BOARD QUESTION PAPER: JULY 2022
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

*The question paper is divided into **FOUR** sections.*

*(1) **Section A:** Q.1 contains **Eight** multiple choice type of questions, each carrying **Two** marks. Q.2 contains **Four** very short answer type questions, each carrying **One** mark.*
*(2) **Section B:** Q.3 to Q. 14 contains **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*
*(3) **Section C:** Q.15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*
*(4) **Section D:** Q.27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*
(5) Use of log table is allowed. Use of calculator is not allowed.
(6) Figures to the right indicate full marks.
(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.
(8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g. (a)……. / (b)…….. /(c)…….. /(d)…….., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.
(9) Start answer to each section on a new page.

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

(i) The negation of $(p \vee \sim q) \wedge r$ is _______.

(a) $(\sim p \wedge q) \wedge r$ (b) $(\sim p \wedge q) \vee r$
(b) $(\sim p \wedge q) \vee \sim r$ (d) $(\sim p \vee q) \wedge \sim r$ (2)

(ii) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) =$ _______.

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$ (2)

(iii) If $|\bar{a}| = 3$ and $|\bar{b}| = 4$, then value of λ for which $\bar{a} + \lambda\bar{b}$ is perpendicular to $\bar{a} - \lambda\bar{b}$ is _______.

(a) $\pm\frac{9}{16}$ (b) $\pm\frac{3}{4}$ (c) $\pm\frac{16}{9}$ (d) $\pm\frac{4}{3}$ (2)

(iv) The equation of plane passing through (2, –1, 3) and making equal intercepts on the co-ordinate axes is _______.

(a) $x + y + z = 1$ (b) $x + y + z = 2$
(c) $x + y + z = 3$ (d) $x + y + z = 4$ (2)

(v) The equation of tangent to the curve $y = 1 - e^{\frac{x}{2}}$ at the point of intersection with Y-axis is _______.

(a) $x + 2y = 0$ (b) $2x + y = 0$
(c) $x - y = 2$ (d) $x + y = 2$ (2)

(vi) The area of the region bounded by the curve $y = \sin x$, X-axis and lines $x = 0$, $x = \frac{\pi}{2}$ is _______ sq. units.

(a) 2 (b) 3 (c) 4 (d) 1 (2)

(vii) The differential equation of $y = c^2 + \frac{c}{x}$ is _______.

(a) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$ (b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $x^3\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y$ (d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$ (2)

(viii) If the mean and variance of a Binomial distribution are 18 and 12 respectively then value of n is _______.
(a) 36 (b) 54 (c) 18 (d) 27 (2)

Q.2. Answer the following questions: **[4]**

(i) If the statement p, q are true statements and r, s are false then determine the truth value of $(p \to q) \vee (r \to s)$. (1)

(ii) Find the direction cosines of the vector $\hat{i} + 2\hat{j} - 2\hat{k}$. (1)

(iii) Evaluate: $\int \frac{1}{x\log x}\,dx$. (1)

(iv) Write the degree of the differential equation
$\frac{dy}{dx} + \frac{3xy}{\frac{dy}{dx}} = \cos x$ (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Without using truth table prove that:
$\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$ (2)

Q.4. Find the inverse of the matrix A = $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ (2)

Q.5. Find the principal solutions of $\sin\theta = \frac{1}{2}$. (2)

Q.6. Find k if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$ (2)

Q.7. Show that the points A(4, 5, 2), B(3, 2, 4) and C(5, 8, 0) are collinear. (2)

Q.8. Find the cartesian equation of the line passing through the point A(2, 1, –3) and perpendicular to the vectors $\bar{b} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{c} = \hat{i} + 2\hat{j} - \hat{k}$. (2)

Q.9. If $y = \tan^{-1}\left(\frac{8x}{1-15x^2}\right)$ then find $\frac{dy}{dx}$. (2)

Q.10. Evaluate: $\int \frac{1}{25 - 9x^2}\,dx$ (2)

Q.11. Evaluate: $\int_{-3}^{3} \frac{x^3}{9 - x^2}\,dx$ (2)

Q.12. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 9$. (2)

Q.13. A particle is moving along the X-axis. Its acceleration at time t is proportional to its velocity at that time. Find the differential equation of the motion of the particle. (2)

Q.14. In a meeting 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposes, and X = 1 if he is in favour. Find E(X) and Var(X). (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. Examine whether the statement pattern $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology, contradiction or contingency. (3)

Q.16. In ΔABC, with usual notations prove that $a^2 = b^2 + c^2 - 2bc \cos A$. (3)

Q.17. In ΔABC, $A = 45°$, $B = 60°$, then find the ratio of its sides. (3)

Q.18. Find the volume of the tetrahedron whose vertices are A(–1, 2, 3), B(3, –2, 1), C(2, 1, 3) and D(–1, –2, 4). (3)

Q.19. Find the angle between two lines:
$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ and $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda'\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ (3)

Q.20. Find the vector equation of the plane passing through the points A(–2, 7, 5) and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$. (3)

Q.21. Find the derivative of $\cos^{-1}x$ w.r.t. $\sqrt{1-x^2}$. (3)

Q.22. If $f(x) = 3x + \frac{1}{3x}$ find the values of x for which function $f(x)$ is decreasing. (3)

Q.23. Evaluate: $\int e^{\sin^{-1}x}\left(\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}}\right) dx$ (3)

Q.24. Solve the differential equation $(x + 1)\frac{dy}{dx} - 1 = 2e^{-y}$. Also find particular solution when $y = 0, x = 1$. (3)

Q.25. Find expected value and variance of X, where X is number obtained on the uppermost face when a fair die is thrown. (3)

Q.26. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles 9 articles are defective? (3)

SECTION – D

Attempt any FIVE of the following questions: **[20]**

Q.27. Solve the following system of equations by method of inversion.
$x + y + z = -1, y + z = 2, x + y - z = 3.$ (4)

Q.28. ΔOAB is formed by the lines $x^2 - 4xy + y^2 = 0$ and the line $2x + 3y - 1 = 0$. Find the equation of the median of the triangle drawn from origin O. (4)

Q.29. If $\bar{a}, \bar{b}, \bar{c}$ are three non co-planar vectors, then prove that any vector $\bar{r}$ in the space can be uniquely expressed as a linear combination of $\bar{a}, \bar{b}, \bar{c}$. (4)

Q.30. Solve the following L.P.P. graphically
Maximize $z = 4x + 3y$
Subject to $3x + y \leq 15$,
$3x + 4y \leq 24$,
$x \geq 0, y \geq 0$ (4)

Q.31. If $y = f(x)$ is a differentiable function of x on interval l and y is one-one, onto and $\frac{dy}{dx} \neq 0$ on l. Also if $f^{-1}(y)$ is differentiable function on $f(l)$ then prove that: $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ where $\frac{dy}{dx} \neq 0$
Hence find the derivative of the inverse of function $y = 2x^3 - 6x$. (4)

Q.32. Prove that: $\int \sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c.$ (4)

Q.33. The profit function $p(x)$ of a firm selling x items per day is given by $p(x) = (150 - x)\,x - 1625$. Find the number of items the firm should manufacture per day to get maximum profit. Also find the maximum profit. (4)

Q.34. Evaluate: $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}}\ dx$ (4)