CSM – 53/18

Mathematics

Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and **three** of the remaining questions, selecting at least **one** from each Section.*

SECTION – A

1. Answer any **three** questions of the following :

   (a) (i) Find the least squares approximation of second degree for the discrete data : 10

x	f(x)
– 2	15
– 1	1
0	1
1	3
2	19

(ii) Find all the roots of $\cos x - x^2 - x = 0$ to five decimal places. 10

(b) (i) Solve the initial value problem that consists of the differential equation $(x^2 + 1)\frac{dy}{dx} + 4xy = x$ and the initial condition $y(2) = 1$. 10

(ii) Solve $\frac{dx}{y - xz} = \frac{dy}{yz + x} = \frac{dz}{x^2 + y^2}$. 10

(c) (i) Let G be a connected graph with at least two vertices. Assume that every edge of G belongs to a unique cycle. Prove that G is Eulerian. 10

(ii) Let G be a connected plane graph with V vertices, E edges and R regions. Then find V – E + R. 10

(d) Find the power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$ in powers of x. 20

2. (a) (i) Show that the equation $\log_e x = x^2 - 1$ has exactly two real roots $\alpha_1 = 0.45$ and $\alpha_2 = 1$. 10

(ii) Using Netwon-Raphson method, find a real root of the equation $x^3 - 2x - 5 = 0$. 10

(iii) Evaluate $\int_{-1}^{1} (1 - x^2)^{3/2} \cos x \, dx$, using Gauss-Legendre three point formula. 10

(b) Given the initial value problem

$$u' = t^2 + u^2, \ u(0) = 0,$$

determine the first three non-zero terms in the Taylor's series for u(t) and obtain the value for u(1). Also, determine t, when the error in u(t) obtained from the first two non-zero terms is to be less than 10^{-6} after rounding. 30

3. (a) (i) Suppose G is a connected planar graph in which every vertex has degree at least 3. Prove that at least two regions of G have at most 5 edges on their boundaries. 15

(ii) Prove that in any graph with more than one vertex there must exist two vertices of the same degree. 15

(b) (i) A graph G has 50 edges, four vertices of degree 2, six of degree 5, eight of degree 4 and the rest of degree 6. How many vertices does G have ? 15

(ii) A connected graph G has 14 vertices and 88 edges. Show that G is Hamiltonian, but not Eulerian. 15

4. (a) (i) Solve the differential equation $y'' + 2y' + 2y = 2$, given that $y(0) = 0$, $y'(0) = 1$, using Laplace Transform method. 15

(ii) Find the complete integral of the equation : 15

$$x\frac{\partial z}{\partial x} + 3y\frac{\partial z}{\partial y} - 2z + 2x^2\left(\frac{\partial z}{\partial y}\right)^2 = 0$$

(b) (i) Solve : 15

$$\frac{d^2y}{dx^2} + 4y = 4\tan 2x$$

(ii) Solve : 15

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \sin x$$

SECTION – B

5. Answer any **three** questions of the following :

(a) Write a FORTRAN program to evaluate $S = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\cdots$, by direct summation of successive terms upto and excluding those terms whose value is less than or equal to 10^{-4} or N terms whichever is earlier, for a given x in radians. 20

(b) Using dual simplex method, solve the following LPP : 20

Maximize $Z = -2x_1 - x_3$

Subject to the constraints :

$$x_1 + x_2 - x_3 \geq 5$$

$$x_1 - 2x_2 + 4x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0$$

(c) A particle is projected vertically upwards with a velocity u in a medium whose resistance varies as the square of the velocity. Investigate the motion. 20

(d) Derive Euler Equation of motion in Cartesian form. 20

6. (a) There are four jobs to be assigned to five machines. Only one job can be assigned to one machine. The amount of time in hours required for the jobs per machine are given in the following matrix :

Jobs	Machines				
	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

Find an optimum assignment of jobs to the machines to minimize the total processing time and also find out for which machine no job is assigned. What is the total processing time to complete all the jobs ? 30

(b) A square of side 2a is placed with its plane vertical between two smooth pegs which are in the same horizontal line at a distance c apart. Show that it will be in equilibrium when the inclination of one of its edges to the horizontal is either $\frac{\pi}{4}$ or $\frac{1}{2}\sin^{-1}\left(\frac{a^2 - c^2}{c^2}\right)$. 30

7. (a) Using Simplex method, solve the following linear programming problem : 30

Minimize $Z = x_1 - 3x_2 + 2x_3$

Subject to the constraints :

$3x_1 - x_2 + 3x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

(b) For a rigid body in general motion, show that there is no point at rest. Show also that, in general, there is only one point and only one, with no acceleration. 30

8. (a) Show that there always exists an optimal solution to a balanced transportation problem. Determine the optimal solution to the following transportation problem : 30

	D_1	D_2	D_3	D_4	D_5	$a_i \downarrow$
S_1	4	7	3	8	2	4
S_2	1	4	7	3	8	7
S_3	7	2	4	7	7	9
S_4	4	8	2	4	7	2
$b_j \rightarrow$	8	3	7	2	2	

(b) (i) Derive the equation of Continuity of a compressible fluid flow. 20

(ii) Write a FORTRAN program which finds all three digit prime numbers. 10