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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 25 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Value of $\sum_{k=0}^{6} {}^{51-k}C_3$ is

(1) ${}^{51}C_3 - {}^{45}C_3$ (2) ${}^{51}C_3 - {}^{45}C_4$ (3) ${}^{51}C_4 - {}^{45}C_4$ (4) ${}^{51}C_4$

Ans. (3)

Sol. ${}^{51}C_3 + {}^{50}C_3 + - - - - - - - - + {}^{45}C_3$

$\Rightarrow {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + - - - - - - - - + {}^{51}C_3 - {}^{45}C_4$

$\Rightarrow {}^{51}C_4 + {}^{51}C_3 + \dots\dots$

$= {}^{51}C_4 + {}^{51}C_3 - {}^{45}C_4 = {}^{51}C_4 - {}^{45}C_4$

2. The value of integral is :- $\int_1^2 \frac{dx}{x^3(x^2+2)^2}$

(1) $\frac{11+6\ln 4}{96}$ (2) $\frac{11-3\ln 4}{96}$ (3) $\frac{11+3\ln 4}{96}$ (4) $\frac{11-6\ln 4}{96}$

Ans. **(4)**

Sol. $\int_1^2 \frac{dx}{x^3 x^4\left(1+\frac{2}{x^2}\right)^2}$

Let $1+\frac{2}{x^2} = t \Rightarrow \frac{-4}{x^3}dx = dt$

$I = \frac{-1}{4}\int_3^{\frac{3}{2}} \left(\frac{t-1}{2}\right)^2 \frac{dt}{t^2} = -\frac{1}{16}\int_3^{\frac{3}{2}} \frac{\left(t^2-2t+1\right)}{t^2}dt$

$= \frac{-1}{16}\int_3^{\frac{3}{2}} \left(1-\frac{2}{t}+\frac{1}{t^2}\right)dt$

$= \frac{-1}{16}\left(t-2\ell n|t|-\frac{1}{t}\right)_3^{\frac{3}{2}}$

$= \frac{-1}{16}\left(\frac{3}{2}-2\ell n\frac{3}{2}-\frac{2}{3}\right)-\left(3-2\ell n2-\frac{1}{3}\right)$

$= \frac{-1}{16}\left(2\left(\ell n2-\ell n\frac{3}{2}\right)-\frac{3}{2}-\frac{1}{3}\right)$

$= \frac{-1}{16}\left(2\ell n3-\frac{11}{6}\right) = \frac{-1}{16}\left(\ell n4-\frac{11}{6}\right)$

$= \frac{11-6\ell n4}{16\times 6}$

$\frac{11-6\ln 4}{96}$

3. Remainder when $(2023)^{2023}$ is divided by 35, is

(1) 28 (2) 7 (3) 14 (4) 0

Ans. **(2)**

Sol. $-7^{2023} = -7 \times 7^{2022}$

$= \frac{-7^{2022}}{5}$

$= \frac{-[50-1]^{1011}}{5}$

$= \frac{[5\lambda-1]}{5}$

remainder = + 1

when divided by 5 now remainder is 7 when divided 35

4. Given, function be $f(x) = 2x^n + \lambda$ for which the value of f(4) = 133, f(5) = 255, find [f(3) – f(2)]

Ans. **(38)**

Sol. $f(4) = 2(4^n) + \lambda = 133$ $f(5) = 2(5^n) + \lambda = 25$

$f(5) - f(4) = 2(5^n - 4^n) = 122$

$\Rightarrow 5^n - (4^n) = 61$

$\Rightarrow n = 3$

$\Rightarrow \lambda = 3$

$\Rightarrow f(3) - f(2) = 2(3^3 - 2^3) = 2(27 - 8)$

$= 2(19) = 38$

5. If the range of the function $f(x) = \log_{\left(\sqrt{2}(\sin x - \cos x) + m - 2\right)}$ is [0, 2] then the possible value of m is

5. If the range of the function $f(x) = \log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + m - 2)$ is $[0,2]$ then the possible value of m is

(1) 0 (2) 3 (3) 5 (4) 7

Ans. (3)

Sol. $\log_{\sqrt{5}}\left(\sqrt{2}(\sin x - \cos x) + m - 2\right) \in [0,2] \in [\log_{\sqrt{5}} 1, \log_{\sqrt{5}} 5]$

$\Rightarrow \sqrt{2}(\sin x - \cos x) + m - 2 = [1, m]$ .....(i)

Range of $(\sin x - \cos x)$ is $[-\sqrt{2}, \sqrt{2}]$

So, from (i)

$[-4 + m, m] \rightarrow [1, m]$

$-4 + m = 1 \Rightarrow m = 5$

6. If $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{10} \end{bmatrix}$ $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$ and $M = A^T BA$ then $A\, M^{2023}\, A^T$ is

(1) B (2) $-B^{2023}$ (3) B^{2023} (4) $-B$

Ans. (3)

Sol. $M^2 = (A^T BA)(A^T BA)$

$= A^T B (AA^T) BA$

$= A^T BI\, BA = A^T B^2 A$

$AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$M^{2023} = A^T B^{2023} A$

$\because B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix} \Rightarrow B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$\because M^{2023} = A^T B^{2023} A$

$A^T M^{2023} A^T = A (A^T B^{2023} A) A^T$

$= B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

7. The number of numbers between 5000 and 10000 by using the digits 1, 3, 5, 7, 9 without repetition is equal to .

(1) 120 (2) 72 (3) 12 (4) 6

Ans. (2)

Sol. Digit in thousand place must be 5, 7 or 9.

= 3 ways

Now Total numbers between 5000 to 10000

are = 3 × 4 × 3 × 2

= 72

8. If $\left|\frac{z + 2i}{z - i}\right| = 2$ is a circle then centre of circle is

(1) (0, 0) (2) (0, 2) (3) (2, 0) (4) (-2, 0)

Ans. (2)

Sol. $\left|\frac{x + iy + 2i}{x + iy - i}\right| = 2 \Rightarrow \frac{x^2 + (y+2)^2}{x^2 + (y-1)^2} = 4$

$\Rightarrow x^2 + y^2 + 4y + 4 = 4x^2 + 4y^2 - 8y + 4$

$\Rightarrow 3x^2 + 3y^2 - 12y = 0$

$x^2 + y^2 - 4y = 0$

Centre = (0 ,2)

9. If roots of $x^2 + (60)^{1/4}x + a = 0$ be α and β and the value of $\alpha^4 + \beta^4 = -30$ then find the product of all

possible values of a

Ans. **(45)**

Sol. Since , $\alpha^4 + \beta^4 = -30$

$\Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = -30$

$\Rightarrow [(-60^{1/4})^2 - 2a]^2 - 2a^2 = -30$

$\Rightarrow \left(\sqrt{60} - 2a\right)^2 = 2a^2 - 30$

$\Rightarrow 60 + 4a^2 - 4a\sqrt{60} = 2a^2 - 30$

$\Rightarrow 2a^2 - 4a\left(\sqrt{60}\right) + 90 = 0$

$\Rightarrow a^2 - 2a\left(\sqrt{60}\right) + 45 = 0$

So, product of value of a = 45

10. If $(p \rightarrow q)\Delta(p \nabla q)$ is a tautology then the ordered pair of Δ, ∇ is

(1) $(\vee,\vee)$ (2) $(\vee,\wedge)$ (3) $(\wedge,\vee)$ (4) $(\wedge,\wedge)$

Ans. **(1)**

Sol. If $\Delta = \nabla = \vee$

$\Rightarrow (p \rightarrow q) \vee (p \vee q)$

$\Rightarrow ((\sim p \vee q) \vee p) \vee q$

$\Rightarrow ((\sim p \vee p) \vee q) \vee q$

$\Rightarrow (t \vee q) \vee q = t \vee q = t$

11. If $1 + \int_{1/3}^{3} |\ell nx| dx = \frac{m}{n} \ell n\left(\frac{n^2}{e}\right)$ Then Value of $m^2 + n^2 - 5$ is equal to

Ans. **(20)**

Sol. $\int_{1/3}^{1} -\ell nx dx + \int_{1}^{3} \ell nx dx$

$-(x\ell nx - x)_{1/3}^{1} + (x\ell nx - x)_{1}^{3}$

$-[(0-1) - (\frac{1}{3}\ell n\frac{1}{3} - \frac{1}{3})] + [(3\ell n3 - 3) - (0-1)]$

$2 - \frac{1}{3} + 3\ell n3 - \frac{1}{3}\ell n3 - 3$

$-\frac{4}{3} + \frac{8}{3}\ell n3 = (2\ell n3 - 1)\frac{4}{3}$

$\frac{4}{3}(\ell n9 - \ell ne) = \frac{4}{3}\ell n\frac{9}{e}$

$m = 4, n = 3$

$m^2 + n^2 - 5 = 4^2 + 3^2 - 5 = 20$

$\vec{a} = -\vec{i} - \vec{j} + \vec{k}$

12. $\vec{a} \times \vec{b} = \vec{i} - \vec{j}$

$\vec{a}.\vec{b} = 1$

Let $\vec{a} - 6\vec{b} - x\hat{i} + y\hat{j} + z\hat{k}$, then x + y + z equal

Ans. **(9)**

Sol. $\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ -1 & -1 & 1 \\ x & y & z \end{vmatrix}$

$= (-z-y)i - j(-z-x) + k(-y+x) = i - j$

$-z-y = 1$, $z + x = -1$, $x = y$

also $(\vec{a}.\vec{b}) = 1$

$-x-y+z = 1$

$-2x + z = 1$

$x + z = -1$

$-3x = 2$

$x = -\frac{2}{3}$

$y = -\frac{2}{3}$, $z = -1 + \frac{2}{3}$

$= -\frac{1}{3}$

now $\vec{a} - 6\vec{b} = \left(-\hat{i}-\hat{j}+\hat{k}\right) - 6\left(-\frac{2\hat{i}}{3} - \frac{2\hat{j}}{3} - \frac{\hat{k}}{3}\right)$

$= 3i + 3j + 3k$

13. The probability of a smoker to get a lung cancer is 27 times the probability of a normal person getting ling cancer. The probability that the man is smoker given that he is suffusing form lung cancer, hers it is given that probability of a person to be smoker is $\frac{1}{4}$, is

(1) $\frac{9}{10}$ (2) $\frac{8}{11}$ (3) $-\frac{9}{10}$ (4) $-\frac{8}{11}$

Ans. **(1)**

Sol. $P(S) = \frac{1}{4}$, $P(N) = \frac{3}{4}$

$P\left(\frac{C}{S}\right) = 27\,P\left(\frac{C}{N}\right)$

$P\left(\frac{S}{C}\right) = \frac{P(S).P\left(\frac{C}{S}\right)}{P(S).P\left(\frac{C}{S}\right) + P(N).P\left(\frac{C}{N}\right)}$

$= \frac{\frac{1}{4}\times 27\times P\left(\frac{C}{N}\right)}{\frac{1}{4}\times 27\times P\left(\frac{C}{N}\right) + \frac{3}{4}\times P\left(\frac{C}{N}\right)}$

$= \frac{27}{27+3} = \frac{27}{30} = \frac{9}{10}$

14. If $y = y(x)$ is the solution of equation $\frac{dy}{dx} + \alpha y = \gamma e^{-\beta x}$ where $\alpha, \beta, \gamma > 0$ and $\alpha \neq \beta$ then $\lim_{x\to\infty} y(x)$ is equal to.

Ans. **(0)**

Sol. I.F. $= e^{\int \alpha dx} = e^{\alpha x}$

Solution $y.e^{\alpha x} = \int e^{\alpha x}.\gamma.e^{-\beta x}\,dx + c$

$\Rightarrow y e^{\alpha x} = \gamma \int e^{(\alpha-\beta)x}\,dx + c$

$= \frac{e^{(\alpha-\beta)x}}{(\alpha-\beta)x}$

$\Rightarrow y(x)\cdot e^{\alpha x} = \gamma\frac{}{(\alpha-\beta)} + c$

$\Rightarrow y(x) = \gamma\frac{e^{-\beta x}}{(\alpha-\beta)} + ce^{-\alpha x}$

$\lim_{x\to\infty} y(x) = \lim_{x\to\infty}\left(\frac{\gamma.e^{-\beta x}}{(\alpha-\beta)} - c.e^{-\alpha x}\right)$

$= 0 - 0 = 0$

15. If a, b, $\frac{1}{18}$ are in G.P. and $\frac{1}{a}$, 10, $\frac{1}{b}$ are in A.P. also a, b, are positive then 12b + 16a =

Ans. **(3)**

Sol. $b^2 = \frac{a}{18}$

$20 = \frac{1}{a} + \frac{1}{b}$

$20 = \frac{1}{18b^2} + \frac{1}{b}$

$360b^2 = 1 + 18b$

$360b^2 - 18b - 1 = 0$

$b = \frac{18 + 42}{360 \times 2} = \frac{1}{12}$

and $a = 18b^2 = \frac{18}{144} = \frac{1}{8}$

So, 12b + 16a = 1 + 2 = 3

16. Two dice are rolled If the probability that the Sum of the numbers on dice is n, where n-2, $\sqrt{3n}$, n + 2 are in geometric progression, is $\frac{x}{48}$, then the value of x is :-

(1) 4 (2) 12 (3) 7 (4) 3

Ans. **(1)**

Sol. n-2, $\sqrt{3n}$, n+2 $\rightarrow$ G.P.

$\left(\sqrt{3n}\right)^2 = (n-2)(n+2)$

$\Rightarrow 3n = n^2 - 4$

$\Rightarrow n^2 - 3n - 4 = 0$

$\Rightarrow (n-4)(n+1) = 0$

$\Rightarrow n = 4, n = -1$

$\Rightarrow n = 4$

$P(S = 4) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48}$

$\Rightarrow x = 4$

17. Consider then function $f(x)\begin{cases}\left(1+|\cos x|\right)^{\frac{\lambda}{|\cos x|}} : x < \frac{\pi}{2}\\ \mu \quad : \quad x = \frac{\pi}{2}\\ e^{\frac{\cot 6x}{\cot 4x}} : x > \frac{\pi}{2}\end{cases}$

If f(x) is continuous at $x = \frac{\pi}{2}$ then (where $\lambda \neq 0$)

(1) $\lambda = \frac{2}{3}$, $\mu = e^{\frac{2}{3}}$ (2) $\lambda = e^{\frac{2}{3}}$; $\mu = \frac{3}{2}$

(3) $\lambda = \frac{3}{2}$ m $= e^{\frac{3}{2}}$ (4) $\lambda = e^{\frac{3}{2}}$, $\mu = \frac{3}{2}$

Ans. **(1)**

Sol. $f\left(\frac{\pi}{2}^+\right) = \lim_{x \to \frac{\pi}{2}^+} e^{\frac{\cot 6x}{\cot 4x}} = \lim_{x \to \frac{\pi}{2}^+} e^{\frac{\sin 4x}{\sin 6x} \cdot \frac{\cos 6x}{\cos 4x}}$

$= \lim_{x \to \frac{\pi}{2}^+} e^{\frac{4\cos 4x}{6\cos 6x}} = e^{\frac{2}{3}}$

LHL $f\left(\frac{\pi}{2}^-\right) = \lim_{x \to \frac{\pi}{2}^-} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} = e^{\lambda}$

$\Rightarrow \lambda = \frac{2}{3}, \mu = e^{\frac{2}{3}}$

18. Let $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$. If maximum value of $f(x)$ occurs at negative x and minimum value of $f(x)$ occurs at positive x, then $p \in$ .....

(1) $\left(-\infty, \frac{9}{2}\right)$ (2) $\left(-\infty, \frac{1}{2}\right)$ (3) $\left(\frac{9}{2}, \infty\right)$ (4) None of these

Ans. (1)

Sol. $f'(x) = 6x^2 + 2(2p-7)x + 3(2p-9)$

$f'(0) < 0$

$3(2p-9) < 0$

$2p - 9 < 0$

$p - \frac{9}{2}$

$p \in \left(-\infty, \frac{9}{2}\right)$

19. It A is a symmetric matrix and B & C are skew symmetric matrices of same order then

(1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric (2) AC – A is symmetric

(3) $A^{13}B^{26} - B^{26}A^{13}$ is skew symmetric (4) AC – A is skew symmetric

Ans. (3)

Sol. $(A^{13}B^{26} - B^{26}A^{13})^T = (A^{13}B26)T - (B^{26}A13)^T$

$\Rightarrow (B^{26})^T (A^{13})^T - (A^{13})^T (B^{26})^T$

$\Rightarrow (B^T)^{26} (A^T)^{13} - (A^T)^{13} (B^T)^{26}$

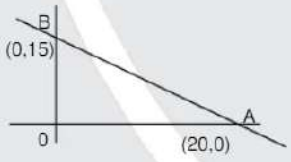
$\Rightarrow (-B)^{26} A^{13} - A^{13} (-B)^{26}$

$\Rightarrow -(A^{13}B^{26} - B^{26}A^{13})$

20. If the straight line $3x + 4y = 60$ makes a triangle with co – ordinate axes. then the number of points (a ,b) lying inside the triangle such that b is a multiple of a (where $a, b \in N$), is

Ans. (31)

Sol. $y = \frac{60-3x}{4}$



(1 , 1), (1 , 2) , ...(1 , 14) → 14

(2 , 2), (2 , 4) , ...(2 , 12) → 6

(3 , 3), (3 , 6) , ...(3 , 12) → 4

(4 , 4), (4 , 8) , ... → 2

(5 , 5), (5 , 10) , ... → 2

(6 , 6), (7 , 7) , (8 , 8) .....→ 3

Total number of points lying inside the triangle = 31