CSM – 53/17

Mathematics

Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any ***three*** *of the remaining questions selecting at least* ***one*** *from each Section.*

SECTION – A

1. Attempt any **three** of the following :

   (a) (i) Obtain the Chebyshev polynomial approximation of the second degree to the function $f(x) = x^3$ on [0, 1]. 10

   (ii) Find $f'(5)$ from the following table : 10

x	f(x)
0	4
2	26

x	f(x)
3	58
4	112
7	466
9	922

(b) Suppose $A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ is the adjacency matrix of a graph. Find out whether the graph is connected or not. 20

(c) (i) Determine the curve for which the radius of curvature is proportional to the slope of the tangent. 10

(ii) Solve $\frac{d^2y}{dx^2} + a^2y = \sec ax$. 10

(d) Transform the differential equation

$$\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2\cos^3 x \cdot y = 2\cos^5 x$$

into one having z as independent variable where $z = \sin x$ and solve it. 20

2. (a) (i) The values of f(x) are given at a, b and c. Show that the maximum is obtained by :

$$x = \frac{f(a)\left(b^2 - c^2\right) + f(b)\left(c^2 - a^2\right) + f(c)\left(a^2 - b^2\right)}{2\left[f(a)(b-c) + f(b)(c-a) + f(c)(a-b)\right]}$$

10

(ii) Find the root of the equation $xe^x = \cos x$ using the Regula-Falsi Method correct to four decimals. 20

(b) Compute the value of the definite integral $\int_{0.2}^{1.4} \left(\sin x - \log_e x + e^x\right) dx$ by (i) the Trapezoidal rule, (ii) Simpson's one-third rule, (iii) Simpson's three-eighth rule and (iv) Weddle's rule. After finding the true value of the integral, compare the errors in the four cases. 30

3. (a) (i) Prove that given any two vertices u and v of a graph G, every u-v walk contains a u-v path. Give an example to illustrate the proof. 15

(ii) Prove that in any graph G there is an even number of odd vertices. 15

(b) (i) Let G be a connected plane graph, and let n, e and f denote the number of vertices, edges and faces of G respectively. Prove that $n - e + f = 2$. 15

(ii) Let G be a simple plannar graph with n vertices and e edges where $n \geq 3$. Prove that $e \leq 3n - 6$. 15

4. (a) (i) Find the complete integral of : 10

$$\left(\frac{\partial z}{\partial x}\right)^2 x + \left(\frac{\partial z}{\partial y}\right)^2 y = z$$

(ii) Find the general solution of : 20

$$(x^2 + 2)y'' - xy' - 3y = 0$$

(b) (i) Solve the simultaneous equation :

$$\frac{dy}{dx} - y = e^t, \frac{dy}{dx} + x = \sin t$$

using the Laplace transform method which satisfies the conditions $x(0) = 1$, $y(0) = 2$. 15

(ii) Find the general solution of : 15

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

SECTION – B

5. Attempt any **three** of the following :

(a) Write a FORTRAN program and flow chart to find the sum $\frac{1}{1-x} = 1 + x + x^2 + \cdots\cdots \; |x| < 1$ to 0.01% accuracy. 20

(b) Derive the equation of continuity of a compressible fluid flow. 20

(c) Find the dual of the following primal problem : 20

Minimize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to the constraints,

$$3x_1 + 5x_2 + 4x_3 \geq 7$$
$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1 - 2x_2 + 5x_3 \geq 3$$
$$4x_1 + 7x_2 - 2x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

(d) Show that the effect of a couple on a rigid body remains the same if the couple is transferred from the given plane into any other parallel plane. 20

6. (a) Solve the following cost minimizing transportation problem using North-West-Corner Rule : 30

	D_1	D_2	D_3	D_4	D_5	D_6	$a_i \downarrow$
S_1	$C_{ij} = 10$	12	13	8	14	19	18
S_2	15	18	12	16	19	20	22
S_3	17	16	13	14	10	18	39
S_4	19	18	20	21	12	13	14
$b_j \rightarrow$	10	11	13	20	24	15	

(b) If the curve is an equiangular spiral $r = ae^{\theta \cot \alpha}$ and if the radius vector to the particle has constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and is of magnitude $\frac{v^2}{r}$, when v is the speed of the particle. 30

7. (a) A uniform beam of length 2a rests in an equilibrium against a smooth vertical wall and upon a peg at a distance b from the wall. Show that the inclination of the beam to the vertical is $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$. 30

(b) Write the structure of the Simplex Algorithm. Using simplex method solve the following linear programming problem : 30

Maximize $Z = 3x_1 + x_2 + 3x_3$

Subject to the constraints,

$$2x_1 + x_2 + x_3 \le 2$$

$$x_1 + 2x_2 + 3x_3 \le 5$$

$$2x_1 + 2x_2 + x_3 \le 6$$

$$x_1, x_2, x_3 \ge 0$$

8. (a) Show that the optimal solution of an assignment problem remains the same if a constant is added or subtracted from any row or column of the cost matrix. Solve the

following cost-minimizing assignment problem whose cost matrix is given below : 30

	M_1	M_2	M_3	M_4
J_1	2	5	7	9
J_2	4	9	10	1
J_3	7	3	5	8
J_4	8	2	4	9

(b) Derive Euler Equation of motion in Cartesian form. 30