

Model Question Paper 2021-22
Mathematics
Class-12

TIME – 3 Hrs 15 Min

Maximum Marks - 100

Note: First 15 minutes are allotted for the candidates to read the question paper.

Instructions :

- (i) There are in all *nine* questions in this question paper.
- (ii) *All* questions are compulsory.
- (iii) In the beginning of each question, the number of parts to be attempted has been clearly mentioned.
- (iv) Marks allotted to the questions are indicated against them.
- (v) Start solving from the first question and proceed to solve till the last one.
- (vi) Do not waste your time over a question you cannot solve.

1. Choose the correct option and write down in your answer sheet.

- (a) Suppose that the function defined as $f(x) = 3x$ is $f: \mathbb{R} \rightarrow \mathbb{R}$, select the correct option. 01
- (i) f is one-one onto (ii) f is many-one onto
(iii) f is one-one but not onto (iv) f is neither one-one nor onto
- (b) If R is a relation on the set N , defined as $R = \{(a, b) : a = b - 2, b > 6\}$, select the correct option from the following. 01
- (i) $(2, 4) \in R$ (ii) $(3, 8) \in R$
(iii) $(6, 8) \in R$ (iv) $(8, 7) \in R$
- (c) Find the value of integral $\int xe^x dx$ 01
- (i) e^x (ii) $(x+1)e^x$ (iii) $(x-1)e^x$ (iv) $\frac{x^2}{2}e^x$

- (d) Order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$ is - 01
- (i) 2 (ii) 1 (iii) 0 (iv) not defined
- (e) If the vector's $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - 4\hat{j} + \lambda\hat{k}$ are mutually perpendicular, then find the value of λ - 01
- (i) 3 (ii) 2 (iii) 4 (iv) 0

2. Attempt all the parts:

- (a) Find the principal value of $\text{Cot}^{-1}\left(\frac{-1}{\sqrt{3}}\right)$. 01
- (b) Show that the function $f(x) = |x|$, is continuous at $x = 0$. 01
- (c) Find the order and power of the differential equation $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - y \frac{dy}{dx} = 0$. 01
- (d) Find the maximum value of $z = 3x + 4y$ subject to the following constraints $x + y \leq 4, x \geq 0, y \geq 0$. 01
- (e) If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$ then find the value of $P(A/B)$. 01

3. Attempts all the parts:

- (a) If $A = \{1,2\}$ and $B = \{3,4\}$ then find the number of relations between A and B. 02
- (b) If $y = A \sin x + B \cos x$ then prove that $\frac{d^2y}{dx^2} + y = 0$. 02
- (c) Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. 02
- (d) A problem of mathematics is given to three students. Probabilities of solving the problem by them are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. If all the three students try their best, then find the probability that problem is solved. 02

4. Attempt all the parts.

- (a) Show that the function defined on \mathbb{R} as $f(x) = 7x - 3$ is an increasing function. 02
- (b) Find the unit vector perpendicular to each of vectors $(\bar{a} + \bar{b})$ and $(\bar{a} - \bar{b})$ where $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$. 02
- (c) Find the area of parallelogram whose adjacent sides are given by vectors $\bar{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\bar{b} = \hat{i} - \hat{j} + \hat{k}$. 02
- (d) A and B are two given events where $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = P$. Find the value of P if events are mutually exclusive. 02

5. Attempt all the parts.

- (a) Prove that the relation R on the set of integers Z is defined as $R = \{(a, b) : (a-b) \text{ is divisible by number } 2\}$ is an equivalence relation. 05
- (b) Prove that
$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$
 05
- (c) Differentiate the function $(\sin x)^{\cos x}$ with respect to x. 05
- (d) Find the
$$\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx.$$
 05
- (e) Find the shortest distance between the lines $\bar{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\bar{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$. 05

6. Attempt all the parts:

- (a) Show that the function $f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is discontinuous at $x = 0$. 05
- (b) Find the area bounded by the parabolas $y = x^2$ and $y^2 = x$. 05
- (c) Find the equation of the plane passing through the intersection of the planes $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\bar{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point $(1, 1, 1)$. 05

- (d) Minimize $z = 3x + 2y$ subject to the constraints; 05
 $x + y \geq 8$ $3x + 5y \leq 15$ $x \geq 0$, $y \geq 0$
- (e) In a hostel 60% students read Hindi newspaper, 40% students read English newspaper and 20% read both newspapers -
- (i) Find the probability of the students who read neither Hindi newspaper nor English newspaper. $2\frac{1}{2}$
- (ii) If she reads Hindi newspaper then what is the probability that she also reads English newspaper. $2\frac{1}{2}$

7. Attempt any one of the following:

- (a) If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 08
then find out the value of $(AB)^{-1}$.

- (b) Solve the following system of linear equations by the matrix method:

$$\begin{aligned} 3x - 2y + 3z &= 8 \\ 2x + y - z &= 1 \\ 4x - 3y + 2z &= 4 \end{aligned} \quad \text{08}$$

8. Attempt any one of the following:

- (a) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. 08
- (b) Find the general solution of the differential equation $\frac{dy}{dx} - y = \text{Cos}x$. 08

9. Attempt any one of the following:

- (a) Find the value of the integral $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$. 08
- (b) Evaluate $\int_0^{\pi} \frac{xdx}{a^2 \text{Cos}^2x + b^2 \text{Sin}^2x}$. 08
