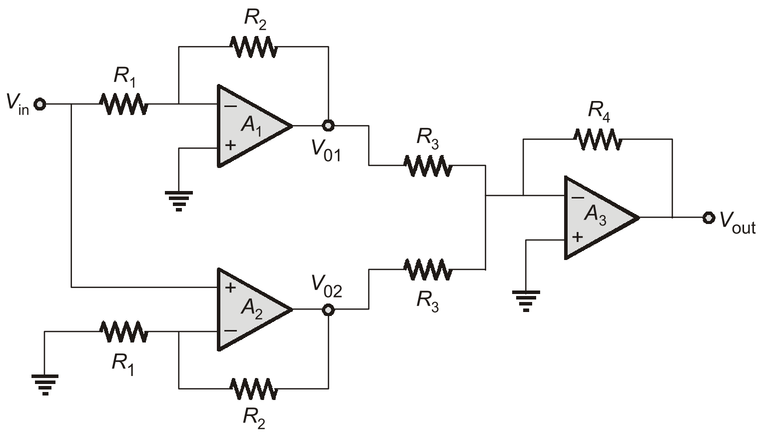

(a) $1+\frac{R_4}{R_3}$

(b) $\frac{R_4}{R_3}$

(c) $\frac{-R_4}{R_3}$

(d) $1-\frac{R_4}{R_3}$

Ans. (c)




Here, $A_1$ is an inverting amplifier and $A_2$ is a non-inverting amplifier.

$$V_{01} = \frac{-R_2}{R_1} V_{in}$$

$$V_{02} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

Also, $A_3$ is an inverting summing amplifier,

$$V_{out} = \frac{-R_4}{R_3} V_{01} - \frac{R_4}{R_3} V_{02}$$

$$= \frac{-R_4}{R_3}\left[\frac{R_2}{R_1} V_{in} + \left(1 + \frac{R_2}{R_1}\right) V_{in}\right]$$

$$V_{out} = \frac{-R_4}{R_3} V_{in}$$

$$\text{Gain, } \frac{V_{out}}{V_{in}} = \frac{-R_4}{R_3}$$

End of Solution

Q.12 For a cascaded common source amplifier having unity open loop gain, which of the following condition is satisfied for oscillation in closed system?

(a) gain greater than unity and phase shift greater than 180°
(b) gain greater than unity and phase shift less than 180°
(c) gain less than unity and phase shift greater than 180°
(d) gain less than unity and phase shift less than 180°

Ans. (a)

For oscillation,

Loop gain, $|A\beta| \geq 1$

Phase shift = 0° or 360°

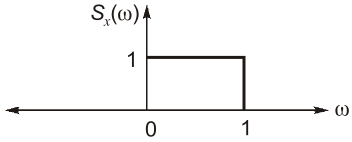
Hence, for common source amplifier gain must be greater than unity and phase shift must be greater than 180°.

End of Solution

**Q.13** For a real signal, which of the following is/are valid power spectral density/densities?

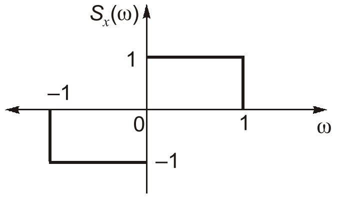
(a) $S_x(\omega) = \dfrac{2}{9+\omega^2}$

(b)




(c) $S_x(\omega) = e^{-\omega^2}\cos^2\omega$

(d)




**Ans.** **(a, c)**

(i) $S_x(\omega) \geq 0$

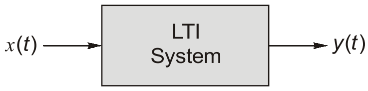
(ii) $S_x(\omega)$ is even function

Hence, options (a) and (c) are valid power spectral densities.

*End of Solution*

**Q.14** Let $X(t)$ be a white Gaussian noise with power spectral density $\frac{1}{2}$ W/Hz. If $X(t)$ is input to an LTI system with impulse response $e^{-t}u(t)$. The average power of the system output is _______ W.

**Ans.** **(0.25)**




$$h(t) = e^{-t}\,u(t)$$

**Given:** Input PSD

$$\Rightarrow \qquad S_X(f) = \frac{1}{2}\ \text{W/Hz}$$

We know output PSD,

$$S_Y(f) = S_X(f)\,|H(f)|^2$$

$$S_Y(f) = \frac{1}{2}|H(f)|^2$$

$$\text{Power }[y(t)] = \int_{-\infty}^{\infty} S_Y(f)df = \int_{-\infty}^{\infty} \frac{1}{2}|H(f)|^2\,df$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} h^2(t)dt = \frac{1}{2}\int_{0}^{\infty} e^{-2t}dt$$

$$= \frac{1}{2}\times\frac{1}{2} = \frac{1}{4} = 0.25\text{ W}$$

*End of Solution*

**Q.15** The signal-to-noise ratio (SNR) of an (ADC) with a full scale sinusoidal input is given to be 61.96 dB. The resolution of ADC is ______ bits.

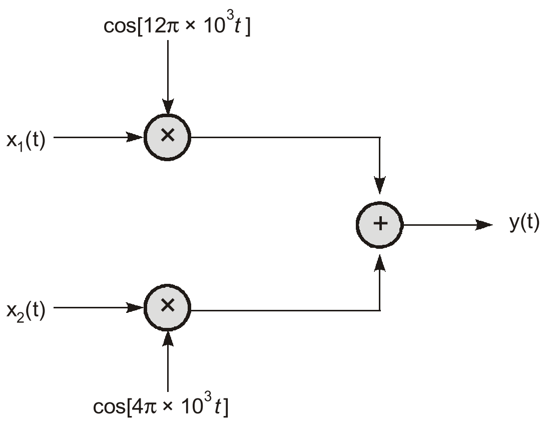
**Ans.** (10)

We know that for sinusoidal input, the signal to noise ratio (SNR) is given as,

$$\text{SNR} = 1.76 + 6.02\, n \text{ dB}$$
$$61.96 \text{ dB} = 1.76 + 6.02\, n \text{ dB}$$
$$6.02\, n = 61.96 - 1.76$$
$$\therefore \quad n = 10 \text{ bits/sample}$$

End of Solution

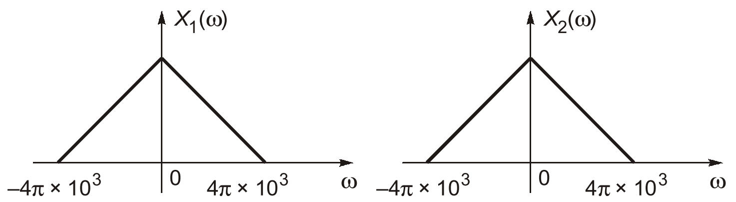
**Q.16** Let $x_1(t)$ and $x_2(t)$ be two band limited signals having bandwidth $B = 4\pi \times 10^3$ rad/s each. In the figure below, the Nyquist sampling frequency in rad/s, required to sample $y(t)$ is



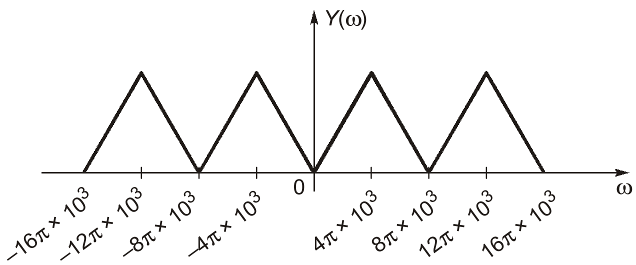

(a) $8\pi \times 10^3$
(b) $20\pi \times 10^3$
(c) $40\pi \times 10^3$
(d) $32\pi \times 10^3$

**Ans.** (d)

Given that, $x_1(t)$ and $x_2(t)$ are two bandlimited signals having bandwidth $B = 4\pi \times 10^3$ rad/sec.




and $$y(t) = x_1(t)\cos(12\pi \times 10^3 t) + x_2(t)\cos(4\pi \times 10^3 t)$$



So, Nyquist rate $= 2\omega_{max}$

$= 2[16\pi \times 10^3]$

$= 32\pi \times 10^3$ rad/sec

Hence, option (d) is correct.

*End of Solution*

**Q.17** If $x(t)$ is an FM modulated signal $x(t) = A_c \cos\left[\omega_c(t) + K_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\right]$. It is passed through non-linear system having $y(t) = 2x(t) + 5(x(t))^2$ and $x(t)$ has $B.W \rightarrow B_T$.
Find minimum value of $\omega_c$ to detect $x(t)$ from $y(t)$ having bandwidth of $m(t)$ is $\omega$.

(a) $B_T + \omega$

(b) $\frac{5}{2}B_T$

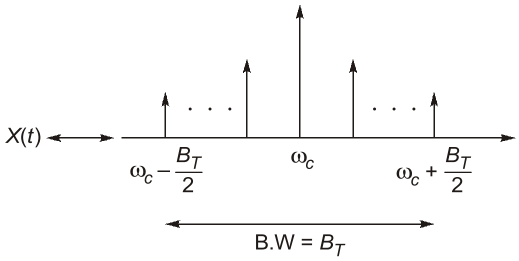
(c) $\frac{3}{2}B_T$

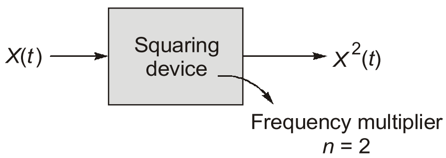
(d) $4B_T$

**Ans.** **(c)**

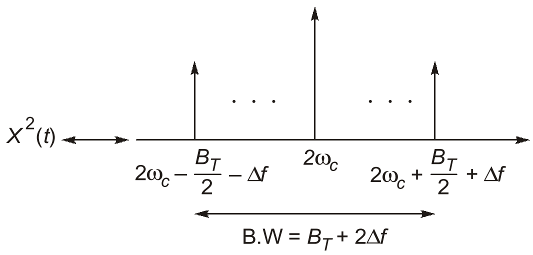
$$X(t) = A_c \cos\left[\omega_c t + K_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\right]$$

B.W. $[X(t)] \rightarrow BT = 2[\Delta f + \omega]$

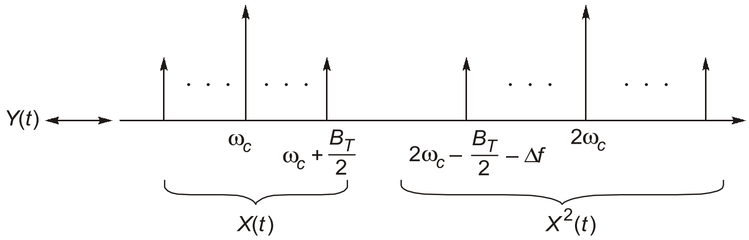



$$X^2(t) \rightarrow \begin{rmatrix} \Delta f' = 2\Delta f \\ \omega_c' = 2\omega_c \end{rmatrix}$$



$$\text{BW}[X^2(t)] = 2[\Delta f' + \omega]$$
$$= 2[2\Delta f + \omega] = BT + 2\Delta f$$
$$Y(t) = 2X(t) + 5X^2(t)$$




To recover $X(t) \rightarrow$ $$2\omega_C - \frac{B_T}{2} - \Delta f > \omega_c + \frac{B_T}{2}$$

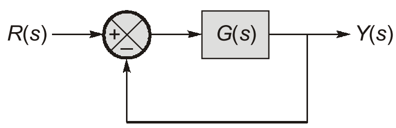
$$\omega_c > \Delta f + BT$$
$$\omega_c > \Delta f + 2\Delta f + 2\omega$$
$$\omega_c > 3\Delta f + 2\omega$$
$$\omega_c > \frac{3}{2}\{2[\Delta f + \omega]\} - \omega$$
$$\omega_c > \frac{3}{2}B_T - \omega$$

Compared to FM BW, message BW is very small. So, that it can be ignored.

$$\omega_C > \frac{3}{2}B_T$$
$$[\omega_c]_{min} = \frac{3}{2}B_T$$

**End of Solution**

Q.18 The open loop transfer function of a unity negative feedback system is $G(s) = \dfrac{K}{s(1+sT_1)(1+sT_1)}$, where $K$, $T_1$ and $T_2$ are positive constants. The phase cross-over frequency in rad/s, is




(a) $\dfrac{1}{T_2\sqrt{T_1}}$

(b) $\dfrac{1}{T_1 T_2}$

(c) $\dfrac{1}{T_1\sqrt{T_2}}$

(d) $\dfrac{1}{\sqrt{T_1 T_2}}$

**Ans.** (d)

We know phase crossover frequency is that frequency at which phase of the open loop transfer function is –180°.

$$\therefore \quad G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

$$G(j\omega) = \frac{K}{(j\omega)(1+j\omega T_1)(1+j\omega T_2)}$$

$$\text{Phase of } G(j\omega) = \phi = -90 - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

$\therefore$ At $\omega = \omega_{pc}$, $\phi = -180$

$$\therefore \quad -180 = -90 - \tan^{-1}(\omega_{pc}T_1) - \tan^{-1}(\omega_{pc}T_2)$$

$$90 = \tan^{-1}(\omega_{pc}T_1) + \tan^{-1}(\omega_{pc}T_2)$$

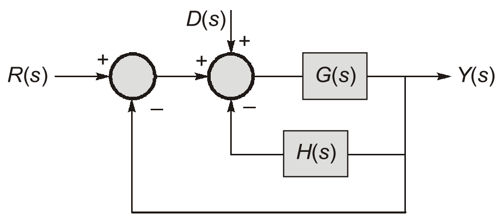
$$\tan^{-1}\left(\frac{\omega_{pc}T_1 + \omega_{pc}T_2}{1-\omega_{pc}^2 T_1 T_2}\right) = 90$$

$$1-\omega_{pc}^2 T_1 T_2 = 0$$

$$\omega_{pc} = \frac{1}{\sqrt{T_1 T_2}}$$

*End of Solution*

**Q.19** Given, $Y(s) = G_1(s)R(s) + G_2(s)D(s)$ find $G_1(s)$ and $G_2(s)$.



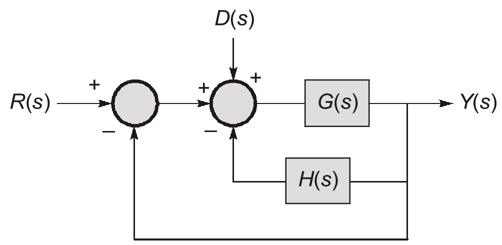

(a) $G(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$; $G_2(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

(b) $G_1(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$; $G_2(s) = \dfrac{G(s)}{1+G(s)+H(s)}$

(c) $G_1(s) = \dfrac{G(s)}{1+G(s)+H(s)}$; $G_2(s) = \dfrac{G(s)}{1+G(s)+G(s)H(s)}$

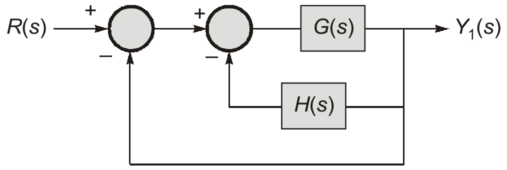
(d) $G(s) = \dfrac{G(s)H(s)}{1+G(s)+H(s)}$; $G_2(s) = \dfrac{G(s)H(s)}{1+G(s)+G(s)H(s)}$

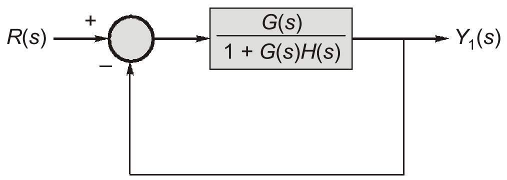
Ans. (a)




$$Y(s) = \underbrace{G_1(s)R(s)}_{Y_1(s)} + \underbrace{G_2(s)D(s)}_{Y_2(s)}$$

Considering first $R(s)$ only, then $Y(s)$ is $Y_1(s)$



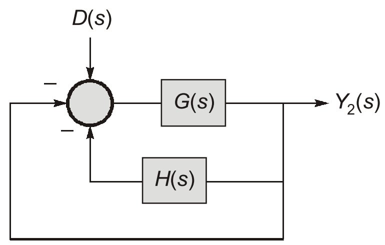

$$\frac{Y_1(s)}{R(s)} = \frac{\dfrac{G(s)}{1+G(s)H(s)}}{1+\dfrac{G(s)}{1+G(s)H(s)}}$$

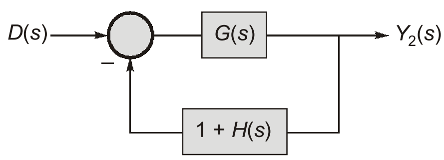
$$\frac{Y_1(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)+G(s)}$$

$$Y_1(s) = \left[\frac{G(s)}{1+G(s)+G(s)H(s)}\right]R(s)$$

$$\therefore \quad G_1(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$$

Now considering $D(s)$ only, then $Y(s)$ is $Y_2(s)$



$$\frac{Y_2(s)}{D(s)} = \frac{G(s)}{1+G(s)[1+H(s)]}$$

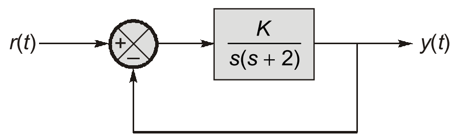
$$Y_2(s) = \left[\frac{G(s)}{1+G(s)+G(s)H(s)}\right]D(s)$$

$$\therefore \quad G_2(s) = \frac{G(s)}{1+G(s)+G(s)H(s)}$$

Hence, $G_1(s)$ and $G_2(s)$ both are equal.

*End of Solution*

**Q.20** For closed loop system shown below, the input is $r(t) = \alpha t u(t)$, the steady state error will be




(a) $\frac{\alpha}{4K}$

(b) $\frac{\alpha}{K}$

(c) $\frac{\alpha}{2K}$

(d) $\frac{2\alpha}{K}$

**Ans. (d)**

Given, input is $r(t) = \alpha t u(t)$

$$R(s) = \frac{\alpha}{s^2}$$

From the figure,

$$G(s)H(s) = \frac{K}{s(s+2)}$$

Now steady state error for Ramp input is

$$e_{ss} = \frac{\alpha}{K_v}, \text{ where } \alpha \text{ is the magnitude of Ramp input}$$

$$K_v = \lim_{s\to 0}[sG(s)H(s)]$$

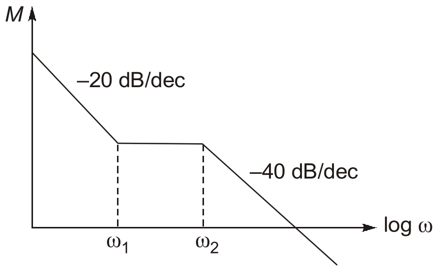
$$K_v = \lim_{s\to 0}\left[\frac{s\times K}{s(s+2)}\right] = \frac{K}{2}$$

$$\therefore \quad e_{ss} = \frac{\alpha\times 2}{K}$$

$$e_{ss} = \frac{2\alpha}{K}$$

*End of Solution*

Q.21 For bode plot shown below. If open loop transfer function $G(s) = \frac{K(s+z)^a}{s^b(s+p)^c}$, then find the value of $a + b + c$ = ________.




Ans. (4)

From the Bode magnitude plot, it is clear that there is one pole at origin,

$\therefore \quad b = 1$

and at frequency $\omega_1$, system has a zero

$\therefore \quad a = 1$

and at frequency $\omega_2$, system have two poles

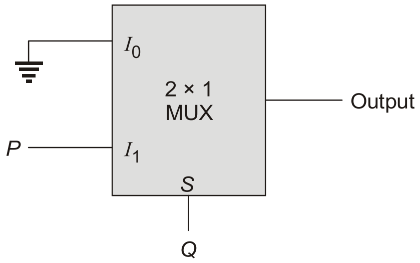
$\therefore \quad c = 2$

$\therefore \quad a + b + c = 1 + 1 + 2$

$a + b + c = 4$

End of Solution

Q.22 The output of the 2 × 1 MUX shown below is



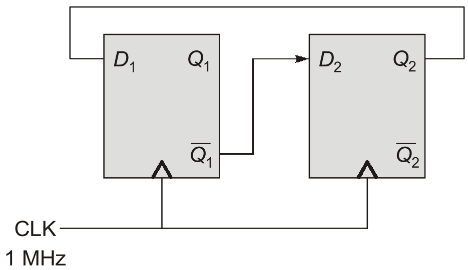

(a) $PQ$
(b) $\bar{P}Q$
(c) $P\bar{Q}$
(d) $\bar{P}\bar{Q}$

Ans. (a)

$$\text{Output} = \bar{Q}\cdot I_0 + Q\cdot I_1$$
$$= \bar{Q}\cdot 0 + Q\cdot P$$
$$= PQ$$

End of Solution

**Q.23** For given connection, if initial state of $Q_1 = 1$ and $Q_2 = 0$,




The output frequency in kHz is _______.

**Ans.** (250)

| Clk | $D_1 = Q_2$ | $D_2 = \overline{Q_1}$ | $Q_1$ | $Q_2$ |
|---|---|---|---|---|
| Initial | | | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 1 | 0 | 1 | 0 |

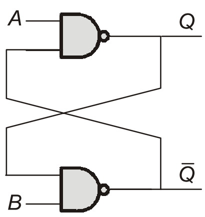
Therefore, the given counter is having MOD-4

$\therefore$ Output frequency $(f_0) = \dfrac{f_i}{4}$

$= \dfrac{1000}{4} = 250$ kHz

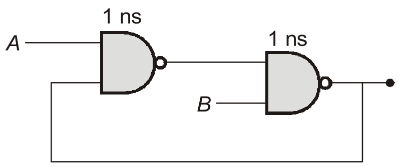
**End of Solution**

**Q.24** For the circuit shown below, the propagation delay of each NAND gate is 1 ns. The critical path delay in *ns* is ______.




**Ans.** (2)

The given circuit can be drawn as;

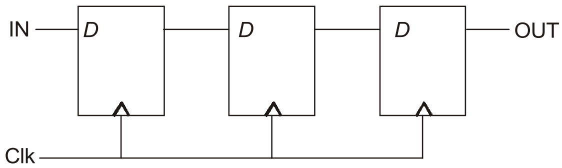



$\therefore$ The critical path delay $= 1 \text{ ns} + 1 \text{ ns}$

$= 2$ ns

**End of Solution**

Q.25 For the circuit shown below




If the clock frequency is 1 GHz then throughput and latency at output will be

(a) 1000 Mbps, 1 ns
(b) 1000 Mbps, 3 ns
(c) 333.33 Mbps, 1 ns
(d) 333.33 Mbps, 3 ns

Ans. (b)

The given circuit is a type of SISO.

$\therefore$ Latency $= n \times T_{clk}$ ..... $n$ = number of flip flops

$= 3 \times 1$ ...... $T_{clk} = \dfrac{1}{f_{clk}} = 1\text{ns}$

$= 3$ ns

Now, Throughput = Number of bits/sec

$\because$ 1 bit = 1 nsec

$\therefore$ Throughput $= 10^9$ bits/sec

$= 1000$ Mbps

End of Solution

Q.26 In a semiconductor, Fermi level lies inside the conduction band then the semiconductor is known as

(a) degenerate *n*-type
(b) non degenerate *n*-type
(c) degenerate *p*-type
(d) non degenerate *p*-type

Ans. (a)

As the Fermi lies inside the conduction band hence it is degenerate *n*-type semiconductor.

End of Solution

Q.27 In an extrinsic semiconductor, the hole concentration is $1.5 \times n_i$ and the intrinsic carrier concentration is $1 \times 10^{10}$ cm$^{-3}$. The ratio of electron to hole mobility for equal hole electron drift current is given as

Ans. (2.25)

Given, intrinsic carrier concentration $n_i = 1 \times 10^{10}$ cm$^{-3}$

Hole concentration, $p = 1.5 \times n_i$

$p = 1.5 \times 10^{10}$ cm$^{-3}$

Given, electron and hole current are equal

$$I_{p\text{ drift}} = I_{n\text{ drift}}$$

$$pq\,\mu_p EA = nq\,\mu_n EA$$

$$1.5 \times 10^{10}\,\mu_p = n\,\mu_n \quad \text{...(i)}$$

But according to mass action law,

$$np = n_i^2$$

$\therefore \quad n = \frac{n_i}{1.5} = \frac{10^{10}}{1.5}\ \text{cm}^{-3}$

Put in equation (i)

$\therefore \quad 1.5 \times 10^{10}\ \mu_p = \frac{10^{10}}{1.5} \times \mu_n$

$$\frac{\mu_n}{\mu_p} = 2.25$$

End of Solution

**Q.28** In a semiconductor device, the fermi-energy level is 0.35 eV above the valence band energy. The effective density of states in the valence band at $T = 300$ K is $1 \times 10^{19}\ \text{cm}^{-3}$. The thermal equilibrium hole concentration is silicon at 400 K ______ $\times 10^{13}\ \text{cm}^{-3}$. Given kT at 300 K is 0.026 eV.

**Ans.** **(63.36)**

Given, $E_F - E_V = 0.35$ eV [Considering it is given at 400 K]

Also, $V_{T_1} = KT_1 = 0.026$ eV at $T_1 = 300$ K

$\therefore \quad \frac{V_{T_1}}{V_{T_2}} = \frac{T_1}{T_2} \Rightarrow V_{T_2} = \frac{T_2}{T_1} \times V_{T_1}$

$\therefore \quad V_{T_2} = \frac{400}{300} \times 0.026$

$$V_{T_2} = 0.03466 \text{ eV at } T_2 = 400 \text{ K}$$

Now, $N_v = 1 \times 10^{19}/\text{cm}^3$ at $T_1 = 300$ K

$$N_v \propto T^{3/2}$$

$$\frac{N_{V_2}}{N_{V_1}} = \left(\frac{T_2}{T_1}\right)^{3/2}$$

$$N_{V_2} = \left(\frac{T_2}{T_1}\right)^{3/2} NV_1 \qquad (\because\ T_2 = 400 \text{ K})$$

$$= \left(\frac{400}{300}\right)^{3/2} N_{V_1}$$

$$N_{V_2} = 1.5396 \times 10^{19}/\text{cm}^3$$

Now, hole concentration at 400 K is given as

$$p = N_v\, e^{-(E_F - E_V)/kT_2}$$

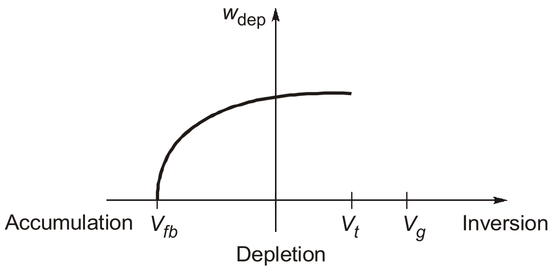
$$= 1.5396 \times 10^{19} \times e^{-0.35\,\text{eV}/0.03466\,\text{eV}}$$

$$p = 63.36 \times 10^{13}\ \text{cm}^{-3}$$

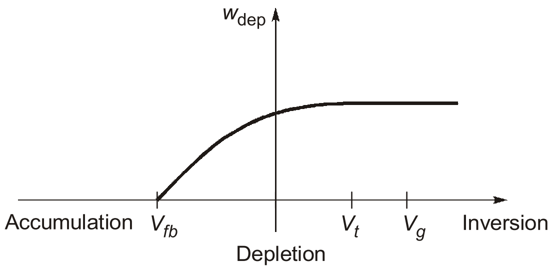
End of Solution

**Q.29** For a MOS capacitor. $V_{fb}$ and $V_t$ are the flat band voltage and the threshold voltage, respectively. The variation of the depletion width ($W_{dep}$) for varying gate voltage ($v_g$) is best represented by

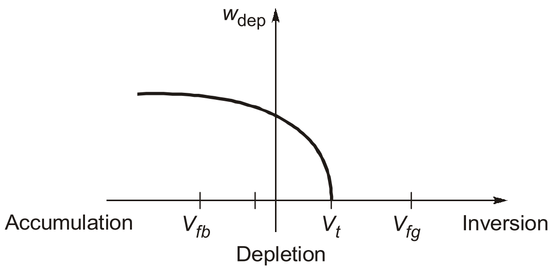
(a)



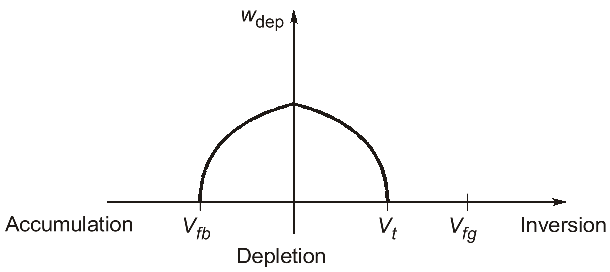

(b)




(c)




(d)




**Ans.** **(b)**

∵ We know $V_G < V_{FB}$ then accumulation mode.

∴ In accumulation mode $\boxed{W_d = 0}$ because there is no depletion charge.

Now, $V_{FB} < V_G < V_T \Rightarrow$ then depletion and inversion mode.

∴ Depletion width is available.

∴ $V_G > V_T \Rightarrow$ Strong inversion.

∴ Depletion width $W_d \Rightarrow$ Constant.

And $W_d = \sqrt{\dfrac{2\epsilon|\phi_s|}{qN_s}}$ and $|\phi_s| \propto V_G$

But after strong inversion, $W_d$ remains constant.

$\therefore$ Correction option is (b).

**End of Solution**

**Q.30** In intrinsic semiconductor at $T$ = 0 K, which condition is satisfied?

(a) Both valence band of conduction are filled with electron.

(b) Conduction band filled with electron and valence band empty with electron.

(c) Valence band is completely filled with electron and conduction band is completely empty.

(d) Valence band are filled with holes and conduction are filled with electrons.

**Ans. (c)**

Intrinsic semiconductor at T = 0 K behaves as an insulator.

Hence, valence band is completely filled with electron and conduction band is completely empty.

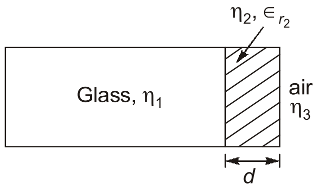
**End of Solution**

**Q.31** A transparent dielectric coating is applied to glass ($\epsilon_r$ = 4, $\mu_r$ = 1) to eliminate the reflection of red light ($\lambda$ = 0.75 $\mu$m). The minimum thickness of the dielectric coating in $\mu$m can be used is (round off 2 decimal places)

**Ans. (0.133)**

For no reflection, impedance must be matched.

Hence, $\eta_2$ acts like a quarter wave impedance transformer.




So,

(i) $\eta_2 = \sqrt{\eta_1 \cdot \eta_3} \Rightarrow \epsilon_{r_2} = \sqrt{\epsilon_{r_1} \cdot \epsilon_{r_3}} \Rightarrow \epsilon_{r_2} = 2$

(ii) For impedance matching,

$$d = (2n+1)\frac{\lambda}{4};\ n = 0, 1, 2...$$

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{\lambda_0}{\sqrt{\epsilon_{r_2}}}$$

Here, $$\lambda = \frac{0.75\times10^{-6}}{\sqrt{2}} = 0.53\times10^{-6}$$

Hence, for minimum distance, $n = 0$

So, $$d = \frac{\lambda}{4} = \frac{0.53 \times 10^{-6}}{4} = 0.133\ \mu\text{m}$$

End of Solution

**Q.32** Consider a narrow band signal, propagating in a lossless dielectric medium ($\varepsilon_r = 4$, $\mu_r = 1$), with phase velocity $V_p$ and group velocity $V_g$. Which of the following statement is true? ($C$ is the velocity of light in vacuum)

(a) $V_p > C$, $V_g < C$
(b) $V_p < C$, $V_g > C$
(c) $V_p > C$, $V_g > C$
(d) $V_p < C$, $V_g < C$

**Ans.** **(d)**

- Phase velocity, $$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \in_0}} \cdot \frac{1}{\sqrt{\mu_r \in_r}} = \frac{C}{\sqrt{\mu_r \in_r}}$$

  $\therefore$ $V_p < C$

- Group velocity, $$V_g = \frac{d\omega}{d\beta} = \frac{V_p}{1 - \frac{\omega}{V_p}\frac{dV_p}{d\omega}}$$

Here, $V_p \neq f(\omega)$

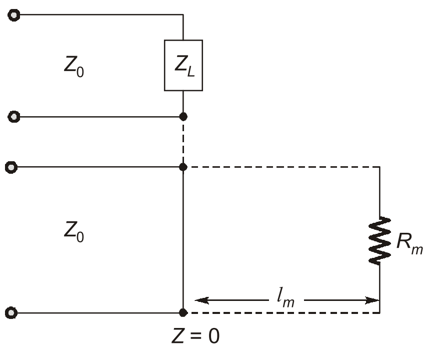
$\therefore$ $V_g = V_p < C$

Hence, $V_p < C$

$V_g < C$

End of Solution

**Q.33** The standing wave ratio on a 50 Ω lossless transmission line transmitted in an unknown load impedance is found to be 2.0. The distance between successive voltage minima is 30 cm and the first minimum is located at 10 cm from the load. $Z_L$ can be replaced by an equivalent, length $l_m$ and terminating resistance $R_m$ of the same line. The value of $R_m$ and $l_m$, respectively are




(a) $R_m = 100\ \Omega$, $L_m = 5$ cm
(b) $R_m = 25\ \Omega$, $L_m = 20$ cm
(c) $R_m = 25\ \Omega$, $L_m = 5$ cm
(d) $R_m = 100\ \Omega$, $L_m = 2$ cm

Ans. (a, b)

Given $S = 2$, $Z_{min} = 10$ cm, $Z_0 = 50\ \Omega$

As we know that, $|\Gamma| = \frac{S-1}{S+1} = \frac{2-1}{2+1} = \frac{1}{3}$

Now, distance between successive voltage minima = 30 cm

$$\Rightarrow \quad \frac{\lambda}{2} = 30 \text{ cm}$$

$$\Rightarrow \quad \lambda = 60 \text{ cm}$$

Also, for minima,

$$2\beta Z_{min} = (2n + 1)\pi + \theta_\Gamma$$

At $n = 0$, 1st minima, $Z_{min} = 10$ cm

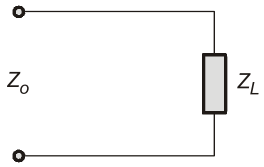
$$\frac{4\pi}{\lambda} Z_{min} = \pi + \theta_\Gamma$$

$$\Rightarrow \quad \frac{4\pi}{60} * 10 = \pi + \theta_\Gamma$$

$$\Rightarrow \quad \frac{2\pi}{3} - \pi = \theta_\Gamma$$

$$\Rightarrow \quad \theta_\Gamma = \frac{-\pi}{3} \qquad \therefore \Gamma = \frac{1}{3}\angle -60^\circ$$

Now, $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

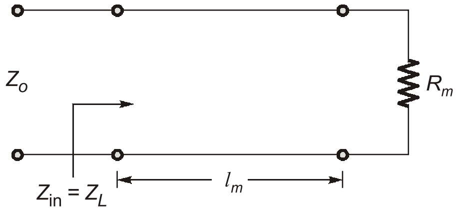



$$\Rightarrow \quad Z_L = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right]$$

$$\Rightarrow \quad Z_L = 50\left[\frac{1+0.33e^{-j\frac{\pi}{3}}}{1-0.33e^{-j\frac{\pi}{3}}}\right]$$

$$\Rightarrow \quad Z_L = 67.97\ \angle -32.67^\circ$$

Now, $Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$




$$\Rightarrow \quad Z_{in} = 50\left[\frac{R_m + j50\tan\beta l_m}{50 + jR_m \tan\beta l_m}\right]$$

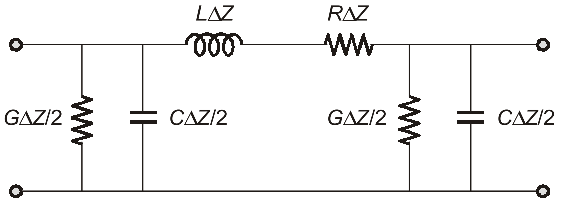
Here, $Z_{in} = Z_L = 67.97\ \angle -32.67^\circ$

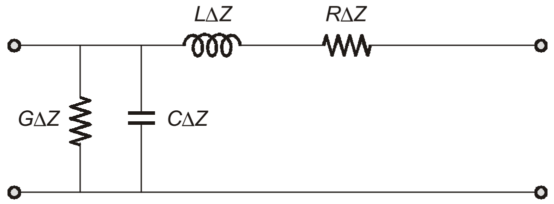
Going through options,

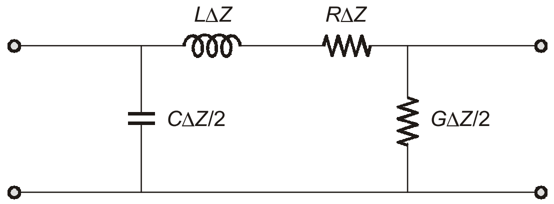
$\left.\begin{array}{l} R_m = 100\ \Omega \text{ and } L_m = 5 \text{ cm} \\ \text{and } R_m = 25\ \Omega \text{ and } L_m = 20 \text{ cm} \end{array}\right\}$ satisfy this identity, hence option (a) and (b) are correct.

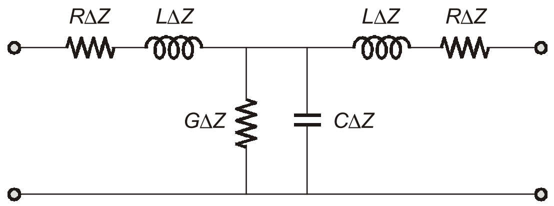
End of Solution

**Q.34** The following circuits representing an a lumped element equivalent of and infinitesimal section of a transmission line is/are

(a)



(b)



(c)



(d)



**Ans.** (a, b)

*End of Solution*

**Q.35** The electric field of a plane electromagnetic wave is

$$\vec{E} = \hat{a}_x\, C_{1x} \cos(\omega t - \beta z) + \hat{a}_y C_{1y} \cos(\omega t - \beta z + \theta) \text{ V/m.}$$

Which of the following combination(s) will give rise to left handed elliptically polarized (LHEP) wave?

(a) $C_{1x} = 1,\ C_{1y} = 2,\ \theta = 3\pi/2$

(b) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = \pi/2$

(c) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = 3\pi/4$

(d) $C_{1x} = 1,\ C_{1y} = 1,\ \theta = \pi/4$

**Ans.** (b, c, d)

Given, $\vec{E} = \hat{a}_x C_{1x} \cos(\omega t - \beta z) + \hat{a}_y C_{1y} \cos(\omega t - \beta z + \theta)$

at $z = 0$

$$\vec{E} = C_{1x} \cos\omega t\ \hat{a}_x + C_{1y} \cos(\omega t + \theta)\hat{a}_y$$

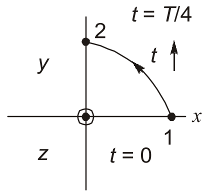
Going by options,

Option (a) $\vec{E} = \cos\omega t\,\hat{a}_x + 2\cos(\omega t + 3\pi/2)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = \hat{a}_x$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = 2\hat{a}_y$

$\Rightarrow$ Hence, it is RHEP.

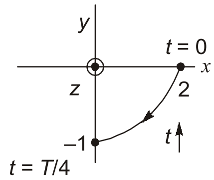



Option (b) $\vec{E} = 2\cos\omega t\,\hat{a}_x + \cos(\omega t + \pi/2)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = 2\hat{a}_x$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = -1\hat{a}_y$

$\Rightarrow$ Hence, it is LHEP.

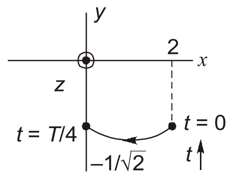



Option (c) $\vec{E} = 2\cos\omega t\,\hat{a}_x + \cos(\omega t + 3\pi/4)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = 2\hat{a}_x - \frac{1}{\sqrt{2}}\hat{a}_y$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = 0 - \frac{1}{\sqrt{2}}\hat{a}_y = \frac{-1}{\sqrt{2}}\hat{a}_y$

$\Rightarrow$ Hence, it is LHEP.



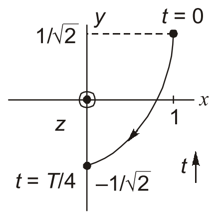

Option (d) $\vec{E} = \cos\omega t\,\hat{a}_x + \cos(\omega t + \pi/4)\hat{a}_y$

at $t = 0$, $\omega t = 0$, $\vec{E} = \hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y$

at $t = T/4$, $\omega t = \pi/2$, $\vec{E} = 0 - \frac{1}{\sqrt{2}}\hat{a}_y$

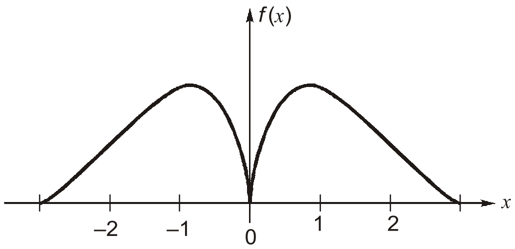
$\Rightarrow$ Hence, it is LHEP.

$\therefore$ Option (b), (c) and (d) are correct.




End of Solution

**Q.36** Which one of the following function represent the below graph




(a) $f(x) = x2^{-|x|}$

(b) $f(x) = |x|2^{-x}$

(c) $f(x) = x2^{-x}$

(d) $f(x) = x^2\,2^{-|x|}$

**Ans.** (d)

Since, the given function is an even function.

Option (d) is only represents the even function.

End of Solution

**Q.37** Let $x$ be an $n \times 1$ real column vector with length $l = \sqrt{x^T x}$. The trace of the matrix $P = xx^T$ is

(a) $l$

(b) $\frac{l^2}{4}$

(c) $\frac{l^2}{2}$

(d) $l^2$

**Ans.** (d)

Given, $l = \sqrt{x^T x},\ P = \left(xx^T\right)_{n \times n}$

Let
$$(x)_{n \times 1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ x_n \end{bmatrix}$$

$$l = \sqrt{x^T x} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots x_n^2}$$

$$P = xx^T$$

$$= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ x_n \end{bmatrix} \begin{bmatrix} x_1\, x_2\, x_3 \ldots x_n \end{bmatrix}$$

$$P = \begin{bmatrix} x_1^2 & & & & \\ & x_1^2 & & & \\ & & - & & \\ & & & - & \\ & & & & x_n^2 \end{bmatrix}$$

$$\text{Trace of } P = x_1^2 + x_2^2 + \ldots\ldots + x_n^2 = l^2$$

End of Solution

**Q.38** Let $V_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$ and $V_2 = \begin{bmatrix}2\\1\\3\end{bmatrix}$ be two vectors. The value of the coefficient $\alpha$ in the expression $V_1 = \alpha V_2 + e$, which minimizes the length of the vector $e$, is

(a) $\frac{-2}{7}$

(b) $\frac{2}{7}$

(c) $\frac{7}{2}$

(d) $\frac{-7}{2}$

**Ans.** (b)

$$e = V_1 - \alpha V_2$$

$$e = (i + 2k + 0k) - \alpha(2i + j + 3k)$$

$$\hat{e} = (1-2\alpha)\hat{i} + (2-\alpha)\hat{j} + (0-3\alpha)\hat{k}$$

$$|\hat{e}| = \sqrt{(1-2\alpha)^2 + (2-\alpha)^2 + (-3\alpha)^2}$$

$$|\hat{e}|^2 = 5 + 14\alpha^2 - 8\alpha \text{ to be minimum at } \frac{\partial e^2}{\partial \alpha} = 28\alpha - 8 = 0$$

$\therefore$ $\alpha = \frac{2}{7}$ stationary point

End of Solution

**Q.39** The rate of increase of a scalar field $f(x, y, z) = xyz$ in the direction $V = (2, 1, 2)$ at a point $(0, 2, 1)$ is

(a) 4

(b) $\frac{4}{3}$

(c) 2

(d) $\frac{2}{3}$

**Ans.** (b)

$$f(x, y, z) = xyz$$

$$\overline{\nabla f} = \hat{i}f_x + \hat{j}f_y + \hat{k}f_z$$

$$= \hat{i}(yz) + \hat{j}(xz) + \hat{k}(xy)$$

$$\overline{\nabla f}_{(0,2,1)} = \hat{i}(2) + 0\hat{j} + 0\hat{k}$$

Directional derivative,

$$D.D = \overline{\nabla f}\cdot\frac{\bar{a}}{|\bar{a}|}$$

$$= \left(2\hat{i} + 0\hat{j} + 0\hat{k}\right)\cdot\frac{\left(2\hat{i} + \hat{j} + 2\hat{k}\right)}{\sqrt{2^2 + 1^2 + 2^2}}$$

$$= \frac{4}{\sqrt{9}} = \frac{4}{3}$$

End of Solution

**Q.40** The value of the line integral $\int_P^Q (z^2dx + 3y^2dy + 2xz.dz)$ along the straight line joining the points $P(1, 1, 2)$ and $Q(2, 3, 1)$ is

(a) 20
(b) 24
(c) 29
(d) –5

**Ans.** (b)

$\int_P^Q z^2dx + 3y^2dy + 2xzdz$ along the line joining the points $P(1, 1, 2)$ and $Q(2, 3, 1)$ is

$$= \int_{P(1,2)}^{P(2,1)} z^2dx + 2xydz + \int_{y=1}^{3} 3y^2dy$$

$$= \left(xz^2\right)_{(1,2)}^{(2,1)} + \left(y^3\right)_1^3$$

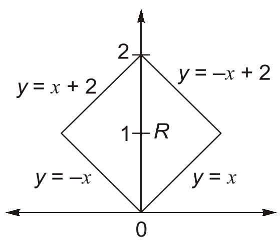
$$= (2 \times 1^2 - 1 \times 2^2) + (3^3 - 1^3)$$

$$= -2 + 26$$

$$= 24$$

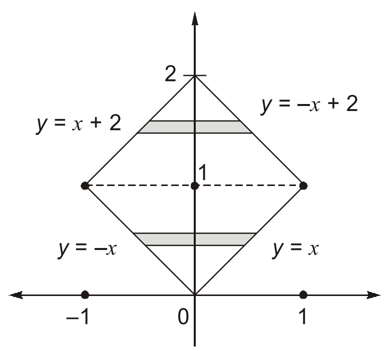
End of Solution

**Q.41** The value of the integral $\int_{S_R} xy\,dx\,dy$ over the region $R$ given in the figure is




**Ans.** (0)

$$I = \iint_R xy\,dx\,dy$$



$$= \int_{y=0}^{1}\int_{x=-y}^{y} xy\,dx\,dy + \int_{y=1}^{2}\int_{x=y-2}^{2-y} xy\,dx\,dy$$

$$= \int_{0}^{1} y\left(\frac{x^2}{2}\right)_{-y}^{y} dy + \int_{1}^{2} y\left(\frac{x^2}{2}\right)_{y-2}^{2-y} dy$$

$$= 0 + 0 = 0$$

End of Solution

**Q.42** Let $\omega^4 = 16j$. Which of the following cannot be value of $\omega$?

(a) $2e^{j\pi/8}$ (b) $2e^{j5\pi/8}$

(c) $2e^{2j\pi/8}$ (d) $2e^{j9\pi/8}$

**Ans.** **(c)**

$$\omega = (2)j^{1/4}$$

$$\omega = 2(0 + j)^{1/4}$$

$$\omega = 2\left[e^{j(2n+1)\pi/2}\right]^{1/4}$$

$$= 2\left[e^{j(2n + j\pi/8)}\right]$$

For $n = 0$, $\omega = e^{j\pi/8}$

For $n = 2$, $\omega = 2e^{5\pi j/8}$

For $n = 4$, $\omega = 2e^{9\pi j/8}$

End of Solution

**Q.43** Find the value of integral, $I = \oint_c \frac{z+2}{z^2+2z+2}dz$ for the given contour $c: \left|z+1-\frac{3}{2}j\right| = 1$

(a) $\pi(1 - j)$ (b) $-\pi(1 - j)$

(c) $-\pi(1 + j)$ (d) $\pi(1 + j)$

**Ans.** **(d)**

$$I = \oint_c \frac{z+2}{z^2+2z+2}dz;\; c = \left|z+1-\frac{3}{2}i\right| = 1$$

Poles are given $(z + 1)^2 + 1 = 0$

$$z + 1 = \pm\sqrt{-1}$$

$$z = -1 + j,\ -1 - j$$

where $-1 - i$ lies outside '$c$'

$z = (-1, 1)$ lies inside '$c$'.

by CRT

$$\oint_c f(z)dz = 2\pi i \text{ Res } (f(z),\ z = -1 + j)$$

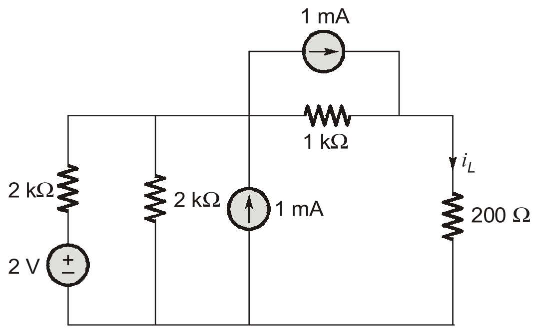
$$= 2\pi i\left(\frac{z+2}{2(z+1)}\right)_{z=-1+i}$$

$$= 2\pi i\left(\frac{-1+j+2}{2(-1+j+1)}\right)$$

$$= \pi(1+j)$$

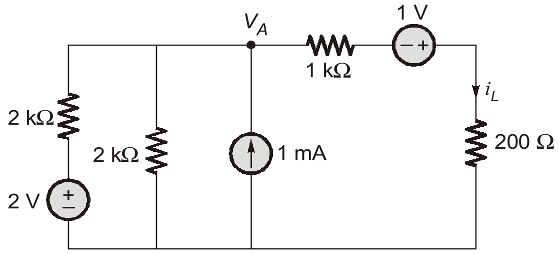
**End of Solution**

Q.44 Find $i_L$ (in mA) through 200 Ω resistor.




**Ans.** (1.36)

By applying source transformation,




Apply nodal at node $V_A$,

$$\frac{V_A-2}{2\,\text{k}\Omega}+\frac{V_A}{2\,\text{k}\Omega}+\frac{V_A+1}{1.2\,\text{k}\Omega} = 1\text{ mA}$$

$$V_A\left[\frac{1}{2}+\frac{1}{2}+\frac{1}{1.2}\right] = 1+\frac{2}{2}-\left(\frac{1}{1.2}\right)$$

$$V_A = 0.636\text{ V}$$

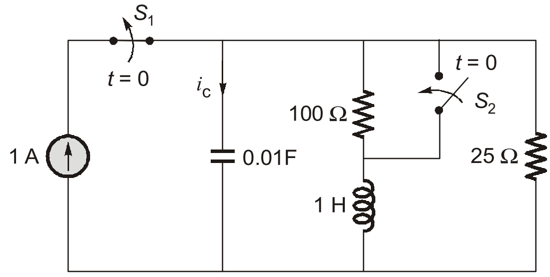
The current through 200 Ω resistor,

$$i_L = \frac{V_A+1}{1.2\,\text{k}\Omega}=\frac{0.636+1}{1.2}$$

$$i_L = 1.36\text{ mA}$$

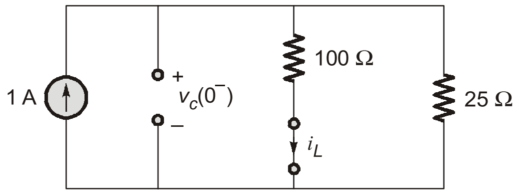
**End of Solution**

**Q.45** The switch $S_1$ was closed and $S_2$ was opened for a long time. At $t = 0$, switch $S_1$ is opened and $S_2$ is closed. The value of $i_C(0^+)$ ________ (in A).




**Ans.** (–1)

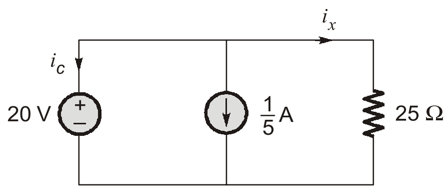
At $t = 0^-$ ; $S_1 \rightarrow$ closed, $S_2 \rightarrow$ opened




$$i_L(0^-) = \frac{1 \times 25}{100+25} = 0.2 \text{ A}$$

$$v_c(0^-) = \frac{1}{5} \times 100 = 20 \text{ V}$$

At $t = 0^+$ ; $S_1 \rightarrow$ opened, $S_2 \rightarrow$ closed



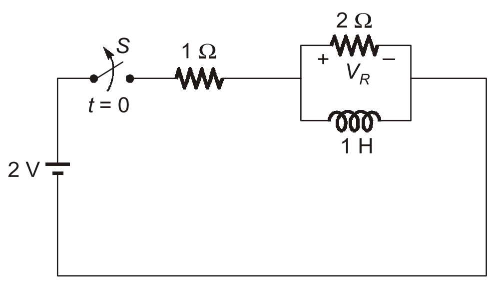

$$i_x = \frac{20}{25} = \frac{4}{5} \text{ A} = 0.8 \text{ A}$$

By KCL: $-i_c = i_x + 0.2 = 0.8 + 0.2$

$\Rightarrow$ $i_c = -1$ A

*End of Solution*

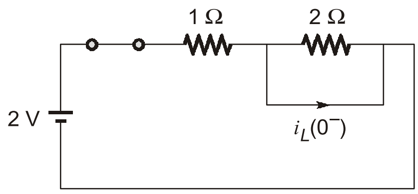
**Q.46** For the circuit shown below, the switch was opened at $t = 0$ ;




The magnitude of $V_R$ is ________ Volts.

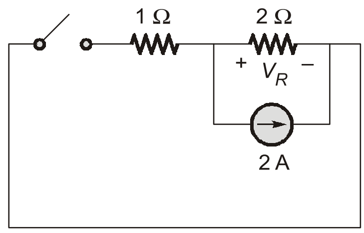
**Ans.** (4)

At $t = 0^-$




$$i_L(0^-) = \frac{2}{1} = 2\text{ A}$$

At $t = 0^+$




$$V_R = -2 \times 2 = -4$$

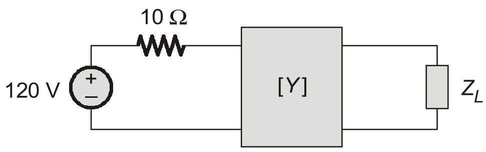
Magnitude of voltage $V_R$,

$$|V_R| = 4$$

End of Solution

---

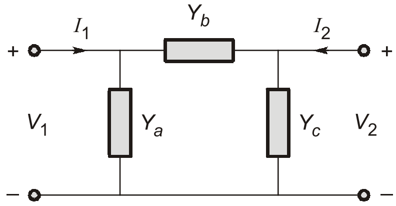
**Q.47** For the two-port network shown below, if $Y = \frac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & \frac{4}{3} \end{bmatrix} S$, then the value of $Z_L$ for maximum power transfer to the load will be ________ Ω.




**Ans.** (80)

$$[Y] = \begin{bmatrix} \frac{2}{100} & -\frac{1}{100} \\ -\frac{1}{100} & \frac{4}{300} \end{bmatrix}$$

For the given $Y$-parameter the two-port network is



$$Y_{11} = Y_a + Y_b = \frac{2}{100}$$

$$Y_{12} = Y_{21} = -Y_b = -\frac{1}{100}$$

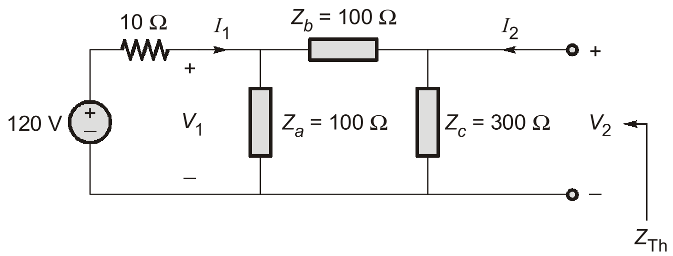
$$Y_{22} = Y_b + Y_c = \frac{4}{300}$$

On solving,
$$Y_b = \frac{1}{100}\,S$$

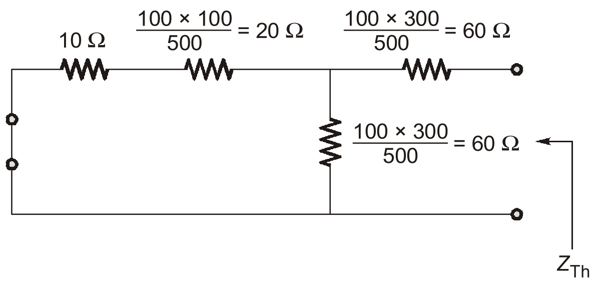
$$Y_a = \frac{1}{100}\,S$$

$$Y_c = \frac{1}{300}\,S$$

The network becomes,




Converting Δ – to ⅄,




$$Z_{Th} = 60 + [(20+10) \,||\, 60]$$

$$= 60 + \frac{30 \times 60}{30 + 60} = 80\ \Omega$$

For maximum power transfer,

$$Z_L = Z_{Th} = 80\ \Omega$$

**End of Solution**

**Q.48** For a series *RLC* circuit, if $Q = 1000$ and resonant frequency $\omega_o = 10^6$ rad/sec then which of the following will be the value of *R*, *L* and *C*?

(a) $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 0.001$
(b) $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 0.01$
(c) $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 0.1$
(d) $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 1$

**Ans.** (a)

**Given:** $Q = 1000$ and $\omega_o = 10^6$ rad/sec

We know, for series $RLC$ circuit,

$$Q = \frac{\omega_o L}{R}$$

Also,
$$\omega_o = \sqrt{\frac{1}{LC}}$$

$$Q = \frac{1}{\sqrt{LC}} \times \frac{1}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

So, $L = 1\ \mu\text{H}$, $C = 1\ \mu\text{F}$ and $R = 0.001$

End of Solution

**Q.49** Let $m(t)$ be a strictly band limited signal with bandwidth $B$ and energy $E$. Assuming $\omega_0 = 10B$ the energy in the signal $m(t)\cos\omega_0 t$ is

(a) $2E$

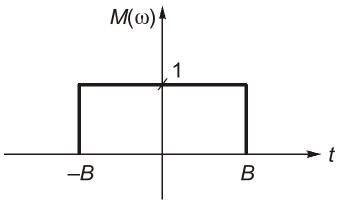
(b) $E$

(c) $\frac{E}{2}$

(d) $\frac{E}{4}$

**Ans.** (c)

$m(t) =$



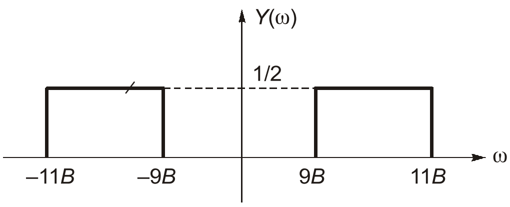

$$\text{Energy } (E) = \frac{1}{2\pi}\int_{-B}^{B} (1)^2 \cdot d\omega$$

$$E = \frac{B}{\pi}$$

Now, let
$$y(t) = m(t)\cos\omega_0 t$$

$$Y(\omega) = \frac{1}{2}[M(\omega - \omega_0) + M(\omega + \omega_0)]$$

Here;
$$\omega_0 = 10B$$




Now,
$$\text{Energy } (E') = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 \cdot d\omega$$

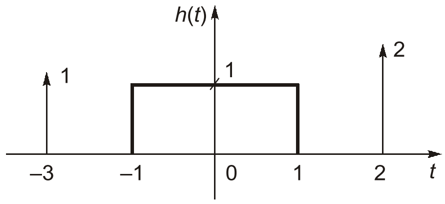
$$= \frac{1}{2\pi}\left[\int_{-11B}^{-9B}\left(\frac{1}{2}\right)^2 \cdot d\omega + \int_{9B}^{11B}\left(\frac{1}{2}\right)^2 \cdot d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{4}\times 2B + \frac{1}{4}\times 2B\right]$$

$$E' = \frac{B}{2\pi} = \frac{1}{2}\left(\frac{B}{\pi}\right) = \frac{E}{2}$$

End of Solution

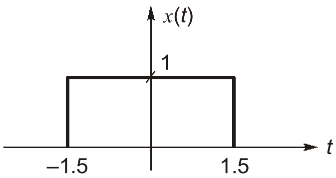
**Q.50** Consider a signal $x(t) = u(t + 1.5) - u(t - 1.5)$ and $h(t)$ is shown below:




if $y(t) = x(t) * h(t)$. Then value of $\int_{-\infty}^{\infty} y(t)\, dt$ is _______.

**Ans.** (15)

$$x(t) = u(t + 1.5) - u(t - 1.5)$$




$$\Rightarrow \quad x(t) = \text{rect}\left(\frac{t}{3}\right)$$

$$x(t) = \text{rect}\left(\frac{t}{3}\right) \xleftrightarrow{FT} 3\,Sa(1.5\omega)$$

Now, $$h(t) = \delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$$

Taking Fourier transform

$$H(\omega) = e^{3j\omega} + 2\,Sa(\omega) + 2e^{-2j\omega}$$

$\because$ $$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

We know, $$Y(\omega) = \int_{-\infty}^{\infty} y(t) \cdot e^{-j\omega t} \cdot dt$$

$\therefore$ $$\int_{-\infty}^{\infty} y(t) = Y(0)$$

$\therefore$ $$Y(0) = X(0) \cdot H(0)$$

$$= 3[1 + 2 + 2] = 15$$

End of Solution

**Q.51** The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$

(a) $\dfrac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

(b) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$

(c) $\sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$

(d) $\sqrt{\pi}\, e^{-\frac{\omega^2}{2}}$

**Ans. (c)**

We know; $e^{-at^2}; a > 0 \xrightarrow{FT} \sqrt{\frac{\pi}{a}} \cdot e^{-\omega^2/4a}$

Here; $a = 1$

$$\therefore \quad X(\omega) = \sqrt{\pi} \cdot e^{-\omega^2/4}$$

*End of Solution*

**Q.52** If input $x[n]$ having DTFT $X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-3j\Omega}$ be passed through as LTI system of frequency response $H(e^{j\Omega}) = 1 - \frac{1}{2} e^{-2j\Omega}$. The output $y(n)$ of the system

(a) $\delta(n) - \delta(n-1) - 0.5\delta(n-2) + 2.5\delta(n-3) - \delta(n-5)$
(b) $\delta(n) + \delta(n-1) - 0.5\delta(n-2) - 2.5\delta(n-3) + \delta(n-5)$
(c) $\delta(n) - \delta(n-1) - 0.5\delta(n-2) - 2.5\delta(n-3) + \delta(n-5)$
(d) $\delta(n) + \delta(n-1) - 0.5\delta(n-2) + 2.5\delta(n-3) + \delta(n-5)$

**Ans. (a)**

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\Omega}) = X(e^{j\Omega}) H(e^{j\Omega})$$

$$= \left[1 - e^{-j\Omega} + 2e^{-3j\Omega}\right]\left[1 - \frac{1}{2} e^{-2j\Omega}\right]$$

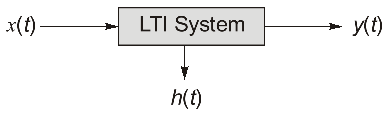
$$= 1 - e^{-j\Omega} + 2.5e^{-3j\Omega} - 0.5e^{-j2\Omega} - e^{-j5\Omega}$$

Taking IDTFT;

$$y[n] = \delta[n] - \delta[n-1] - 0.5\delta[n-2] + 2.5\delta[n-3] - \delta[n-5]$$

*End of Solution*

**Q.53** If $x(t) = 10\cos(10.5\omega_o t)$ and $h(t) = \pi\left(\frac{\sin\omega_o t}{\pi t}\right)^2 \cdot \cos 10\omega_0 t$ then the output $y(t)$ is ______.




(a) $\frac{15}{2}\omega_0 \cos 10.5\,\omega_0$

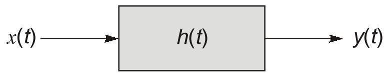
(b) $\frac{15}{4}\omega_0 \cos 10.5\,\omega_0$

(c) $\frac{5}{2}\omega_0 \cos 10.5\,\omega_0$

(d) $15\omega_0 \cos 10.5\,\omega_0$

**Ans.** **(b)**

Given $h(t)$ is Real and Even. When sinusoidal input applied to LTI system having even impulse response, then output will also be sinusoidal.

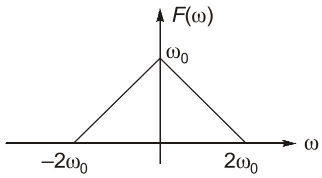



here, $y(t) = H(\omega)\big|_{\omega=10.5\omega_0} \cdot 10\cos(10.5\omega_0 t)$

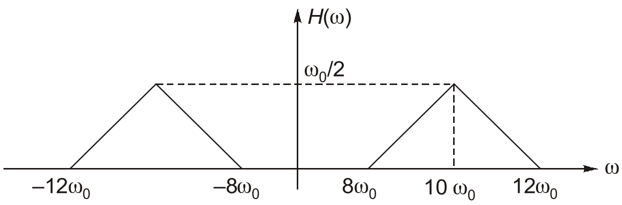
let, $h(t) = f(t) \times \cos 10\omega_0 t$

where, $f(t) = \pi\left(\frac{\sin\omega_0 t}{\pi t}\right)^2$

$f(t) \xrightarrow{FT}$




Now; $H(\omega) = \frac{1}{2}[F(\omega + 10\omega_0) + F(\omega - 10\omega_0)]$




$\therefore \quad H(\omega)\big|_{\omega = 10.5\omega_0} = \frac{3}{8}\omega_0$

Hence, $y(t) = \left(\frac{3}{8}\omega_0\right)(10\cos 10.5\omega_0)$

$= \frac{15}{4}\omega_0 \cos 10.5\omega_0$

*End of Solution*

Q.54 For LTI system having input $x(t)$ and output $y(t)$. If output related with input as $y(t) = x(e^t)$, then select the correct?

(a) Causal, Time variant
(b) Non-casual, Time variant
(c) Causal, Time invariant
(d) Non-causal, Time invariant

Ans. (b)

We have, $y(t) = x(e^t)$

At $t = 0$

$$y(0) = x(1)$$

i.e. present value of output depends on future value of input, hence it is non-causal.

**For Time Variant:**

Delay the input,

$$y(t) = x(e^t - t_o) \quad ...(i)$$

Delay the output,

$$y(t - t_o) = x(e^{t-t_o}) \quad ...(ii)$$

i.e. equations (i) ≠ (ii)

Hence, it is time variant system.

*End of Solution*

Q.55 Match the following:

| | Signal types | | Spectral characteristics |
|---|---|---|---|
| 1. | Continuous and aperiodic | a. | Continuous and aperiodic |
| 2. | Continuous and periodic | b. | Continuous and periodic |
| 3. | Discrete and aperiodic | c. | Discrete and aperiodic |
| 4. | Discrete and periodic | d. | Discrete and periodic |

(a) 1–a, 2–c, 3–b, 4–d
(b) 1–a, 2–c, 3–d, 4–b
(c) 1–c, 2–a, 3–b, 4–d
(d) 1–b, 2–c, 3–d, 4–a

Ans. (a)

| Signal Types | Spectral Characteristics |
|---|---|
| • Continuous and aperiodic | • Aperiodic and continuous |
| • Continuous and periodic | • Aperiodic and discrete |
| • Discrete and aperiodic | • Periodic and continuous |
| • Discrete and periodic | • Periodic and discrete |

*End of Solution*

■■■■