BOARD QUESTION PAPER : JULY 2018
MATHEMATICS AND STATISTICS

Time: 3 Hours **Total Marks: 80**

Note:
i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answer to every new question must be written on a new page.
v. Answers to both sections should be written in the same answer book.
vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: **(6) [12]**

i. If the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product, then the value of 'k' is _______.

(A) 2 (B) 1
(C) –1 (D) –2

ii. If the vectors $\hat{i} - 2\hat{j} + \hat{k}, a\hat{i} - 5\hat{j} + 3\hat{k}$ and $5\hat{i} - 9\hat{j} + 4\hat{k}$ are coplanar, then the value of a is _______.

(A) 3 (B) –3
(C) 2 (D) –2

iii. The acute angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 8$ is _______.

(A) $\sin^{-1}\left(\frac{8}{21}\right)$ (B) $\cos^{-1}\left(\frac{8}{21}\right)$
(C) $\sin^{-1}\left(\frac{1}{8}\right)$ (D) $\cos^{-1}\left(\frac{1}{8}\right)$

(B) Attempt any THREE of the following: **(6)**

i. Write the dual of each of the following statements :
a. $\sim p \wedge (q \vee c)$
b. "Shweta is a doctor or Seema is a teacher."

ii. In Δ ABC, prove that $ac \cos B - bc \cos A = a^2 - b^2$.

iii. Show that the equation $2x^2 + xy - y^2 + x + 4y - 3 = 0$ represents a pair of straight lines.

iv. If $\overline{a}, \overline{b}, \overline{c}$ are the position vectors of the points A, B, C respectively such that $3\overline{a} + 5\overline{b} = 8\overline{c}$, then find the ratio in which C divides AB.

v. If points A (5, 5, λ), B (–1, 3, 2) and C (–4, 2, –2) are collinear, then find the value of λ.

Q.2. (A) Attempt any TWO of the following: **(6) [14]**

i. Using truth table, examine whether the following statement pattern is a tautology, a contradiction or a contingency:
$(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$

ii. Find the inverse of the matrix A where $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ by using adjoint method.

iii. If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$.

(B) Attempt any TWO of the following: **(8)**

i. Prove that three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ are coplanar if and only if there exists non-zero linear combination $x\bar{a} + y\bar{b} + z\bar{c} = \bar{0}$.

ii. Maximize $z = 6x + 4y$ subject to constraints, $x \le 2$, $x + y \le 3$, $-2x + y \le 1$, $x, y \ge 0$. Also find the maximum value of 'z'.

iii. Find the general solution of $\sin 2x + \sin 4x + \sin 6x = 0$.

Q.3. (A) Attempt any TWO of the following: **(6) [14]**

i. Write the negations of the following statements:

a. If diagonals of a parallelogram are perpendicular, then it is a rhombus.

b. Mangoes are delicious, but expensive.

c. A person is rich if and only if he is a software engineer.

ii. Express the following equations in matrix form and solve them by the method of reduction: $x + y + z = 6$, $3x - y + 3z = 6$ and $5x + 5y - 4z = 3$

iii. Find the vector equation of the line passing through the point $(-1, -1, 2)$ and parallel to the line $2x - 2 = 3y + 1 = 6z - 2$.

(B) Attempt any TWO of the following: **(8)**

i. A plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r). Show that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$.

ii. If α, β, γ are direction angles of the line l, then prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Hence deduce that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

iii. Using the Sine rule, prove the Cosine rule.

SECTION – II

Q.4. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: **(6) [12]**

i. Equation of the tangent to the curve $2x^2 + 3y^2 - 5 = 0$ at (1, 1) is _______.

(A) $2x - 3y - 5 = 0$ (B) $2x + 3y - 5 = 0$
(C) $2x + 3y + 5 = 0$ (D) $3x + 2y + 5 = 0$

ii. The order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} - \left(\frac{dy}{dx}\right)^{\frac{1}{3}} = 0$ are respectively _______.

(A) 3, 2 (B) 2, 3
(C) 6, 3 (D) 3, 1

iii. Given $X \sim B(n, p)$. If $p = 0.6$, $E(X) = 6$, then the value of Var (X) is _______.

(A) 2.4 (B) 2.6
(C) 2.5 (D) 2.3

(B) Attempt any THREE of the following: **(6)**

i. The displacement s of a particle at time t is given by $s = t^3 - 4t^2 - 5t$. Find its velocity and acceleration at $t = 2$.

ii. If $y = \cos^{-1}(1 - 2\sin^2 x)$, find $\frac{dy}{dx}$.

iii. Evaluate : $\int \frac{1}{\sin x \cdot \cos^2 x} dx$

iv. Solve the differential equation $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

v. Obtain the probability distribution of the number of sixes in two tosses of a fair die.

Q.5. (A) Attempt any TWO of the following: **(6) [14]**

i. If $f(x) = \frac{1-\sqrt{3}\tan x}{\pi - 6x}$, for $x \neq \frac{\pi}{6}$ is continuous at $x = \frac{\pi}{6}$, find $f\left(\frac{\pi}{6}\right)$.

ii. If $\sec^{-1}\left(\frac{x+y}{x-y}\right) = a^2$, show that $\frac{dy}{dx} = \frac{y}{x}$.

iii. Evaluate : $\int \frac{e^x}{(1+e^x)(2+e^x)} dx$

(B) Attempt any TWO of the following: **(8)**

i. A stone is dropped into a pond. Waves in the form of circles are generated and the radius of the outermost ripple increases at the rate of 2 inch/sec. How fast will the area of the wave increase

a. when the radius is 5 inch?

b. after 5 seconds?

ii. Evaluate : $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}} dx$

iii. The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria and the number doubles in 1 hour, find the number of bacteria after 2½ hours. [Take $\sqrt{2} = 1.414$]

Q.6. (A) Attempt any TWO of the following: **(6) [14]**

i. Discuss the continuity of the following function in its domain, where

$f(x) = x^2 - 4$, for $0 \leq x \leq 2$

$= 2x + 3$, for $2 < x \leq 4$

$= x^2 - 5$, for $4 < x \leq 6$

ii. If $y = f(u)$ is differentiable function of u, and $u = g(x)$ is a differentiable function of x, then prove that $y = f[g(x)]$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

iii. Suppose that 80% of all families own a television set. If 10 families are interviewed at random, find the probability that at most three families own a television set.

(B) Attempt any TWO of the following: **(8)**

i. If u and v are integral functions of x, then show that $\int u \cdot v\, dx = u \int v\, dx - \int \left[\frac{du}{dx} \int v\, dx\right] dx$

Hence evaluate $\int \log x\, dx$.

ii. Prove that :

$$\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a - x)\, dx$$

iii. Find k if the function $f(x)$ is defined by

$f(x) = kx(1-x)$, for $0 < x < 1$

$= 0$, otherwise,

is the probability density function (p.d.f.) of a random variable (r.v.) X. Also find $P\left(X < \frac{1}{2}\right)$