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PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 24 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If $1.\ ({}^{30}C_1)^2 + 2.\ ({}^{30}C_2)^2 + 3\ .\ ({}^{30}C_3)^2 + \ldots\ldots\ldots + 30.\ ({}^{30}C_{30})^2 = \dfrac{\alpha 60!}{30!\ 30!}$ then value of α is.

(1) 12 (2) 15 (3) 18 (4) 30

Ans. **(2)**

Sol. $\text{LHS} = \sum_{r=1}^{30} r \cdot \left({}^{30}C_r\right)^2$

$= \sum_{r=1}^{30} r \cdot {}^{30}C_r \cdot {}^{30}C_r$

$= \sum_{r=1}^{30} r \cdot \frac{30}{r} \cdot {}^{29}C_{r-1} \cdot {}^{30}C_r$

$= 30 \sum_{r=1}^{30} {}^{29}C_{r-1} \cdot {}^{30}C_r$

$= 30 \times$ coefficient of x^{29} in $(1+x)^{59}$

$= 30 \times {}^{59}C_{29} \times \frac{60}{30} \times \frac{30}{60}$

$= \frac{900}{60} \times {}^{60}C_{30} = 15 \cdot {}^{60}C_{30}$

$\Rightarrow \alpha = 15$

2. Number of 5×5 matrices, elements of which is either 0 or 1, such that the sum of elements in each row and column is unity, is

Ans. **(120)**

Sol. Total number of matrices = 5.4.3.2.1 = 5! = 120

3. Let $a_1, a_2, a_3, a_4, a_5, a_6$ are in AP whose mean is $\frac{19}{2}$ and $a_1 + a_3 = 10$ then find 8(variance) .

Ans. **(10)**

Sol. $\frac{\frac{6}{2}[2a+5d]}{6} = \frac{19}{2}$

$2a + 5d = 19$.....(i)

$a_1 + a_3 = 10$

$a + a + 2d = 10$

$2a + 2d = 10$.....(ii)

from (i) & (ii)

$d = 3$

$a = 2$

Numbers are 2, 5, 8, 11, 14, 17

$\sigma^2 = \frac{2^2+5^2+8^2+11^3+14^2+17^2}{6} - \left(\frac{19}{2}\right)^2$

$= \frac{699}{6} - \left(\frac{19}{2}\right)^2 = \frac{233}{2} - \frac{361}{4}$

$\sigma^2 = \frac{466-361}{4} = \frac{105}{4}$

$\Rightarrow 8\sigma^2 = 210$

4. $\sim (P \wedge (P \rightarrow \sim q))$ is logically equivalent to-

(1) $\sim P \rightarrow q$ (2) $\sim P \wedge q$ (3) $P \rightarrow q$ (4) $P \vee \sim q$

Ans. **(3)**

Sol. $\sim P \vee \sim (p \rightarrow \sim q)$

$\Rightarrow \sim P \vee (p \wedge q)$

$\Rightarrow (\sim p \vee p) \wedge (\sim p \vee q)$

$\Rightarrow t \wedge (\sim p \vee q)$

$\Rightarrow \sim p \vee q \Rightarrow p \to q$

5. $\vec{p} = \hat{i} + 2m\hat{j} + m\hat{k}$ & $\vec{Q} = 4\hat{i} - 2\hat{j} + m\hat{k}$ are orthogonal then m is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(2)**

Sol. $\vec{P}.\vec{Q} = 0$

$4 - 4m + m^2 = 0$

$m^2 - 4m + 4 = 0$

$(m-2)^2 = 0 \quad m = 2$

6. The set of values of a for which $\lim\limits_{x \to a}([x-5]-[2x+2]) = 0$ where $[\alpha]$ is greatest integer smaller than or equal to α is

(1) (–7.5, –6.5) (2) [–7.5, –6.5) (3) (–7.5, –6.5] (4) [–7.5, – 6.5]

Ans. **(2)**

Sol. $\lim\limits_{x \to a}([x-5]-[2x+2]) = 0$

$\lim\limits_{x \to a}([x]-5-[2x]-2) = 0$

$\lim\limits_{x \to a}([x]-[2x]) = 7$

Let $a \in [n, n+\frac{1}{2})$ then $n - 2n = 7$

$n = -7$

$\Rightarrow a \in [-7, -6.5)$

Let $a \in [n+\frac{1}{2}, n+1)$ then

$n - (2n+1) = 7$

$-n = 8$

$n = -8$

$a \in [-7\frac{1}{2}, -7)$

Hence $a \in [-7.5, -6.5)$

7. If y=y[x] is the solution of equation $(x^2-3y^2)\,dx + 3xydy = 0$ where y(1) = 1 then $6y^2(e)$ is equal to

(1) e^2 (2) $2e^2$ (3) $\frac{e^2}{2}$ (4) $\frac{e^2}{3}$

Ans. **(2)**

Sol. D.E. $\to \frac{2ydy}{dx} - \frac{2y^2}{x} = \frac{-2x}{3}$

Let $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

D.E. $\to \frac{dt}{dx} - \frac{2t}{x} = -\frac{2x}{3}$

I.F. $= e^{\int -\frac{2}{x}dx} = e^{-2\ell n|x|} = e^{\ell n\frac{1}{x^2}} = \frac{1}{x^2}$

solution $\Rightarrow t.\frac{1}{x^2} = \int \frac{1}{x^2}\left(\frac{-2x}{3}\right)dx + c$

$\Rightarrow \frac{y^2}{x^2} = \frac{-2}{3}\ell n|x| + c$

$\therefore x = 1, y = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$

$\Rightarrow \frac{y^2}{x^2} = -\frac{2}{3}\ell n|x| + 1$

$x = e \Rightarrow \frac{y^2(e)}{e^2} = \frac{-2}{3} + 1 \Rightarrow y^2(e) = \frac{e^2}{3}$

$\Rightarrow 6y^2(e) = 2e^2$

8. Given S = {(a, b), (b, c), (b, d)}. How many minimum ordered pairs must be added to S so that S becomes

8. Given S = {(a,b), (b,c) , (b,d)} . How many minimum ordered pairs must be added to S so that S becomes a equivalence relation.

Ans. **(13)**

Sol.

Reflexive	(a,a),	(b,b),	(c,c),	(d,d)
Symmetric	(b,a),	(c,b),	(d,b)	
Transitive	(a,c)	(a,d)	(d,c)	
	↓	↓	↓	
	(c,a)	(d,a)	(c,d)	

So 13 ordered pair

9. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $-\frac{\sqrt{3}}{2}+i\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}-i\frac{1}{2}$

(3) $-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

Ans. **(1)**

Sol. $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

$=\left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$

$=\left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}\right)^3$

$=\left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3=\left(\cos\frac{10\pi}{36}+i\sin\frac{10\pi}{36}\right)^3$

$=\left(\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}\right)^3$

$=\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

$=-\frac{\sqrt{3}}{2}+i\frac{1}{2}$

10. Number of real solution of equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5$ is

(1) 0 (2) 2 (3) 4 (4) 1

Ans. **(1)**

Sol. $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$

$3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left(x+\frac{1}{x}\right)+5=0$

Let $x+\frac{1}{x}=t$; $t\in(-\infty,-2]\cup[2,\infty)$

$3t^2-6-2t+5=0$

$3t^2 - 2t - 1 = 0$

$t = 1, t = \frac{-1}{3}$ But $t \in (-\infty,-2] \cup [2,\infty)$

$\Rightarrow$ No. real solution

11. If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in R$ then the value of $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+.....+f\left(\frac{2022}{2023}\right)$ is

(1) 2011 (2) 1011 (3) 1010 (4) 2021

Ans. (2)

Sol. $f(x) = \frac{4^x}{4^x+2}$

$\because \quad f(x)+f(1-x) = \frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2} = \frac{4^x+2}{4^x+2} = 1$

$f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+.....+f\left(\frac{2022}{2023}\right)$

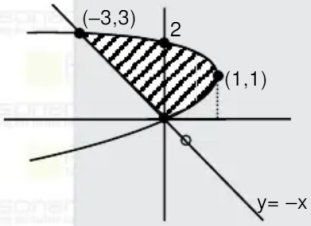
$= 1 + 1 + \ldots\ldots\ldots\ldots\ldots + 1 = 1011$

(1011 time)

12. If A is the area bonded by $y^2 - 2y = -x$ & $x + y = 0$ then 8A equals

Ans. 36

Sol.



$A = \int_0^3 \left[\left(2y-y^2\right)-(-y)\right]dy = \int_0^3 \left(3y-y^2\right)dy = \left[\frac{3y^2}{2}-\frac{y^3}{3}\right]_0^3$

$= \frac{3\times 9}{2}-\frac{27}{3} = \frac{9}{2}$ So, 8A = 36

13. Find the value of n for which $\frac{1^3+2^3+3^3+....+n^3}{1.3+2.5+3.7.....\text{upto n terms}} = \frac{9}{5}$

Ans. (5)

$= \frac{\left(\frac{n(n+1)}{2}\right)^2}{\sum_{r=1}^{n} r(2r+1)} = \frac{9}{5}, \frac{\left(\frac{n(n+1)}{2}\right)^2}{2\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r} =$

$\Rightarrow \frac{\left(\frac{n(n+1)}{2}\right)^2}{2\left(\frac{n(n+1)(2n+1)}{6}\right)+\left(\frac{n(n+1)}{2}\right)} = \frac{9}{5}$

$\Rightarrow \frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n(n+1)}{2}\left[\frac{2(2n+1)}{3}+1\right]} = \frac{9}{5}$

$$\Rightarrow \frac{\left[\frac{n(n+1)}{2}\right]}{\frac{4n+5}{3}} = \frac{9}{5}$$

$\Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow (5n + 6)(n-5) = 0$

$\Rightarrow n = \frac{-6}{5}, \quad n = 5$

14. If the sum of coefficient of first three terms of $\left(x - \frac{3}{x^2}\right)^n$ is 376 then the coefficient of x^4 is-

(1) 405 (2) 810 (3) 0 (4)135

Ans. **(1)**

Sol. ${}^nC_0 - {}^nC_1.3 + {}^nC_2.3^2 = 376$

$\Rightarrow n=10$

Now $T_{r+1} = {}^{10}C_r.\left(\frac{-3}{x^2}\right)^r.(x)^{10-r}$

$= {}^{10}C_r.(-3)^r.x^{10-3r}$

For $x^4 \Rightarrow 10 - 3r = 4 \Rightarrow r = 2$

Coff. of $x^4 = {}^{10}C_2.(-3)^2 = 405$

15. A is a 3 × 3 matrix such that $|\text{Adj (Adj (AdjA))}| = 12^4$ Then $|A^{-1} \text{ adj } A| =$

(1) $\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $4\sqrt{3}$

Ans. **(2)**

Sol. $|\text{Adj (Adj (Adj A))}|$

$=|\text{Adj (Adj A)}|^2$

$=|\text{Adj A}|^4$

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A| = 12^{1/2}$

Now, $|A^{-1} \text{ Adj A}| = |A^{-1}|\,|\text{Adj A}|$

$= \frac{1}{|A|^2}|A|^2 = |A| = 12^{1/2} = 2\sqrt{3}$

16. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$ is equal to

(1) $\frac{\pi}{3}$ (2) 2π (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

Ans. **(2)**

Sol. $\frac{48}{2}\left(\text{Sin}^{-1}\left(\frac{2x}{3}\right)\right)_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} = 24\left(\text{Sin}^{-1}\frac{\sqrt{3}}{2} - \text{Sin}^{-1}\frac{1}{\sqrt{2}}\right)$

$= 24\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 24\left(\frac{\pi}{12}\right) = 2\pi$

17. The number of integers greater than 7000 formed with digits 3, 5, 6, 7, 8 without repetition, is
(1) 168 (2) 84 (3) 21 (4) 44

Ans. **(1)**

Sol. Number can be 5 digits of 4 digits
5 digits numbers = 5!
4 digits numbers = 2 × 4 × 3 × 2 = 48
Total numbers = 120 + 48 = 168

18. If $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$, $x \in R$ then
(1) $f(3) - f(2) = f(1)$
(2) $2f(0) - f(1) + f(3) = f(2)$
(3) $3f(1) + f(2) = f(3)$
(4) $f(1) + f(2) + f(3) = f(0)$

Ans. **(2)**

Sol. $f(x) = x^3 - x^2 f'(1) + x f''(2) - f'''(3)$ .....(1)
$f'(x) = 3x^2 - 2x f'(1) + f''(2)$ .....(2)
$f''(x) = 6x - 2f'(1)$ .....(3)
$f'''(x) = 6 \Rightarrow f'''(3) = 6$
from (3)$\rightarrow f''(2) = 12 - 2f'(1)$ .....(4)
from (2) $\rightarrow f'(1) = 3(1)^2 - 2f'(1) + f''(2)$
$\Rightarrow f''(2) = 3f'(1) - 3$ .....(5)
$\Rightarrow 12 - 2f'(1) = 3f'(1) - 3 \Rightarrow f'(1) = 3$
$f''(2) = 12 - 6 = 6$
$f(x) = x^3 - 3x^2 + 6x - 6$, $f(0) = -6$
$f(1) = -2$, $f(2) = 2$, $f(3) = 12$

19. There are three urns. First urn contains 4 Red , 6 black balls, Second urn contains 5 Red , 5 black balls and third urn contains λ red and 4 black balls. An urn is selected and a ball is drawn and found to be red. The probability that it is drawn from third urn is 0.4 . Then square of the side length of an equilateral triangle inscribed in parabola $y^2 = \lambda x$ is –

Ans. **432**

Sol. Probability $= \frac{\frac{1}{3}\cdot\frac{\lambda}{10}}{\frac{1}{3}\cdot\frac{4}{10} + \frac{1}{3}\cdot\frac{5}{10} + \frac{1}{3}\cdot\frac{\lambda}{10}} = 0.4$

$\Rightarrow \frac{\lambda}{9+\lambda} = \frac{2}{5} \Rightarrow 5\lambda = 18 + 2\lambda$

$\Rightarrow \lambda = 6$

Parabola $y^2 = 6x$

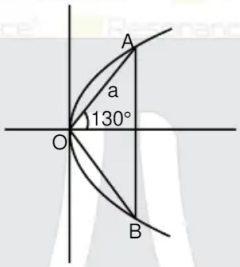
Let side length of triangle be a

$\Rightarrow$ A (a cos 30°, a Sin 30°) lies on $y^2 = 6x$

$\Rightarrow \left(\frac{a}{2}\right)^2 = 6\left(\frac{\sqrt{3}a}{2}\right) \Rightarrow \frac{a}{4} = 3\sqrt{3}$

$\Rightarrow a = 12\sqrt{3}$

$\Rightarrow a^2 = 432$

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