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PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 24 January, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. The value of $8\int_0^{\pi/2} (\cos x)^{2023}$ dx is equal to

1. The value of $\frac{8}{\pi}\int_0^{\pi/2}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ is equal to

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(2)**

Sol. $I=\frac{8}{\pi}\int_0^{\pi/2}\frac{(\cos x)^{2023}}{(\cos x)^{2023}+(\sin x)^{2023}}dx$ .....(i)

Applying $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, we get

$I=\frac{8}{\pi}\int_0^{\pi/2}\frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ .....(ii)

add (i) and (ii), we get

$2I=\frac{8}{\pi}\int_0^{\pi/2}(1)dx=\frac{8}{\pi}\left(\frac{\pi}{2}\right)$

$\therefore I=2$

2. $\int_0^3\left|x^2-3x+2\right|dx$ is equal to

(1) $\frac{11}{6}$ (2) $\frac{24}{2}$ (3) $\frac{15}{8}$ (4) $\frac{10}{2}$

Ans. **(1)**

Sol. $I=\int_0^1\left(x^2-3x+2\right)dx+\int_1^2\left(-x^2+3x-2\right)dx+\int_2^3\left(x^2-3x+2\right)dx$

$=\left[\frac{x^2}{3}-\frac{3x^2}{2}+2x\right]_0^1-\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_1^2+\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_2^3$

$=2\left(\frac{1}{3}-\frac{3}{2}+2\right)-2\left(\frac{8}{3}-\frac{12}{2}+4\right)+\left(\frac{27}{3}-\frac{27}{2}+6\right)$

$=\frac{2}{3}-\frac{6}{2}+4-\frac{16}{3}+\frac{24}{2}-8+\frac{27}{3}-\frac{27}{2}+6$

$=\frac{13}{3}-\frac{9}{2}+2$

$=\frac{26-27}{6}+2=\frac{11}{6}$

3. Number of real values of x satisfying $x^2-4x+[x]+3=x[x]$, is (where [t] is the greatest integer less than or equal to t)

(1) 2 (2) 3 (3) 1 (4) 4

Ans. **(3)**

Sol. $x^2-x[x]-(x-[x])-3(x-1)=0$

$x(x-[x])-1(x-[x])-3(x-1)=0$

$(x-1)(x-[x])-3(x-1)=0$

$(x-1)(x-[x]-3)=0$

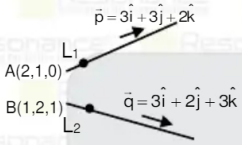
$\Rightarrow x=1$ as $\{x\}\neq 3$

4. Let $L_1: \frac{x-2}{3}=\frac{y-1}{3}=\frac{z-0}{2}$ and $L_2: \frac{x-1}{3}=\frac{y-2}{2}=\frac{z-1}{3}$ are two straight lines , then shortest distance between them is

(1) $\frac{5}{\sqrt{43}}$ (2) $\frac{6}{\sqrt{43}}$ (3) $\frac{1}{\sqrt{43}}$ (4) $\frac{11}{\sqrt{43}}$

Ans. **(4)**

Sol.



Required Shortest Distance = $\left|\overline{AB}.(\vec{p}\times\vec{q})\right|$

Required Shortest Distance $= \left| \frac{|\vec{p}\times\vec{q}|}{|\vec{p}\times\vec{q}|} \right|$

$\because \vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix}$

$= \hat{i}(5) - \hat{j}(3) + \hat{k}(-3)$

$= 5\hat{i} - 3\hat{j} - 3\hat{k}$

$\because \overline{AB} = -\hat{i} + \hat{j} + \hat{k}$

$\because$ distance $= \left|\frac{-5-3-3}{25+9+9}\right| = \frac{11}{\sqrt{43}}$ Ans.

5. The value of $\sum_{r=0}^{22} {}^{22}C_r \, {}^{23}C_r$ is equal to

(1) ${}^{45}C_{22}$ (2) ${}^{44}C_{22}$ (3) ${}^{45}C_{21}$ (4) ${}^{44}C_{22}$

Ans. **(1)**

Sol. $\sum_{r=0}^{22} {}^{22}C_{22-r} \, {}^{23}C_r$

${}^{22+23}C_{22} = {}^{45}C_{22}$

6. If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then

(1) $y''(0) = 0$

(2) $|y'(0)| + |y''(0)| = 3$

(3) $|y''(0)| = 2$

(4) $|y'(0)| + y'(0) = 3$

Ans. **(3)**

Sol. $x = 0 \Rightarrow y^2 + \ell og\,(1) = y \Rightarrow y^2 = y$

$= y = 0$ or 1

$y^2 + \log_e(\cos^2 x) = y$

$\Rightarrow 2yy' + \frac{2\cos x(-\sin x)}{\cos^2 x} = y'$

$y' = \frac{2\tan x}{2y-1}$

$x = 0 \Rightarrow y'(0) = 0$

Again $2yy' = 2\tan x + y'$

$\Rightarrow 2yy'' + 2(y')^2 = 2\sec^2 x + y''$

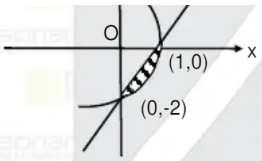
$y''(2y-1) + 2(y')^2 = 2\sec^2 x$

$y'' = \frac{2\sec^2 x - 2(y')^2}{2y-1} \Rightarrow y''(0) = 2$ or -2

7. Area bounded between the curves $y^2+4x=4$ and $2x=y+2$ is

(1) $\frac{16}{3}$ sq. unit (2) $\frac{4}{3}$ sq. unit (3) $\frac{8}{3}$ sq. unit (4) $\frac{1}{3}$ sq. unit

Ans. **(4)**



Sol.

$$\text{Required area} = \frac{1}{4}\int_{-2}^{0}\left(4-y^2\right)dy - \frac{1}{2}(1)(2)$$

$$= \frac{1}{4}\left(4y - \frac{y^3}{3}\right)_{-2}^{0} - 1$$

$$= \frac{1}{4}\left(\frac{16}{3}\right) - 1$$

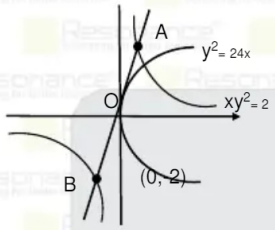
$$= \frac{4}{3} - 1 = \frac{1}{3} \text{ sq. unit}$$

8. A tangent to the parabola $y^2 = 24x$ intersects the hyperbola $xy = 2$ at points A and B, then locus of mid-point of AB is

(1) $y^2 = 3x$ (2) $y^2 = -3x$ (3) $y^2 = 6x$ (4) $y^2 = -6x$

Ans. **(2)**

Sol.



$y^2 = 24x$ ..... (1)

$xy = 2$ ..... (2)

let mid-point of chord AB of $xy = 2$ be m (x_1, y_1)

$\therefore$ equation of AB be $T = S_1$

$$\frac{x(y_1) + y(x_1)}{2} - 2 = x_1y_1 - 2$$

$$\Rightarrow x(y_1) + y(x_1) = 2x_1y_1 \Rightarrow y(x_1) = -x(y_1) + 2x_1y_1$$

$$\Rightarrow y = x\left(\frac{-y_1}{x_1}\right) + 2y_1 \quad ...(3)$$

$\because$ (3) is target to (2)

$$\therefore\ c = \frac{a}{m} \Rightarrow 2y_1 = -\frac{6x_1}{y_1}$$

$\therefore\ y_1^2 = -3x$

$\therefore$ locus of mid-point m (x_1, y_1) is

$y^2 = -3x$

9. If $f(x) = \begin{cases} x^2.\sin(1/x) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$, then

(1) f(x) is not differentiable at x = 0

(2) f(x) Continuous but not differentiable at x = 0

(3) f(x) is differentiable and f'(x) is continuous at x = 0

(3) f(x) is differentiable and f'(x) is continuous at x = 0
(4) f(x) is differentiable and f'(x) is discontinuous at x = 0

Ans. **(4)**

Sol. $= \text{LHD} = \lim\limits_{h\to 0^+} \frac{f(0-h)-f(0)}{-h} = \lim\limits_{h\to 0^+} \frac{-h^2\sin(1/h)}{-h} = \lim\limits_{h\to 0} \frac{\sin(1/h)}{(1\backslash h)} = 0$

$\text{RHD} = \lim\limits_{h\to 0^+} \frac{f(o+h)-f(0)}{h} = \lim\limits_{h\to 0^+} \frac{h^2\sin(1/h)}{h} = \lim\limits_{h\to 0^+} \frac{\sin(1/h)}{1/h} = 0$

$\Rightarrow$ f(x) is continuous and differential at x = 0

Now $f'(x) = \begin{cases} 2x\sin(1/x)-\cos(1/x) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$

clearly f' (x) is discontinuous at x = 0

10. If R = {(a,b) : gcd(a,b) = 1 ; a,b $\in$ z} then the relation R is
(1) Symmetric (2) Reflexive (3) Transitive (4) None of these

Ans. **(1)**

Sol. (1) Reflexive
Let a $\in$ z

aRa $\Rightarrow$ gcd(a,a) = a

$\Rightarrow$ R is not reflexive

(2) Let a, b $\in$ z and aRb

So, aRb $\Rightarrow$ gcd (a,b) = 1

$\Rightarrow$ gcd (b,a) = 1, $\forall$ a, b $\in$ z $\Rightarrow$ b Ra, $\forall$ a, b $\in$ z $\Rightarrow$ R is symmetric Relation on z

11. If tangent to the curve $16y^2 + 9x^2 = 144$, intersects the axes at A and B, then the minimum length of the segment AB.
(1) 5 (2) 8 (3) 7 (4)4

Ans. **(3)**

Sol. $\frac{x^2}{16}+\frac{y^2}{9}=1$

Equation of tangent $\frac{x}{4}\cos\theta+\frac{y}{3}\sin\theta=1$

A (4secθ, 0) B(0, 3(cosecθ)

$AB = \sqrt{16\sec^2+a\operatorname{cosec}^2\theta} = \sqrt{25+(4\tan\theta-3\cot\theta)^2+24} \geq \sqrt{49} \geq 7$

$AB_{min} = 7$

12. If $\sum\limits_{r=0}^{2023} r^2.{}^{2022}C_r = 2023 \times \alpha \times 2^{2023}$ then α is equal to

(1) $\frac{1011}{4}$ (2) $\frac{1011}{2}$ (3) 2022 (4) 2020

Ans. **(1)**

Sol. $\sum r.r^{2022}C_r = \sum r.(2022)^{2021}C_{r-1}$

$= 2022\sum(r-1+1).^{2021}C_{r-1}$

$= 2022\left(\sum\limits_{r=2}^{2022} 2021.^{2020}C_{r-2}+2^{2021}\right)$

$= 2022\left((2021)\sum\limits_{r=2}^{2022} {}^{2020}C_{r-2}+2^{2021}\right) = 2022\left(2021.\left(2^{2020}\right)+2^{2021}\right)$

$= 2^{2020}.2022(2021+2) = 2^{2021}(1011)(2023) = 2^{2023}.(2023)\left(\frac{1011}{4}\right)$

13. The value of $\tan^{-1}\frac{1+\sqrt{3}}{3+\sqrt{3}}+\sec^{-1}\left(\frac{8+4\sqrt{3}}{6+3\sqrt{3}}\right)^{1/2}$ is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{5}$ (4) $\frac{\pi}{3}$

Ans. **(4)**

Sol. $E = \tan^{-1}\left(\frac{1+\sqrt{3}}{\sqrt{3}\left(\sqrt{3}+1\right)}\right) + \sec^{-1}\left(\frac{16+8\sqrt{3}}{12+6\sqrt{3}}\right)^{1/2}$

$\because \frac{16+8\sqrt{3}}{12+6\sqrt{3}} = \frac{4\left(\sqrt{3}+1\right)^2}{\left(3+\sqrt{3}\right)^2}$

$\therefore \left(\frac{16+8\sqrt{3}}{12+6\sqrt{3}}\right)^{1/2} = \frac{2\left(\sqrt{3}+1\right)}{3+\sqrt{3}} = \frac{2}{\sqrt{3}}$

$E = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$E = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

14. From the digits of 224411133. How many 9 digits numbers can be formed so that even digits are at even places.

Ans. **(60)**

Sol. Four even digits at four places in $\frac{4!}{2!\,2!}$ = 6 ways

Five odd digits at odd places = $\frac{5!}{3!\,2!}$ = 10 ways

Required number of numbers = 60 ways.

15. If $\left(1-\sqrt{3}i\right)^{200} = 2^{199}\,(p+iq)$ then the quadratic equation having roots($p - q + q^2$) & $(p + q + q^2)$ is

(1) $x^2-4x+1=0$ (2) $x^2+4x-1=0$ (3) $x^2+4x+1=0$ (4) None of these

Ans. **(1)**

Sol. $2^{200}\left(\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\right)^{200} = 2^{200}\left(\cos\frac{200\pi}{3} - i\left(\sin\frac{200\pi}{3}\right)\right)$

$= 2^{200}\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$

$= 2^{200}\left(\frac{-1}{2} - \frac{\sqrt{3}i}{2}\right)$

$= 2^{199}\left(-1-\sqrt{3}i\right) \Rightarrow P = -1,\ q = -\sqrt{3}$

roots are $(p - q + q^2,\ p + q + q^2) = \left(2+\sqrt{3},\ 2-\sqrt{3}\right)$

equation is $x^2 - 4x + 1 = 0$

16. $\sim(\sim p \wedge q) \Rightarrow (\sim p \vee q)$ is equivalent to

(1) $\sim p \wedge q$ (2) $\sim p \vee q$ (3) $p \vee q$ (4) $p \wedge q$

Ans **(2)**

Sol. $(\sim p \wedge q) \vee (\sim p \vee q)$

$\Rightarrow ((\sim p \wedge q) \vee \sim p) \vee q$

$\Rightarrow \sim p \vee q$

17. There are 12 courses available, in which 5 courses are of language, find the number of ways to select any 5 courses so that at most 2 language courses are always these

Ans. **546**

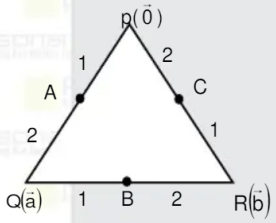
Sol. Required number of ways to select = $^{7}C_5 . {}^{5}C_0 + {}^{7}C_4 . {}^{5}C_1 + {}^{7}C_3 . {}^{5}C_2$

$= 21 + 175 + 350$

$= 546$

18. Let PQR be a triangle and A,B,C are three points on sides PQ, QR & PR respectively such that $\frac{PA}{AQ} = \frac{QB}{BR} = \frac{RC}{CP} = \frac{1}{2}$ then the ratio of the area of ΔPQR to the area of ΔABC is

Ans. **(3)**



Let position vector of P, Q R be $\bar{o}, \bar{a}$ & $\bar{b}$ respectively

$\Rightarrow$ P.V of A = $\frac{\bar{a}}{3}$, P.V of B = $\frac{2\bar{a}+\bar{b}}{3}$ and P.V of C = $\frac{2\bar{b}}{3}$

$\therefore \overline{AB} = \frac{\vec{a}+\vec{b}}{3}$ & $\overline{BC} = \frac{\bar{b}-2\bar{a}}{3}$

$\Delta PQR = \frac{1}{2}\left|\overline{PQ} \times \overline{PR}\right| = \frac{1}{2}\left|\bar{a} \times \bar{b}\right|$

$\Delta ABC = \frac{1}{2}\left|\overline{AB} \times \overline{BC}\right| = \frac{1}{2}\left|\left(\frac{(\bar{a}+\bar{b})}{3} \times \frac{(\bar{b}-2\bar{a})}{3}\right)\right| = \frac{1}{2}\left|\frac{\bar{a} \times \bar{b}}{3}\right|$

$= \frac{\Delta PQR}{\Delta ABC} = 3$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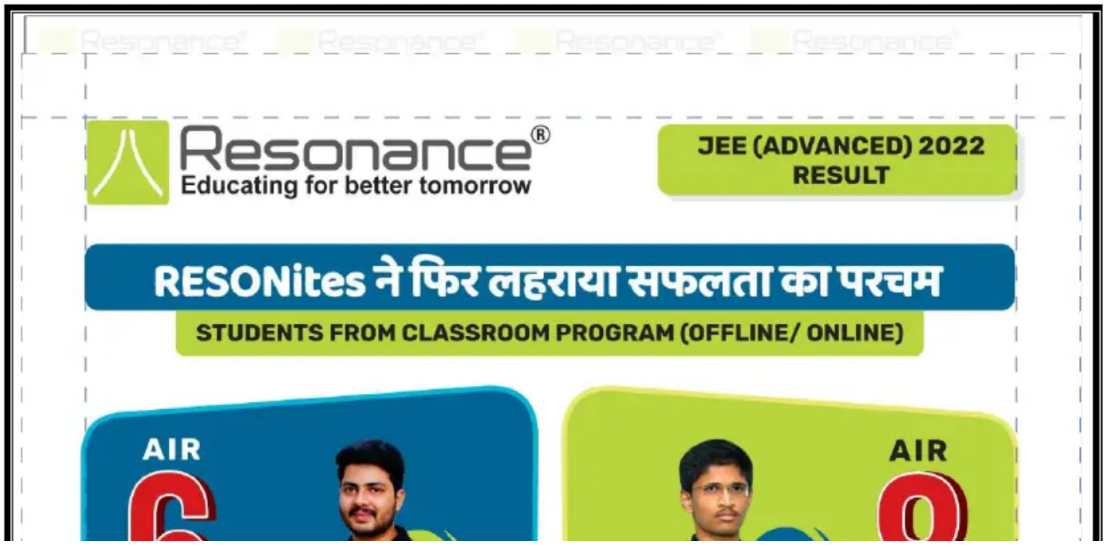
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